Jour. Korean For. Soc. 89(4) : 463 - 469. 2000
韓國林學會誌 89(4) : 463 - 469. 2000

Derivation of Basal Area Projection Function for Forest Plantation Using Medium (3-5years) Measurement Cycles[1]

Sang-Hyun Lee[2]

中間 測定 週期 (3-5년)를 利用한 人工林의 胸高斷面積 推定 函數의 誘導[1]

李 尙 鉉[2]

ABSTRACT

Douglas-fir (*Pseudotsuga menziesii* Mirb. Franco) is highly regarded as a commercial timber species throughout the world in part due to its fast growth relative to many other species. In this study, basal area per hectare equation for Douglas-fir plantations in Southland of New Zealand has been developed based on medium measurement cycles of permanent sample plots data set. The function was developed using the algebraic difference equation method, and various sigmoid-shaped projection equations were used. Parameter estimation was obtained by non-linear routine of the SAS. As a result, of the functions tested a variant of the Schumacher polymorphic function including site index and thinning term as predictor variables showed the higher precision of the fitting. The results indicate that site index is positively correlated with basal area growth. And the thinning term was found to be useful to increase precision of the model.

Key words : *basal area, algebraic difference equation, site index, thinning term*

要　約

이 연구는 다른 수종에 비해 상대적으로 빠른 생장을 보여 상업적으로 중요하게 여겨지는 뉴질랜드 사우스랜드 지역에 조림된 美松 (*Pseudotsuga menziesii* Mirb. Franco)의 胸高斷面積 추정 함수 유도에 관한 것이다. 胸高斷面積 함수를 도출하기 위하여 중간 측정 주기의 영구 표본점 데이터가 사용되었고, 代數差分 방정식을 이용하여 胸高斷面積 함수식을 유도하였다. 母數 추정은 SAS의 비선형 루틴에 의하여 수행하였다. 다양한 생장 추정 함수 모델을 적용한 후 잔차를 분석하여 평균제곱오차가 가장 작고 잔차 패턴이 편의가 없는 생장식을 선발하여, 추가 독립변수를 적용하여 모델의 추정 정도를 분석하였다. 그 결과 여러 추정 생장 함수 중 地位指數 및 간벌주기를 독립변수로 포함한 Schumacher 多形曲線 생장식이 가장 정밀한 추정을 나타내었다. 이 결과로 胸高斷面積 생장과 地位指數 사이에는 양(+)의 相關關係가 있음을 알 수 있었다. 그리고 정의된 간벌주기는 胸高斷面積 식의 精度를 높이는데 유용한 것으로 나타났다.

[1] Received on February 26, 2000.
[2] Faculty of Forest Science (The Institute of Agricultural Science & Technology), College of Agriculture, Chonbuk National University, Chonju 561-756, Korea 전북대학교 산림과학부 (농업과학기술연구소).

INTRODUCTION

A model has been defined as a simplification of a complex reality. A model is also abstract, and so a model is an abstraction or simplification of a system (Hall and Day, 1977; Botkin, 1993). Hall and Day (1977) stated that one of the most important use of models is in assisting a scientist with conceptualizing, organizing and communicating complicated phenomena to help in understanding and assessing these phenomena. Another prime use of models is to test the validity of field measurements and the assumptions derived from them. Growth models, nowadays normally refer to mathematical equations or systems of equations, are used to relate actual growth rates to measured trees, stands and site factors (Bruce and Wensel, 1987). In other words, they are used to predict growth and yield of forest stands under various conditions (Vanclay, 1994).

The purpose of growth and yield models can be regarded as; given set of stand or tree characteristics, such as basal area and stems per hectare at one point in time to determine by how much these will have changed at a future time. Such a quantitative understanding has been crucial to forest managers in helping them make important forest management decisions. Effective decision-making for forest management may depends on reliable forecasts of the growth and yield. Modeling growth and yield forecasting, therefore, is an essential for exploring silvicultural options and harvest planning.

Satisfactory of growth models may depend on the availability of high quality data. Data resulting from re-measured permanent sample plots of trees is referred to as a real growth series. Real growth series data are available for many tree species in plantations. Such real growth series data have extensively been used for investigating tree and stand dynamics as well as for modeling growth. An often used growth modeling formulation was developed by Clutter (1963), and has been referred to as the algebraic differential equation (ADE), (Borders et al., 1984; Ramirez et al., 1987; Lee, 1998). In using projection equations of this functional form, Lee (1998) approved that real growth series data available from a system of permanent sample plots (PSP) for Douglas-fir plantations was appropriate to obtain sample estimates of the parameters of equations. In Lee's study (1998), to build models three projection data sets were used; 1) a short interval data set which contained measurement intervals shorter than 3 years between times T_1 and T_2; 2) a long interval data set which contained measurement intervals longer than 8 years and 3) mixed short and long intervals data. However, he did not use medium measurement intervals, which are contained from 3 to 5 years between T_1 and T_2.

Thus, in this study medium measurement cycles data was investigated to build basal area per hectare equation. The variable to be modeled is basal area per hectare for some reasons. Firstly, when a good fit for basal area equation is obtained with height equation, then volume should follow naturally. Also, much of the variation associated with other stand variables such as, diameter and stocking that are absorbed with basal area used.

The objectives of this study, therefore, are to develop biologically and mathematically reliable basal area per hectare equation of Douglas-fir plantation forest in the Southland of New Zealand based on medium measurement intervals data set. And to provide basic information to build precise yield model based on the basal area equation.

MATERIALS AND METHODS

Data for this study came from a database of Douglas-fir (*Psedostsuga menziesii* Mirb. Franco) permanent sample plots grown in the Southland region of South Island of New Zealand, and maintained by New Zealand Forest Research Institute. All of 101 permanent sample plots, 218 sets of measurements, were used for analysis and measurement intervals were varied from 3 to 5 years. Plot sizes ranged from 0.01 to 0.2 hectare with a mean

Table 1. A summary of permanent sample plots data.

Variables	Mean	Minimum	Maximum
Age (years)	32.0	7.0	76.0
Top height (m)	23.1	4.3	45.4
Basal area (m^2/ha)	52.4	1.1	57.4
Stocking (n/ha)	1032	130	3168

of 0.04 hectare. Mean age of trees was 32 years, while minimum and maximum ages were 7 and 76 years, respectively. A summary of relevant plot statistics is given in Table 1.

The methods used for this study were difference equation (Borders et al., 1984) which has been used widely for growth and yield modeling studies. The main standard statistical procedures used were non-linear least-squares regression based on PROC NLIN in Statistical Analysis System (SAS Inc, 1990). Among the algorithms of PROC NLIN procedures used to estimate parameters the derivative-free method (DUD), which was found to be best in convergence, was adopted for non-linear least-squares regression (Ralston and Jennrich, 1979).

The PROC UNIVARIATE procedure was also used to examine the residuals and provide several statistics that are valuable for making inferences about residuals patterns. The important values utilized the analysis of this study among them were such as, mean of residuals, skewness, kurtosis and extreme values. In addition, graphical charts and plots were used to check the distributions of residuals with regard to normality of errors. Residual errors were plotted against predicted values to determine goodness of fit. Because whether or not the residual patterns lay normally about the zero references line was the important criterion for judging the independent distribution.

The commonly adopted projection equations are log-reciprocal (Schumacher, 1939; Piennar and Turnbull, 1973; Goulding, 1979), Gompertz (Whyte and Woollons, 1990), Weibull (Yang et al., 1978; Goulding and Shiley, 1979) and Hossfeld (Liu, 1990). There are two types of projection functions used for tree growth models, namely anamorphic and polymorphic functions. Firstly, several frequently used and their accuracy of estimation proved anamorphic equations were assayed such as, Schumacher, Chapman-Richard, Hossfeld and Gompertz functions. The functional forms of anamorphic projection equations used are presented in Table 2. Then, polymorphic forms of Schumacher, Chapman-Richard, Hossfeld and Gompertz equations were fitted to the data. The functional forms of polymorphic projection equations are presented in Table 3.

Table 2. General form of anamorphic projection equations applied to data.

Equation name	Equation Forms*
Schumacher anamorphic	$G_2 = G_1 \exp(-\beta(1/T_1^{\gamma} - 1/T_2^{\gamma}))$
Hossfeld anamorphic	$G_2 = 1/((1/G_1) + \beta(1/T_2^{\gamma} - 1/T_1^{\gamma}))$
Chapman-Richards Anamorphic	$G_2 = G_1((1-\exp(-\beta T_2))/(1-\exp(-\beta T_1)))^{\gamma}$
Gompertz anamorphic	$G_2 = G_1 \exp(-\beta(\exp(\gamma T_2) - \exp(\gamma T_1)))$

* G_1 = net basal area of the stand at age T_1
G_2 = net basal area of the stand at age T_2
Exp = exponential function
ln = natural logarithm
α, β, γ are coefficients to be estimated

Table 3. General form of polymorphic projection equations applied to data.

Equation name	Equation Forms
Schumacher	$Y_2 = \exp(\ln(Y_1)(T_1/T_2^{\beta}) + \alpha(1 - (T_1/T_2)^{\beta}))$
Chapman-Richards	$Y_2 = (\alpha/\gamma)^{[1/(1-\beta)]}(1 - (1 - (\gamma/\alpha)Y_1^{(1-\beta)}) \exp(-\gamma(1-\beta)(T_2 - T_1))^{[1/(1-\beta)]}$
Gompertz	$Y_2 = \exp(\ln(Y_1) \exp(-\beta(T_2 - T_1) + (T_2^2 - T_1^2) + (1 - \exp(-\beta(T_2 - T_1) + \gamma(T_2^2 - T_1^2))))$
Hossfeld	$Y_2 = 1/((1/Y_1)(T_1/T_2)^{\gamma} + (1/\alpha)(1 - (T_1/T_2)^{\gamma}))$

* G_1 = net basal area of the stand at age T_1 Exp = exponential function
G_2 = net basal area of the stand at age T_2 ln = natural logarithm
α, β, γ are coefficients to be estimated

RESULTS AND DISCUSSION

The anamorphic equations generally produced biased residuals patterns, Hossfeld equation showed largely biased, though Schumacher anamorphic function proved little bit superior in statistics of residuals and residuals patterns to other anamorphic functions. The statistics of residuals of the anamorphic equations fitted are presented in Table 4 with corresponding mean square error values (MSE).

Most of the polymorphic equations generally fitted well without apperent bias in residuals pattern, and showed better fit than anamorphic forms of equations. Comparing residuals pattern and mean square error values the Schumacher polymorphic function, equation (1), with the lowest mean square error (MSE) 7.073 was found to represent better than the other equation. Therefore, this equation was selected for further analysis. The fitted coefficients and mean square errors of applied functions are shown in Table 5.

$$G_2=\exp(\ln(G_1)(T_1/T_2)^{\beta}+\alpha(1-(T_1/T_2)^{\beta})) \qquad (1)$$

The variables used in formulating functional forms were basal area/ha (G), age of stand (T), altitude (ALT) and site index (SI), thinning index (Xt) and thinning age (Tt). Numerous modifications to equation (1), with the addition and subtraction of predictor variables and other alterations were tested to effect further improvements. Site index (SI), which is possibly the most common of all additional variables used in other studies, was included as a predictor variable. Equation (2) was, hence, fitted to the data.

$$G_2=\exp(\ln(G_1)(T_1/T_2)^{\beta 1}+(\alpha+\beta_2 SI)(1-(T_1/T_2)^{\beta 1})) \qquad (2)$$

The result of including site index in the basal area equation revealed that the coefficient of site index was positive suggesting that higher site indices had higher basal area production. That notion is biologically acceptable for a relationship between site index and growth. The equation (2) gave some improvement of mean square error, and the coefficients for which were different from zero at the α =0.05 level. Altitude is sometimes substituted for site index, as they have almost the same predicting effect in New Zealand and other temperate countries (Temu, 1992), and it has been found be an important variable for explaining variation in basal area and top height growth as shown in Woollons and Hayward (1985), and Whyte et al. (1992). Altitude was, thus, introduced into the basic form, equation (3), instead of the site index.

Table 4. Statistics of residuals with the anamorphic equations fitted to data.

Equation name	MSE	Mean of residuals	Skewness	Kurtosis
Schumacher	14.45	1.303	-0.415	-0.178
Chapman-Richard	16.31	1.448	-0.448	-0.178
Hossfeld	23.56	3.624	-0.892	1.989
Gompertz	38.91	1.427	-0.305	-0.112

Table 5. Coefficients for polymorphic equations fitted to data.

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	5.264	1.105	-	7.073
Chapman-Richards	2.335	0.043	0.004	20.609
Gompertz	4.980	0.093	0.001	10.937
Hossfeld	126.44	-	2.595	8.081

$$G_2=\exp(\ln(G_1)(T_1/T_2)^{\beta 1}+(\alpha+\beta_2 ALT)(1-(T_1/T_2)^{\beta 1})) \quad (3)$$

The result of including altitude showed that the asymptote of basal area growth increased with altitude, which is a biologically unrealistic trend. And the coefficient of altitude was not significant at the $\alpha = 0.05$ level.

A thinning index was employed based on that defined by Murphy and Farrar (1988), to examine the effect of thinning on basal area growth. The thinning index (Xt) employed was;

$$Xt = 1 - (Dt/Db) \quad Xt>0 \quad \text{if } Dt/Db \neq 0$$
$$= 0 \quad \text{if } Dt/Db = 0$$

where

Dt = quadratic mean diameter of trees removed in thinning

Db = quadratic mean diameter before thinning.

Table 6. Successive improvement in fitting basal area/ha equation.

Input variables *	Error of Sum Squares	Mean Square Errors
Basic form (B)	1520.7984	7.0735
B + SI	1452.3533	6.7958
B + Xt	1438.5908	6.7224
B + SI + Xt	1275.8576	6.4437

* SI = Site Index * Xt = Thinning Index

Equation (4) represents the inclusion of this thinning index and ages of thinning.

$$G_2=\exp(\ln(G_1)(T_1/T_2)^{\beta}+\alpha(1-(T_1/T_2)^{\beta})) + \beta_3 Xt(1/T_2-1/T_1)Tt/T_2) \quad (4)$$

The equation (4) includes thinning as a component of the thinning term. A development of the basal area per hectare equation depends on the kind of thinning and times elapsed since the last thinning occurred. The thinning term was helpful in the model fitted with this study data set, while it was not helpful with long measurement interval data (Lee, 1998) because the effect of thinning on productivity are short-lived.

Further analysis, adding site index to the equation (4), was tried and it showed reductions in the residual error of sum squares (RSS) and improvement of mean square errors (MSE). Table 6 shows successive improvement from successive additional variables.

Therefore, equation which are including site index and thinning term was selected best fitting model. Substitution of the coefficients into equation resulted in :

$$G_2=\exp(\ln(G_1)(T_1/T_2)^{1.6429}+(4.6431+0.0004SI)(1-(T_1/T_2)^{1.6429}))+0.0058Xt(1/T_2-1/T_1)Tt/T_2) \quad (5)$$

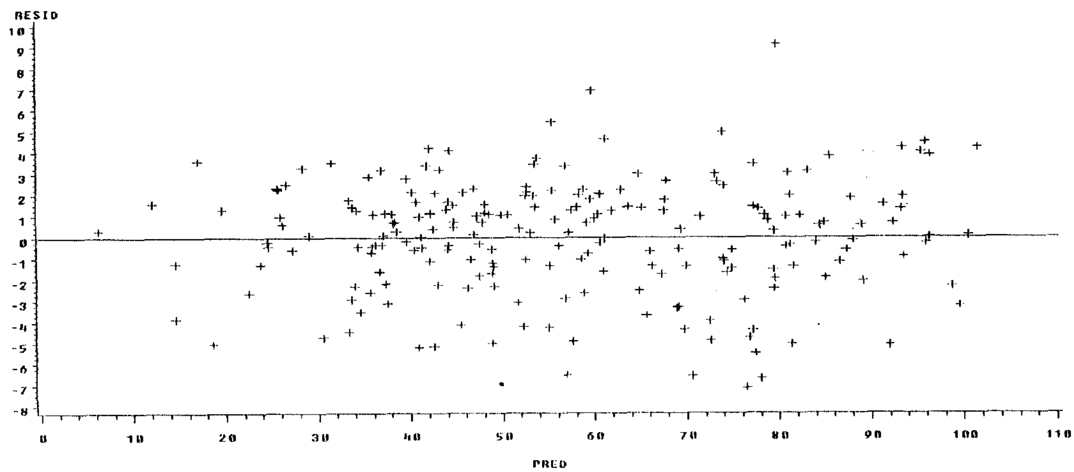

Fig. 1. A plot of residuals against the predicted for basal area projection equation.

A plotting of residuals against predicted values indicated that a random pattern around zero with no detectable biased trend as shown in Fig. 1.

The PROC UNIVARIATE statistics in Table 7 shows proof that the equation provides an unbiased precise estimate of stand basal area per hectare as it contained - 0.245 value for skewness which indicated little bit long tails to the left and 0.349 value for kurtosis, the heaviness of tails in a distribution. The skewness and kurtosis of a normal distribution is zero, but in practice values of these lesser or greater than zero result from least-square regression. A Shapiro - Wilk test for normality was totally accepted as 0.978 that is very closed to 1 of normal distribution.

Table 7. Summary of statistics of residual values for basal area per hectare projection equation.

Mean	0.0211
Absolute mean	2.0789
Skewness	-0.2456
Kurtosis	0.3498
W : Normal	0.9780

The mean residual and absolute mean residual values for the best basal area/ha, equation (5), were 0.02 and 2.08 m^2/ha, respectively showing that slightly under-prediction with an average deviation of 2.08 m^2/ha. This equation had a maximum residual of 9.08 m^2/ha and a minimum residual of -7.11 m^2/ha, while 99% of residuals were contained within $\pm$5.43 m^2/ha.

CONCLUSION

Various sigmoid-shaped projection equations were adopted to derive for basal area per hectare model of Douglas-fir plantation forest. Among the applied projection equations, Schumacher polymorphic function was proved better result than the other functions. Other predictor variables representing site specific factors were included to basic Schumacher function. In the analysis, all predictor variables, the coefficients for which not different from zero at the α = 0.05 probability level were excluded, and coeffi α cients of variables that were not in line with expectations of growth relationships were excluded. The result of including predictor variables showed that the asymptote of basal area growth was correlated with site index. The thinning term was useful in the model fitted with medium measurement cycles data set of this study, while it was not helpful with long measurement interval data (Lee, 1998) because the effect of thinning on productivity are short-lived.

ACKNOWLEDGEMENTS

The author wishes to thank Prof. E. G. Mason for his valuable guidance for growth modeling study and critical comments on the manuscript. Many thanks to the New Zealand Forest Research Institute for providing the Permanent Sample Plots data also, deeply appreciated Carter Harvey Ltd. for permission to use the data.

LITERATURE CITED

1. Borders, B.E., R.L. Bailey and K.D. Ware. 1984. Slash pine index from a polymorphic model joining (splining) nonpolynominal segments with an algebraic difference method. Forest Science 30(2) : 411-423.
2. Botkin, D.B. 1993. Forest dynamics : an ecological model. Oxford University press, Inc. 309 pp.
3. Bruce, D. and L.G. Wensel. 1987. Modeling fores growth : approachs, definitions, and problems. In : Ek, A.R., S.R., Shiffely and T.E. Burks (Eds.). Proceedings of the IUFRO symposium on Forest growth modeling and prediction. USDA, Forest Service general Technical Report. NC 120 : 1-8.
4. Clutter, J.L. 1963. Compatible growth and yield models for loblolly pine. Forest Science 9 : 354-371.

5. Goulding, C.J. 1979. Validation of growth models for pinus radiata in New Zealand. New Zealand Journal of Forestry 24(1) : 108-124.
6. Goulding, C.J. and J.W. Shirey. 1979. A method to predict the yield of log assorments for long term planning. In Elliott, D.A. (Ed.). Mensuration of management planning of exotic forest plantations. New Zealand Forestry Service and Forestry Research Institute Symposium No. 20 : 301-314.
7. Hall, C.A.S. and Jr. J. W. Day. 1977. Systems and models : terms and basic principles. In Charles A.S. Hall and John. W. Day Jr. Ecosystem modelling, in theory and practice : an introduction with case histories. University Press of Colorado. 684 pp.
8. Lee, S.H. 1998. Modelling growth and yield of Douglas-fir using different interval lengths in the South Island of New Zeraland. Ph.D Thesis, School of Forestry, University of Canterbury, New Zealand, 239 pp.
9. Liu, Xu. 1990. Growth and yield of Douglas fir plantations in the Central North Island of New Zealand. Ph.D Thesis. School of Forestry, University of Canterbury, New Zealand. 244 pp.
10. Murphy, P.A. and R.M. Jr. Farrar. 1988. Basal area projection equation for thinned natural evenaged forest stands. Canadian Journal of Forest Research 18 : 827-832.
11. Pienaar, L.V. and K.J. Turnbull. 1973. The Chapman-Richards generalization of Von Bertalanffy's model for basal area growth and yield in even-aged stands. Forest Science 19 : 2-22.
12. Ralston, M.L. and R.I. Jennrich. 1979. Dud, a Borders. 1987. Some implications of the algebraic difference approach for developing growth models. In : Ek, A.R., S.R., Shiffely derivative free algorithm for nonliner least squares. Technometrics 20(1) : 7-13.
13. Ramirez-Maldonado, H., R.L. Bailey and B.E.and T.E. Burks (Eds). Proceedings of the IUFRO symposium on Forest growth modeling and prediction. USDA, Forest Service general Technical Report. NC 120 : 731-738.
14. SAS Institute Inc. 1990. SAS/STAT Procedures guide, Version 6, NC.
15. Schumacher, F.X. 1939. A new growth curve and its application to timber-yield studies. Journal of Forestry 37 : 819-820.
16. Temu, M.J. 1992. Forecasting yield of Douglas fir in the South Island of New Zealand, Ph.D Thesis, School of Forestry, University of Canterbury, New Zealand, 246 pp.
17. Vanclay, J.K. 1994. Modelling forest growth and yield : application to mixed tropical forests. CAB International Wallingford, UK. 312 pp.
18. Whyte, A.G.D., M.J. Temu and R.C.Woollons. 1992. Improving yield forecasting realiblity through aggregated modelling. In : G.B. wood, and B.J. Turner(Eds). proceedings IUFRO-In tegrating Information Over space and Time; Australian National University Canberra, Jan; 13-17, 1992 : 81-88.
19. Whyte, A.G.D. and R.C. Woollons. 1990. Modelling stand growth of radiata pine thinned to varying densities. Canadian Journal of Forest Research 20 : 1069-1076.
20. Woollons, R.C. and W.J. Hayward. 1985. Revision of a growth and yield model for radiata pine in New Zealand. Forest Ecology and Management 11 : 191-202.
21. Yang, R.L., A. Kozak and H.J.G. Smith. 1978. The potential of Weibull-type functions as flexible growth functions. Canadian Journal of Forest Research 8 : 424-431.