⊠연구논문

이차 자기회귀오차 구조를 갖는 선형회귀모형의 자료영향도 평가

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Assessing Local Influence in Linear Regression Models with Second-Order Autoregressive Error Structure

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Abstract

This paper discusses the local influence approach to the linear regression models with AR(2) errors. Diagnostics for the linear regression models with AR(2) errors are proposed and developed when simultaneous perturbations of the response vector are allowed. That is, the direction of maximum curvature of local influence analysis is obtained by studying the curvature of a surface associated with the overall discrepancy measure.

1. Introduction

Outliers or influential points can have a disproportionate influence on the estimated parameters or predictions. And so detecting perturbed observations is very important. For this reason, in recent years interest in diagnostic analysis has

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grown steadily. Cook and Weisberg(1982) and Chatterjee and Hadi(1988) proposed and comprehensively surveyed regression diagnostics for assessing the effects on the estimated coefficients. Rather than omission approaches, Cook(1986) developed a general method for assessing the local influence of a model perturbation. A distinguishing feature of this method is using log-likelihood contours to assess the local influence.

Applications of regression models using time series data can often be found in economics, business, and some fields of engineering. Usually the errors in such time series data show serial correlation. Such observations could be easily collected for a single economic unit or they could be aggregate quantities for a whole region or an economy. For example, time series data on consumption, investment, and income, among other variables, are frequently used to estimate macroeconomic relationships, such as consumption and investment functions.

Yet in the autocorrelated regression models, there has been almost no development for assessing the influence of local departures from model assumptions. Of course, it is not appropriate to apply and discuss case-deletion diagnosis in linear regression model in which the errors are autocorrelated. It is due to the fact that the dependency structure of the autoregressive model will not be valid after deleting a single observation from the data, except the last observation. To overcome this problem for case-deletion diagnosis, in this paper we examine the local change in the autocorrelated parameter estimate caused by a small perturbation. Beach and MacKinnon(1978) have developed a computationally efficient technique for maximizing the full likelihood function in the linear regression with autocorrelated errors. In a recent study, Putterman(1988) has showed the first transformed observation can have a large influence on parameter estimates. In the context of ARMA time series, Schall and Dunne(1991) discussed diagnostics corresponding to four perturbation schemes, respectively. Kim and Huggins(1998) discussed the local influence approach to the linear regression model with AR(1) errors.

This paper treats the effects of simultaneous perturbations of the response vector on all the parameters in the linear regression model with AR(2) errors as an extension of the result of Kim and Huggins(1998). In Section 2 the model is formulated and diagnostics for the linear regression model with AR(2) errors are proposed. Furthermore, the direction of maximum curvature of local influence analysis is obtained. Section 3 presents an illustrative example.

2. The Local Influence Approach for the Linear Regression Models with AR(2) Errors

2.1 Model

We studied the general linear statistical model

$$y = X\beta + \varepsilon,$$

where y is an $n\times 1$ observable random vector, X is an $n\times(k+1)$ nonstochastic matrix of explanatory variables, β is a $(k+1)\times 1$ vector of parameters to be estimated, and ε is an $n\times 1$ unobservable random vector with

$$E(\varepsilon)=0 \text{ and } E(\varepsilon\varepsilon')=\sigma^2\Psi.$$

The disturbances are assumed to follow a second-order autoregressive process

$$\varepsilon_t = \rho_1\varepsilon_{t-1}+\rho_2\varepsilon_{t-2}+a_t \qquad (2.1)$$

where $E(a_t)=0$, $E(a_t a_s)=0$ for $t\neq s$, $E(a_t^2)=\sigma^2$. This process will be stationary if $-1<\rho_2<1-|\rho_1|$.

The inverse of Ψ is given by (see Judge et al. (1985))

$$\Psi^{-1}=\begin{pmatrix} 1 & -\rho_1 & -\rho_2 & \cdots & 0 & 0 \\ -\rho_1 & 1+\rho_1^2 & -\rho_1+\rho_1\rho_2 & \cdots & 0 & 0 \\ -\rho_2 & -\rho_1+\rho_1\rho_2 & 1+\rho_1^2+\rho_2^2 & \cdots & 0 & 0 \\ 0 & -\rho_2 & -\rho_1+\rho_1\rho_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+\rho_1^2 & -\rho_1 \\ 0 & 0 & 0 & \cdots & -\rho_1 & 1 \end{pmatrix}.$$

2.2 Local Influence Approach

To develop diagnostics for the linear regression models with AR(2) errors, we use the local influence approach proposed by Cook(1986). Cook presents a general method for assessing the local influence of minor perturbations of a statistical model. The method relies on a well-behaved likelihood and certain elementary ideas from differential geometry.

To assess the influence of varying ω throughout Ω, Cook(1986) proposed the likelihood displacement

$$LD(\omega) = 2[L(\widehat{\theta}) - L(\widehat{\theta}_\omega)] \tag{2.2}$$

and the associated influence graph

$$\alpha(\omega) = \begin{pmatrix} \omega \\ LD(\omega) \end{pmatrix}, \tag{2.3}$$

where L denotes the log-likelihood corresponding to the postulated model, $\widehat{\theta}$ and $\widehat{\theta}_\omega$ are maximum likelihood estimates for θ in the unperturbed and the perturbed model, respectively, and θ is a $p \times 1$ vector of unknown parameters. Cook proposed to assess the local influence of ω at the postulated model $(\omega = \omega_0)$ by the curvature

$$C_l = 2|l'\ddot{F}l| = 2|l'\Delta'\ \ddot{L}^{-1}\Delta l|$$

of the influence graph (2.3) in direction $l \in R^q$, where l is a fixed nonzero vector of unit length, $\ddot{F}$ is the $q \times q$ matrix with (k, j)th element $\partial^2 L(\widehat{\theta}_\omega)/\partial\omega_k \partial\omega_j$, Δ is the $p \times q$ matrix with elements

$$\Delta_{ij} = \frac{\partial^2 L(\theta|\omega)}{\partial\theta_i \partial\omega_j}\bigg|_{\widehat{\theta}, \omega_0},$$

$L(\theta|\omega)$ denotes the log-likelihood corresponding to the perturbed model for a given ω in Ω, and $-\ddot{L}$ is the $p \times p$ observed information matrix for the postulated model $(\omega = \omega_0)$

$$\ddot{L} = \left(\frac{\partial^2 L(\theta)}{\partial\theta_i \partial\theta_j}\right)\bigg|_{\widehat{\theta}}. \tag{2.4}$$

The vector ω_0 is called *the null vector* or *null perturbation*. The direction of maximum curvature of the likelihood displacement surface is used as the main diagnostic tool in the local influence method.

2.3 Diagnostic for Second-Order Autocorrelation Models with Two Distinct Roots

Now suppose that the response vector y is perturbed according to

$$\tilde{y} = y + \omega = y + a\,l,$$

where $l' = (l_1, \ldots, l_n)$ denotes a directional vector with unit length and the quantity a measures the distance from y along the direction l. Therefore, this perturbation scheme produces an n-dimensional space, referred to as the ω-space. The likelihood displacement $LD(\omega)$ is a function on the ω-space, and it forms a surface $\alpha(\omega)$ which is in an $n+1$ dimensional space:

$$\alpha(\omega) = \begin{pmatrix} \omega \\ LD(\omega) \end{pmatrix}.$$

The relevant parts of the log-likelihood function for the unperturbed and the perturbed model are, respectively,

$$\begin{aligned} &L(\rho_1, \rho_2, \sigma^2, \beta) \\ &= -\frac{n}{2}\log\sigma^2 + \frac{1}{2}\log|\Psi^{-1}| - \frac{(y-X\beta)'\Psi^{-1}(y-X\beta)}{2\sigma^2} \\ &= -\frac{n}{2}\log\sigma^2 + \frac{1}{2}\log[(1+\rho_2)^2\{(1-\rho_2)^2-\rho_1^2\}] - \frac{(y-X\beta)'\Psi^{-1}(y-X\beta)}{2\sigma^2}, \end{aligned} \qquad (2.5)$$

$$\begin{aligned} &L(\rho_1, \rho_2, \sigma^2, \beta|\omega) \\ &= -\frac{n}{2}\log\sigma^2 + \frac{1}{2}\log[(1+\rho_2)^2\{(1-\rho_2)^2-\rho_1^2\}] - \frac{(\tilde{y}-X\beta)'\Psi^{-1}(\tilde{y}-X\beta)}{2\sigma^2}. \end{aligned} \qquad (2.6)$$

Then, Cook's (1986) likelihood displacement is defined as

$$LD(\omega) = 2[L(\hat{\theta}) - L(\hat{\theta}_\omega)],$$

where $\widehat{\theta}$ and $\widehat{\theta_\omega}$ are maximum likelihood estimates for $\theta=(\rho_1,\rho_2,\sigma^2,\beta')'$ in the unperturbed and the perturbed model, respectively.

Differentiating (2.6) with respect to θ and ω, and evaluating at $\widehat{\theta}$ and $\omega_0=0$, we find

$$\Delta=\left.\begin{pmatrix} -(y-X\beta)'\left(\frac{\partial}{\partial\rho_1}\Psi^{-1}\right)\Big/\sigma^2 \\ -(y-X\beta)'\left(\frac{\partial}{\partial\rho_2}\Psi^{-1}\right)\Big/\sigma^2 \\ (y-X\beta)'\Psi^{-1}/(\sigma^2)^2 \\ X'\Psi^{-1}/\sigma^2 \end{pmatrix}\right|_{\theta=\widehat{\theta}}$$

where $\widehat{\beta}=(X'\Psi^{-1}X)^{-1}X'\Psi^{-1}y$. The observed information matrix, $-\ddot{L}$, may be found by taking minus the matrix of second derivatives of the log-likelihood function (2.5), with respect to θ and all derivatives are evaluated at the maximum likelihood estimates $\widehat{\theta}$, but to simplify the notation we do not write the ^ hereafter. Let x_i' denote the ith row vector of X. Then, $\ddot{L}$ is

$$\ddot{L}=\left.\begin{pmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & 0' \\ l_{41} & l_{42} & 0 & L_{44} \end{pmatrix}\right|_{\widehat{\theta}},$$

where

$$l_{11} = -\frac{(1-\rho_2)^2+\rho_1^2}{\{(1-\rho_2)^2-\rho_1^2\}^2}-\frac{1}{\sigma^2}\sum_{i=2}^{n-1}(y_i-x_i'\beta)^2$$

$$l_{21} = -\frac{2(1-\rho_2)\rho_1}{\{(1-\rho_2)^2-\rho_1^2\}^2}-\frac{1}{\sigma^2}\sum_{i=2}^{n-2}(y_i-x_i'\beta)(y_{i+1}-x_{i+1}'\beta)$$

$$l_{22} = -\frac{(1-\rho_2)^2+\rho_1^2}{\{(1-\rho_2)^2-\rho_1^2\}^2}-\frac{1}{(1+\rho_2)^2}-\frac{1}{\sigma^2}\sum_{i=3}^{n-2}(y_i-x_i'\beta)^2$$

$$l_{31} = \frac{-\rho_1}{\{(1-\rho_2)^2-\rho_1^2\}\sigma^2}$$

$$l_{32} = \left\{\frac{1}{1+\rho_2} - \frac{1-\rho_2}{(1-\rho_2)^2-\rho_1^2}\right\}\frac{1}{\sigma^2}$$

$$l_{33} = -\frac{n}{2(\sigma^2)^2}$$

$$l_{41} = \frac{X'\left(\frac{\partial}{\partial\rho_1}\Psi^{-1}\right)(y-X\beta)}{\sigma^2}$$

$$l_{42} = \frac{X'\left(\frac{\partial}{\partial\rho_2}\Psi^{-1}\right)(y-X\beta)}{\sigma^2}$$

and

$$L_{44} = -\frac{X'\Psi^{-1}X}{\sigma^2}.$$

By that, the observed Hessian matrix $\ddot{F}$ takes the form

$$\ddot{F} = \Delta' \ddot{L}^{-1}\Delta. \tag{2.7}$$

But explicit analytic expressions of the direction of maximum curvature can not be derived for (2.7). And, c_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$ in (2.7) with corresponding eigenvector l_{max}. Thus the maximum curvature occurs in the direction l_{max}. Note that the ith diagonal element of the Hessian matrix $\ddot{F}$ becomes the curvature of the likelihood displacement (2.2), when only a single observation, say the ith, is perturbed according to $\tilde{y}_i = y_i + \omega$.

2.4 Diagnostic for Second-Order Autocorrelation Models with Coincident Roots

Assume the characteristic equation of (2.1) has coincident roots. Then, $-\rho_2 = \rho_1^2/4$ should be satisfied. And we get the following expression for the log-likelihood functions (2.5) and (2.6):

$$L(\rho_1, \sigma^2, \beta) = -\frac{n}{2}\log\sigma^2 + 2\log\left(1-\frac{\rho_1^2}{4}\right) - \frac{(y-X\beta)'\Psi^{-1}(y-X\beta)}{2\sigma^2}$$

and

$$L(\rho_1, \sigma^2, \beta|\omega) = -\frac{n}{2}\log\sigma^2 + 2\log\left(1-\frac{\rho_1^2}{4}\right) - \frac{(\tilde{y}-X\beta)'\Psi^{-1}(\tilde{y}-X\beta)}{2\sigma^2}.$$

Similar calculation gives

$$\varDelta = \left.\begin{pmatrix} -(y-X\beta)'\left(\frac{\partial}{\partial\rho_1}\Psi^{-1}\right)\Big/\sigma^2 \\ (y-X\beta)'\Psi^{-1}/(\sigma^2)^2 \\ X'\Psi^{-1}/\sigma^2 \end{pmatrix}\right|_{\theta=\hat{\theta}}$$

and

$$\ddot{L} = \left.\begin{pmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & \mathbf{0}' \\ l_{31} & \mathbf{0} & L_{33} \end{pmatrix}\right|_{\hat{\theta}},$$

where

$$l_{11} = -\frac{1+\rho_1^2/4}{(1-\rho_1^2/4)^2} - \frac{1}{2\sigma^2}\left\{\sum_{i=1}^{n-2}(y_i-x_i'\beta)(y_{i+2}-x_{i+2}'\beta) + 2\sum_{i=2}^{n-1}(y_i-x_i'\beta)^2 \right.$$

$$\left. -3\rho_1\sum_{i=2}^{n-2}(y_i-x_i'\beta)(y_{i+1}-x_{i+1}'\beta) + \frac{3\rho_1^2}{4}\sum_{i=3}^{n-2}(y_i-x_i'\beta)^2\right\}$$

$$l_{21} = \frac{-\rho_1}{(1-\rho_1^2/4)\sigma^2}$$

$$l_{22} = -\frac{n}{2(\sigma^2)^2}$$

$$l_{31} = \frac{X'\left(\frac{\partial}{\partial\rho_1}\Psi^{-1}\right)(y-X\beta)}{\sigma^2}$$

and

$$L_{33} = -\frac{X'\Psi^{-1}X}{\sigma^2}.$$

Then, the remainder follows the procedures given in Section 2.1.

3. Example

As a numerical illustration, we generate 30 samples of y for the model

$$y_t = \beta x_t + \varepsilon_t,$$

where $\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + a_t$ and the a_t's are independent standard normal random variables. The normal random numbers $\{a_t\}$ were generated by the subroutine DRNNOA of the IMSL subroutine library. The value of β was set to 4.5, and the values of ρ_1 and ρ_2 were set to 0.42 and 0.55, respectively. The design matrix X and observation vector are given in <Table 1>. Of course, they have indicated that the errors exhibit second-order autoregressive structure.

The diagnostics of local influence, $l_{\max}$, on all the parameters in AR(2) model are given in <Table 2>, in case the response vector y is perturbed $\tilde{y}$. Note that the ith diagonal element of the Hessian matrix $\ddot{F}$ becomes the curvature of the likelihood displacement (2.2), when only a single observation, say the ith, is perturbed according to $\tilde{y}_i = y_i + \omega$.

< Table 1 > Design matrix X and observation vector y

Case	X	y	Case	X	y
1	2	9.4506917	16	32	143.38236
2	4	19.297263	17	34	151.49609
3	6	26.764237	18	36	151.60114
4	8	35.287758	19	38	169.52662
5	10	43.869907	20	40	179.23352
6	12	52.068514	21	42	187.02976
7	14	52.029498	22	44	196.58665
8	16	70.316612	23	46	205.68527
9	18	79.335775	24	48	214.65864
10	20	88.097048	25	50	223.47479
11	22	97.449896	26	52	224.00739
12	24	104.68197	27	54	242.71498
13	26	115.35925	28	56	253.27025
14	28	123.92458	29	58	262.26113
15	30	133.98929	30	60	270.46071

< Table 2 > Local influence of a single observation

Case	(1) $\tilde{y}=y_7-\delta$		(2) $\tilde{y}=y_{18}-\delta$		(3) $\tilde{y}=y_{26}-\delta$	
	f_{ii}	l_{max}	f_{ii}	l_{max}	f_{ii}	l_{max}
1	-0.013166	0.0332676	-0.014123	0.0304544	-0.013645	0.0359904
2	-0.030187	0.0726128	-0.030495	0.0684809	-0.029352	0.0600097
3	-0.009139	0.0357758	-0.009487	0.0330276	-0.009223	0.0360706
4	-0.003948	-0.038789	-0.003943	-0.036163	-0.003776	-0.036668
5	-0.526982	0.445106	-0.011449	-0.054326	-0.010999	-0.056765
6	-0.410042	-0.193337	-0.010166	-0.045044	-0.009701	-0.03964
7	-1.490673	-0.695095	-0.007852	0.0180038	-0.007541	0.0124056
8	-0.359515	-0.110951	-0.006898	0.0320255	-0.006655	0.0352918
9	-0.565331	0.459436	-0.006372	-0.040997	-0.006118	-0.042717
10	-0.01673	0.0692022	-0.017252	0.0643637	-0.016602	0.0677767
11	-0.017509	-0.020224	-0.019115	-0.018254	-0.018444	-0.026163
12	-0.035748	-0.082993	-0.036483	-0.078259	-0.034848	-0.067504
13	-0.034747	-0.066676	-0.037258	-0.061553	-0.035938	-0.070293
14	-0.008238	0.0114156	-0.00901	0.0102544	-0.008697	0.0158255
15	-0.005306	0.0469917	-0.00517	0.0439946	-0.004934	0.0424879
16	-0.006507	0.0438943	-0.76981	0.5105941	-0.00617	0.0375367
17	-0.012335	0.0029141	-0.350976	-0.130702	-0.012987	0.0093456
18	-0.014638	-0.002111	-1.4915	-0.672142	-0.015393	-0.009164
19	-0.023246	0.0527783	-0.302003	-0.084167	-0.024118	0.0560604
20	-0.013215	-0.013587	-0.596484	0.4571896	-0.013932	-0.019183
21	-0.008434	-0.031998	-0.008783	-0.030285	-0.008408	-0.024593
22	-0.006846	-0.042574	-0.007093	-0.039544	-0.006821	-0.041953
23	-0.004517	0.0420493	-0.004495	0.039241	-0.004308	0.03959
24	-0.012701	-0.066302	-0.012843	-0.061744	-0.494499	0.4151679
25	-0.00971	-0.047757	-0.009799	-0.044968	-0.430545	-0.210924
26	-0.014031	-0.022243	-0.015306	-0.020238	-1.492888	-0.639901
27	-0.012039	0.0115263	-0.013161	0.0102379	-0.336229	-0.154161
28	-0.01701	0.0813039	-0.016598	0.0761902	-0.834897	0.5508095
29	-0.028271	0.0709111	-0.027934	0.0660609	-0.027354	0.0630991
30	-0.054741	-0.038229	-0.055083	-0.036538	-0.055166	-0.033464

NOTE : $\delta=10$, $f_{ii}=$ the ith diagonal element of matrix $\ddot{F}$ and l_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

Next, consider the joint influence provided by l_{max}, when both y_7 and y_{18} are perturbed $\widetilde{y_7}=y_7-\delta$ and $\widetilde{y_{18}}=y_{18}-\delta$, respectively. The diagnostics of local influence, l_{max}, on all the parameters in AR(2) model are given in <Table 3> and <Table 4>, respectively, in case two or three response observations are perturbed.

For example, when two observations y_7 and y_{18} are perturbed $\widetilde{y_7}$ and $\widetilde{y_{18}}$, respectively, we can find the fact that the elements of l_{max} corresponding to $\widetilde{y_7}$ and $\widetilde{y_{18}}$ are the largest among those of l_{max}. Two stage before and after the perturbed observations, $((y_5, y_9), (y_{16}, y_{20}))$, have larger influential effects than any other elements, as seen from <Table 3>. When there are only one or three perturbed observations, the same results can be obtained as the example given above (see Table 2 and Table 4). This means that the errors follow a second-order autoregressive process as was assumed in subsection 2.1. In general, the observation(s) corresponding to dominating element(s) of l_{max} have much possibility of being perturbed and influential observation(s).

< Table 3 > Local influence of two observations

	(1) $\tilde{y}=y_7-\delta$, $\tilde{y}=y_{18}-\delta$		(2) $\tilde{y}=y_7-\delta$, $\tilde{y}=y_{26}-\delta$		(3) $\tilde{y}=y_{18}-\delta$, $\tilde{y}=y_{26}-\delta$	
Case	f_{ii}	l_{max}	f_{ii}	l_{max}	f_{ii}	l_{max}
1	-0.007529	0.0232728	-0.007405	0.025368	-0.007685	0.0240448
2	-0.018208	0.047576	-0.017916	0.0442479	-0.018017	0.0434709
3	-0.006177	0.0254283	-0.0061	0.0265038	-0.006202	0.0252727
4	-0.002107	-0.027224	-0.002061	-0.027382	-0.002067	-0.026463
5	-0.283822	0.3140068	-0.277784	0.3162945	-0.006061	-0.040837
6	-0.229398	-0.144371	-0.225222	-0.157184	-0.005061	-0.028895
7	-0.760978	-0.470983	-0.742723	-0.449493	-0.004009	0.0095583
8	-0.200613	-0.087267	-0.196988	-0.101012	-0.003717	0.0255882
9	-0.304844	0.3243233	-0.298385	0.3268822	-0.003356	-0.030627
10	-0.009268	0.0497104	-0.00909	0.050995	-0.009271	0.0491727
11	-0.009749	-0.016158	-0.009571	-0.019263	-0.010031	-0.018165
12	-0.01841	-0.054704	-0.017989	-0.050354	-0.018198	-0.049322
13	-0.019388	-0.04893	-0.019029	-0.052316	-0.019777	-0.050104
14	-0.0046	0.0098385	-0.004518	0.0120362	-0.004742	0.0113444
15	-0.00278	0.0326502	-0.002717	0.0320073	-0.002686	0.031072
16	-0.414114	0.3852659	-0.003281	0.0282834	-0.405886	0.3741388
17	-0.180704	-0.108383	-0.006682	0.0070731	-0.185571	-0.114839
18	-0.773211	-0.486339	-0.007918	-0.006679	-0.749126	-0.453504
19	-0.154356	-0.073394	-0.012814	0.0420821	-0.158856	-0.08097
20	-0.320978	0.3440815	-0.007212	-0.014154	-0.31371	0.3332634
21	-0.0044	-0.020568	-0.004304	-0.018279	-0.004403	-0.018004
22	-0.003751	-0.030312	-0.003676	-0.03122	-0.003757	-0.030062
23	-0.002449	0.029744	-0.002398	0.0298403	-0.0024	0.028868
24	-0.006852	-0.046775	-0.27158	0.3112406	-0.272975	0.300777
25	-0.004977	-0.031801	-0.226752	-0.157309	-0.236595	-0.150297
26	-0.007836	-0.017518	-0.793421	-0.479255	-0.787499	-0.46668
27	-0.006689	0.0100016	-0.175909	-0.114955	-0.184089	-0.109415
28	-0.008703	0.0556585	-0.458301	0.4126068	-0.458804	0.3989775
29	-0.022073	0.04739	-0.021922	0.0459109	-0.021868	0.0440864
30	-0.052898	-0.028259	-0.053155	-0.02741	-0.053627	-0.027594

NOTE : $\delta=10$, $f_{ii}=$ the ith diagonal element of matrix $\ddot{F}$ and l_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

< Table 4 > Local influence of three observations

	$\tilde{y}=y_7-\delta$ $\tilde{y}=y_{18}-\delta$ $\tilde{y}=y_{26}-\delta$	
Case	f_{ii}	$l_{\max}$
1	-0.0053	0.0191591
2	-0.014024	0.0362932
3	-0.005014	0.0208104
4	-0.001431	-0.022216
5	-0.194033	0.2574452
6	-0.157558	-0.125944
7	-0.50743	-0.36834
8	-0.137473	-0.080067
9	-0.208522	0.2660805
10	-0.006408	0.0415341
11	-0.006671	-0.015081
12	-0.012258	-0.041355
13	-0.013306	-0.041958
14	-0.00316	0.0096725
15	-0.001892	0.0262633
16	-0.282638	0.31503
17	-0.124022	-0.096249
18	-0.515908	-0.38148
19	-0.105754	-0.067735
20	-0.218814	0.2806575
21	-0.002941	-0.015053
22	-0.002563	-0.025172
23	-0.001689	0.0244121
24	-0.189767	0.2532972
25	-0.158654	-0.126087
26	-0.542615	-0.392623
27	-0.122963	-0.091788
28	-0.319289	0.3357782
29	-0.019968	0.0358984
30	-0.053004	-0.025301

NOTE : $\delta=10$, $f_{ii}=$ the ith diagonal element of matrix $\ddot{F}$ and $l_{\max}$ corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

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