Effect of Specimen Thickness on the Statistical Properties of Fatigue Crack Growth Resistance in BS4360 Steel

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In this paper the effect of specimen thickness on fatigue crack growth with the spatial distribution of material properties is presented. Basically, the material resistance to fatigue crack growth is treated as a spatial stochastic process, which varies randomly on the crack surface. The theoretical autocorrelation functions of fatigue crack growth resistance with specimen thickness are discussed for several correlation lengths. Constant ΔK fatigue crack growth tests were also performed on CT type specimens with three different thicknesses of BS 4360 steel. Applying the proposed stochastic model and statistical analysis procedure, the experimental data were analyzed for different specimen thicknesses for determining the autocorrelation functions and probability distributions of the fatigue crack growth resistance.

Key Words : Specimen Thickness, Spatial Stochastic Process, Fatigue Crack Growth Resistance, Autocorrelation Function, Correlation Length, Weibull Distribution

1. Introduction

Experimental data obtained from test specimens under various loading conditions constitute the main source of information about fatigue crack growth of engineering materials. However, these data, regardless of how carefully they are generated, show significant scatter which depends on various uncontrolled factors such as material properties, metallurgical structure, type of loading, environment, and so on. Recently, this scatter of fatigue crack growth data is commonly regarded as an inherent feature of fatigue crack growth process in real engineering materials (Sobczyk, 1993).

Experimental and theoretical studies on the randomness of fatigue crack growth have been reported (Virker, et al., 1979; Tanaka, et al., 1981; Kozin and Bogdanoff, 1989; Ortiz and Kiremidjian, 1989; Mayo and Dominguez, 1996; Kim, 1999). Such a model assumes the fatigue crack growth as a stochastic process and therefore, the stochastic nature of material parameters before any practical application should be known. To this end, the spatial distribution of the material resistance as well as the mean and variance of the fatigue crack growth rate are needed.

Effects of specimen thickness on fatigue crack growth have been investigated (Putatunda and Rigsbee, 1985; Mcmaster et al., 1998; Shim and Kim, 1998). Some have reported that the specimen thickness had no effect, whereas others have reported either an increase or decrease in the crack growth rate with the specimen thickness. It was found that there is scatter in their results. And also, the variability of fatigue crack growth life seems to increase with decreasing specimen thickness. However, most of the studies were done under a constant amplitude loading, and they did not consider the statistical properties of fatigue crack growth resistance. Since the material resis-

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tance to the fatigue crack growth rate depends on the local material properties, the spatial distribution of material properties may be related to the effect of the specimen thickness in the fatigue crack growth rate and scatter of the fatigue crack growth. Hence, the understanding of the spatial distributions in the local strength of materials is valuable information for researchers studying the effect of specimen thickness on fatigue crack growth rate and scatter.

The aim of the present study is, therefore, to investigate the effect of the specimen thickness on the statistical properties of fatigue crack growth resistance considering material inhomogeneity and the fact that material properties obtained from experiment are local average values in a finite region. The theoretical autocorrelation functions of the material resistance to fatigue crack growth with the specimen thickness are discussed for several correlation lengths. Constant ΔK fatigue crack growth tests were performed on CT type specimens with three different thicknesses of BS 4360 steel. Applying the proposed stochastic model and statistical analysis procedure, the experimental data were analyzed for the different specimen thicknesses to determine the autocorrelation functions and probability distribution functions of the fatigue crack growth resistance.

2. A Stochastic Model and the Statistical Analysis Procedure

Assuming Paris law (Paris and Erdogan, 1963), the crack growth rate is

$$\frac{da}{dN}=C(\Delta K)^m \tag{1}$$

where, C and m are the material constants, a the fatigue crack length, and N the number of cycles to load. The material constants m and C, hereinafter, called the growth rate exponent and coefficient, respectively, are assumed random. For one dimensional model of fatigue crack growth, they are the random functions of crack length. It is, however, very difficult to determine separately these two random variables from the observed crack growth data even under the test for the condition of constant stress intensity factor range (Kim, 1999). And for this purpose many experimental data are essential. Since the present study is to investigate the effect of specimen thickness on inhomogeneity of fatigue crack growth resistance, it seems unnecessary to use the stochastic model. Therefore, in the present study, for random variables m and C, taking expectation of da/dN gives

$$\overline{\frac{da}{dN}}=\int_0^\infty\int_0^\infty\left(\frac{da}{dN}\right)\cdot f_c(C)\cdot f_m(m)\,dCdm \tag{2}$$

where, $f_c(\bullet)$ and $f_m(\bullet)$ are the probability density function of random variables C and m, respectively. If ΔK is in constant condition ahead of the crack path, the expectation values are constant. Determining the value of $\overline{da/dN}$ from experimental data for each specimen thickness, we introduce the dimensionless parameter, $S(x)$, which means the inhomogeneity of material properties to fatigue crack growth in front of the crack, then da/dN is written by

$$\frac{da}{dN}=\frac{1}{S(x)}\cdot\overline{\frac{da}{dN}} \text{ or } S(x)=\frac{dN}{da}\overline{\frac{da}{dN}} \tag{3}$$

hereinafter, $S(x)$, is called as the growth resistance coefficient of material to fatigue crack growth, namely the crack growth resistance coefficient.

The spatial stochastic process $S(x)$ is assumed to be a stationary and ergodic process but not necessarily Gaussian, and its autocorrelation function and probability distribution function are determined from experimental data. Considering finite specimen thickness, h, $S(x)$ obtained by Eq. (3) is likely to observe the statistical variation of the two-dimensional local averaged value.

Next, consider the statistical analysis of the fatigue crack growth resistance data. The growth resistance coefficient $S(x_i)$ at crack length ai is given by

$$S(x_i)=\left(\frac{dN}{\Delta a}\right)_i^j\Big/\left(\overline{\frac{dN}{\Delta a}}\right) \tag{4}$$

where, x_i is the position of a crack tip.

Increment, Δa, is constant and especially, 0.4 mm in this experiment.

The residual, ξ_i^j, of the growth resistance coefficient $S(x_i)$ at crack length a_i for each specimen in j thickness specimen is

$$\xi_i^j = S(x_i^j) - \overline{S(x_i^j)} \tag{5}$$

where, $\overline{S(x_i^j)}$ is the mean value of $S(x_i^j)$ namely the local mean value.

The autocorrelation function of the residual $R_{S'S'}(\tau)$, is given by

$$R_{S'S'}(\tau) = \frac{1}{n-k}\sum_{i=1}^{n-k} \xi_i \xi_{i+k} \tag{6}$$

where, $\tau = k\Delta a$, and k is lag.

Using the inverse Fourier transfer, the power spectral density function could be also obtained from autocorrelation function.

The ensemble autocorrelation function $\overline{R_{S'S'}}(\tau)$ for the same thickness specimens can be determined by the following equation.

$$\overline{R_{S'S'}}(\tau) = \frac{1}{M_h}\sum_{i=1}^{M_h}[R_{S'S'}(\tau)] \tag{7}$$

where, M_h is the number of specimen with the same thickness.

As to the probability distribution of the crack growth resistance $S(x)$, the assumption that $S(x)$ follows an extreme distribution may be reasonable, because the fatigue crack growth resistance is a parameter of the material strength (Ortiz and Kiremidjian, 1986). In the present study, the probability distribution function of $S(x)$ is assumes to follow 3-parameter Weibull distribution,

$$F_s(S|\alpha,\beta,\gamma) = 1 - \exp\left[-\left(\frac{S-\gamma}{\beta}\right)^{\alpha}\right] \tag{8}$$

where, α is the shape parameter, β is the scale parameter and γ is the location parameter.

3. Theoretical Analysis for Autocorrelation Function

The crack growth resistance is the local average of the process denoted $(dN/da)_{\Delta a,h}$ over the length Δa, that is: (we treat two-dimensional problems considering specimen thickness, h)

$$\left(\frac{dN}{da}\right)_{\Delta a,h} = \frac{1}{\Delta a \cdot h}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\Delta a}\frac{dN}{da}\,da\,dh \tag{9}$$

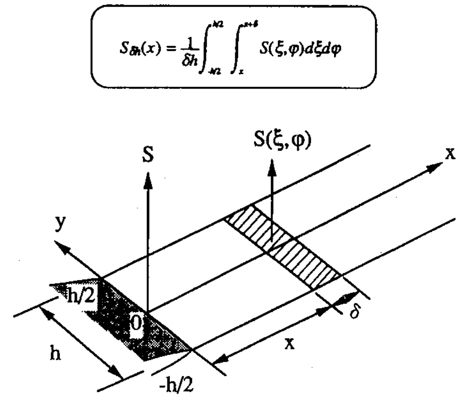

Fig. 1 Local average of material resistance considering specimen thickness

The subscript Δa indicates the length of the averaging interval, and h is the specimen thickness. In a specimen with particular thickness, h, measurement interval Δa is equal to δ, then the crack growth resistance coefficient, $S_{\delta h}(x)$, is as follows:

$$S_{\delta h}(x) = \frac{1}{\delta \cdot h}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\delta} S(\xi,\phi)\,d\xi d\varphi \tag{10}$$

The two-dimensional schematic of the local average of the fatigue crack growth resistance considering specimen thickness is shown in Fig. 1.

$S(\xi,\varphi)$ is assumed to be a continuous two dimensional function and, hereinafter, called the theoretical value of two dimensional fatigue crack growth resistance coefficient. The average of $S_{\delta h}(x)$ coincides with that of $S(\xi,\varphi)$. Namely, the expected value of Eq. (10) is then expressed as

$$E[S_{\delta h}(x)] = E\left[\frac{1}{\delta \cdot h}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\delta} S(\xi,\phi)\,d\xi d\varphi\right] = E[S(\xi,\varphi)] \tag{11}$$

where, $E[-]$ means the expectation. We treat the residual, that is, $S'(\xi,\varphi) = S(\xi,\varphi) - \overline{S}(\xi,\varphi)$ and $S'_{\delta h}(x) = S_{\delta h}(x) - E[S_{\delta h}(x)]$.

The variance of $S'_{\delta h}$ is not necessarily same as that of S. The difference depends on the property of the original spatial random process.

If the residuals, $S'(\xi,\varphi)$, is assumed to be a two-dimensional stationary spatial stochastic process, then its autocorrelation function is given by

$$R_{S'S'}(\tau_1,\tau_2)=E[S'(x,y)S'(x+\tau_1,y+\tau_2)] \tag{12}$$

The autocorrelation function of the local averaged process, $S'(\xi,\varphi)$ is easily shown to be

$$R_{\delta h}(\tau)=\frac{1}{\delta^2\cdot h^2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\delta}d\xi d\varphi\cdot$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x+\tau}^{x+\tau+\delta}R_{S'S'}(\eta-\xi,\zeta-\varphi)\,d\eta d\zeta,$$

$$\tau\geq k\delta \tag{13}$$

In the following, the mean is treated as zero. Then, the variance would be

$$Var[S'_{\delta h}]=\frac{1}{\delta^2\cdot h^2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\delta}d\xi d\varphi\cdot$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{x}^{x+\delta}R_{S'S'}(\eta-\xi,\zeta-\varphi)\,d\eta d\zeta,$$

$$\tau=0 \tag{14}$$

The application of the above calculations is demonstrated by adapting the following autocorrelation function. That is

$$R_{S'S'}(\tau_1,\tau_2)=e^{-(a_0|\tau_1|+b_0|\tau_2|)} \tag{15}$$

where, a_0 and b_0 are positive parameters such that the larger it is, the more rapidly the correlation disappears. In fact, for the autocorrelation function, a_0 and b_0 could be defined as the scale of correlation in each of the directions.

In this case, we obtain the autocorrelation function of $S'_{\delta h}$ using the Eq. (13) and also we can obtain the variance from Eq. (14)

$$R_{\delta h}(\tau)=\frac{16}{a^2_0\delta^2b^2_0h^2}e^{-a_0\tau}(a_0\delta+e^{-a_0\delta}-1)\cdot$$

$$\left(\frac{b_0h}{2}+e^{-bh/2}-1\right),$$

$$\tau\geq\delta \tag{16}$$

$$Var[S'_{\delta h}]=\frac{16}{a^2_0\delta^2b^2_0h^2}(a_0\delta+e^{-a_0\delta}-1)\cdot$$

$$\left(\frac{b_0h}{2}+e^{-b_0h/2}-1\right),$$

$$\tau=0 \tag{17}$$

It is plotted in Fig. 2 with the arbitrarily chosen parameter values ($a_0=b_0=0.01$, 0.1 and 1.0), and a specimen thickness of 6, 12 and 18 mm. Figure 3 shows the effect of variance on the specimen thickness. As shown in Fig. 2, when the parameter $a_0=b_0$ is 0.01, the autocorrelation functions are almost same, even if the specimen thickness is change to 6, 12 and 18 mm. The above results show that the variance decreases with increasing specimen thickness. The shorter the correlation length is the larger the specimen thickness effect on the variance. A short correlation length means that the nearby material properties are quite different from each other.

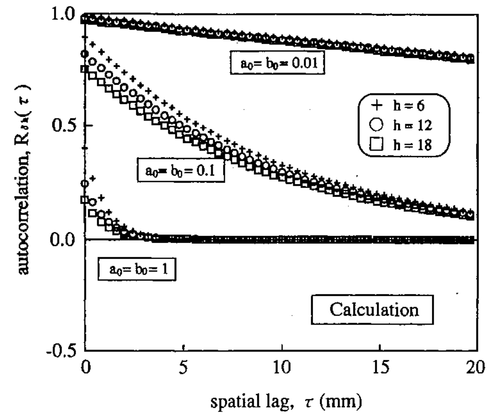

Fig. 2 Calculated autocorrelation functions

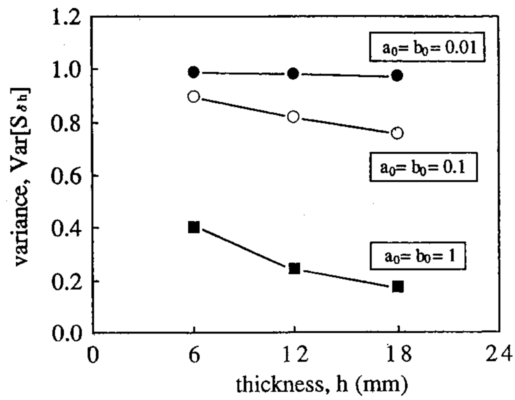

Fig. 3 Effect of the specimen thickness on the variance (calculation)

Regarding the above results, the effect of specimen thickness on the statistical properties of fatigue crack growth resistance depends on the correlation parameter and it is the possibility of a false indication when one estimates the statistical variability using the local averaged values. It is, therefore, necessary to know the spatial distributions of material strength for the simulation of a

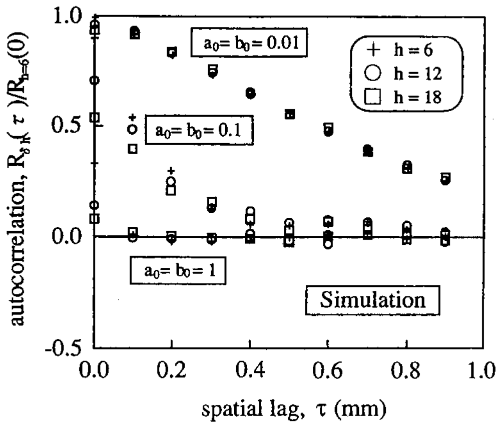

Fig. 4 Simulated autocorrelation functions

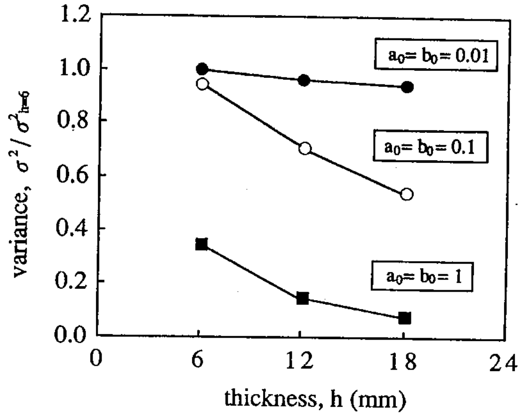

Fig. 5 Effect of the specimen thickness on the variance (simulation)

probabilistic fracture process to obtain numerically the probability distribution function of the macroscopic strength of the material.

4. Numerical Examples

Consider the two-dimensional numerical simulation of non-Gaussian stochastic process for the crack growth resistance. The method proposed by Yamazaki (Yamazaki and Shinozuka, 1986) are applied. To simulate the non-Gaussian stochastic process of $S(x, y)$.

As the numerical examples, the fatigue crack growth resistance is a two-dimensional spatial stationary stochastic process whose probabilistic properties are shown in Eqs. (8) and (15). That is, the probability distribution function of the fatigue crack growth resistance is 3-parameter Weibull distribution and its autocorrelation function is exponential. Given the parameters α, β, γ, a_0 and b_0, the material properties $S(x, y)$ can be simulated by the above method. In the following the shape, scale and location parameters are 3.7, 1.5×10^6 and 2.4×10^6, respectively. Several values are chosen for the correlation parameters a_0 and b_0 to observe its effect on the autocorrelation function of given sized specimens. A Fast Fourier Transform of size 4096 ($2^6\times2^6$) is used and the interval Δx, Δy is 0.1 mm for all correlation parameters.

The autocorrelation functions of the fatigue crack growth resistance obtained from the simulation results are presented in Fig. 4. They correspond to the theoretical results as shown in Fig. 2. The autocorrelation functions of the fatigue crack growth resistance are almost independent of the specimen thickness.

Figure 5 shows the relationship between the variance and specimen thickness. The results show that the variance decreases according to the increase of the specimen thickness. This is a good agreement with the theoretical results shown in Fig. 3. The variance on the specimen thickness is smaller as the correlation parameter value is larger. If the correlation parameter is very small, for example $a_0=b_0=0.01$, the simulated data can be regarded as mutually independent random variable and therefore the dependence of variance on the specimen thickness may be small. The simulated results well coincide with the above calculated results.

5. Experimental Procedure

The chemical composition and the mechanical properties of BS 4360 steel, are shown in Tables 1 and 2, respectively. Compact tension specimens (CT) with an LT orientation were prepared as the various recommendations of ASTM E647. In all, three series of specimens were prepared. In all specimens, the specimen width was held constant at $W=100$ mm, and the thickness was varied to 18, 12 and 6 mm.

Table 1 Chemical composition of BS 4360 steel

C(%)	Si(%)	Mn(%)	P(%)	S(%)	Nb(%)
0.15	0.40	1.32	0.016	0.004	0.26

Table 2 Mechanical properties of BS 4360 steel

UTS(MPa)	YS(MPa)	%Elongation
522	371	30.0

In order to align the specimens, circular washers were used to hold the specimens in the midplane of the clevis grips during fatigue crack growth testing. All the specimens were initially precracked to produce a sharp crack front. The fatigue cracking was carried out in an MTS servo-hydraulic testing machine at room temperature. All tests were performed in tension-tension at constant ΔK level, 28.0 $MPa\sqrt{m}$, the stress ratio, 0.2 and the frequency, 5 Hz. The ΔK level was found to be within ±0.2% of the range, and this was considered to be satisfactory. The micro computer generates time series which is used as the input to a multiplying DA-converter. The crack-opening displacement is measured by a non-contact optimal extension meter. This signal and the load cell output are simultaneously digitalized and taken into a micro computer. The crack lengths were mainly measured by the compliance method. And also, the crack lengths were measured on both specimen surfaces with the help of a travelling microscope(×100) by one person. Data are converted from two consecutive cycles, 200 samples per cycle, and the least square error method is applied to obtain a sample of V/P. From twenty samples, the average value of V/P is estimated. The same data is used to determine the average load amplitude in the duration. This process is repeated continuously. If the computed ΔK deviates from the specified value more than 0.5 percent, the multiplying DAC is gradually adjusted for ΔK to approach the prescribed value. When the crack length increases to 0.4 mm, the length of crack, the number of cycles, the maximum and minimum of load and crack opening displacement are recorded by the computer.

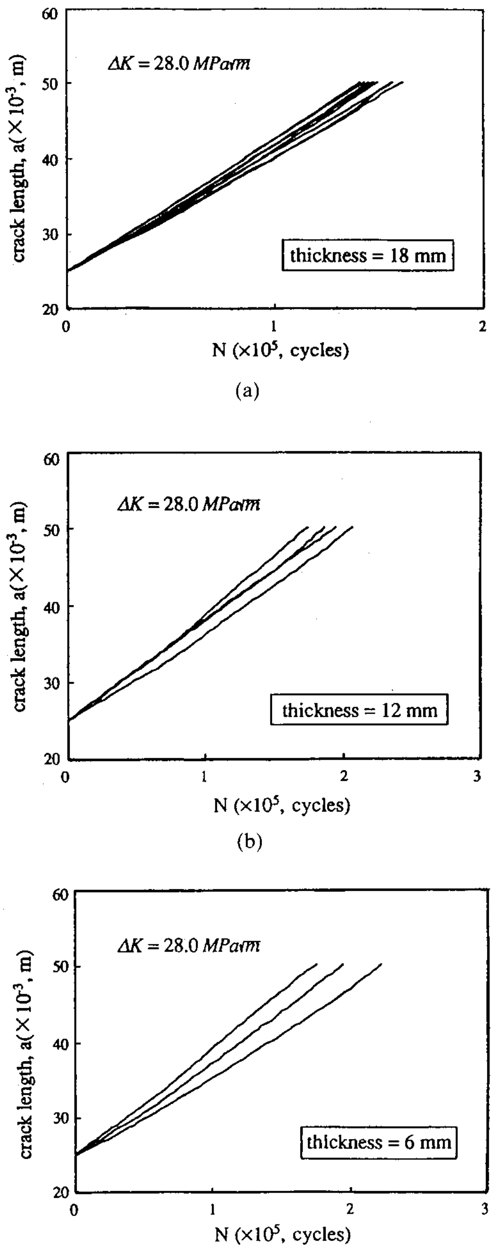

Fig. 6 a-N curves

6. Analysis of Experimental Results

Figure 6 provides a diagram of the crack length, a, against the number of cycles, N, for each specimen thickness. Though each relation is approximately linear, the difference of the inclination of each curve for the same specimen thickness is observed. It is evident from these figures that there is a variation of the local mean value of the growth resistance of material to fatigue crack growth from specimen to specimen. As shown in the figure, some of the a-N curves intersect each other, showing that the crack growth resistance is dependent on the location within a specimen. It seems to be that m varies from specimen to specimen, and a higher frequency fluctuation is due to C. This problem will, however, not be discussed in the present investigation, because the present study investigates the effect of specimen thickness on the statistical properties of fatigue crack growth resistance.

One of the examples of the obtained data is shown in Fig. 7, the growth rate, da/dN is plotted against crack length, a, together with $\Delta K/\overline{\Delta K}$. As shown in the figure, the range of the stress intensity factor is well controlled and its coefficient of variation is about 0.2 percent. Even under these carefully controlled conditions, the observed crack growth rates have remarkable fluctuations. Throughout the tests all the same figures are obtained.

Figure 8 provides a diagram of the mean growth rate,, $\overline{da/dN}$, versus the specimen thickness for $h=18$, 12, and 6 mm. As is seen in the figure, there is no negligible influence of specimen thickness on the mean crack growth rate. And,

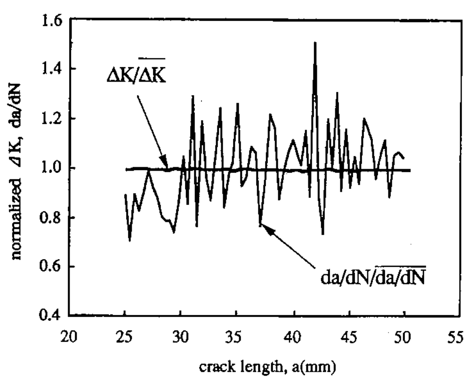

Fig. 7 Crack growth rate versus crack length

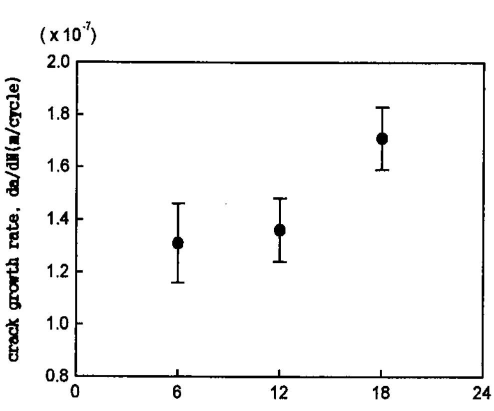

Fig. 8 Mean crack growth rate versus specimen thickness

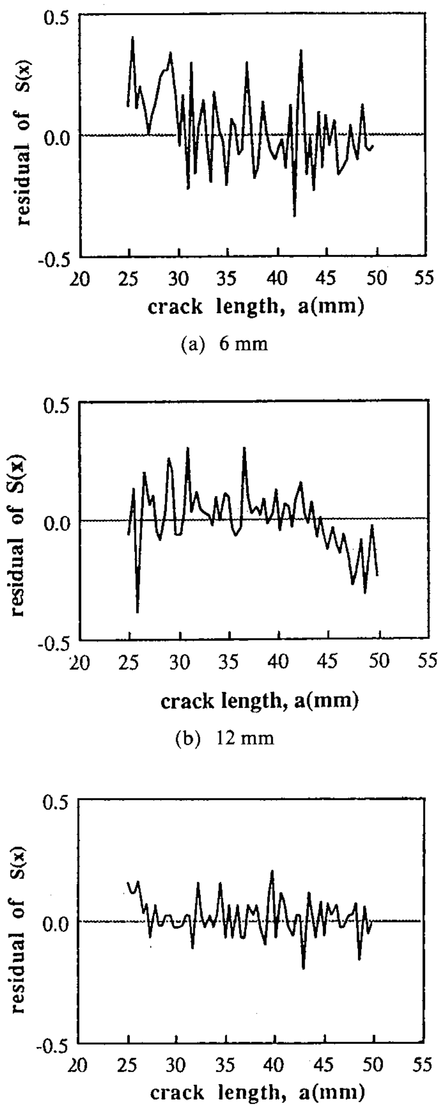

Fig. 9 Residuals of the material resistance

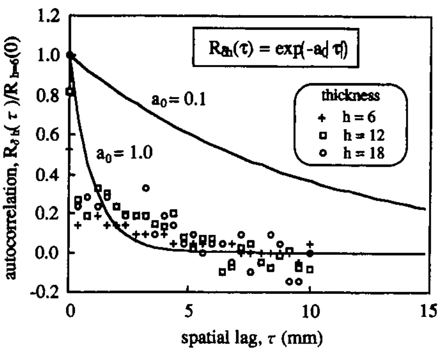

Fig. 10 The experimentally obtained autocorrelation functions and computed ones

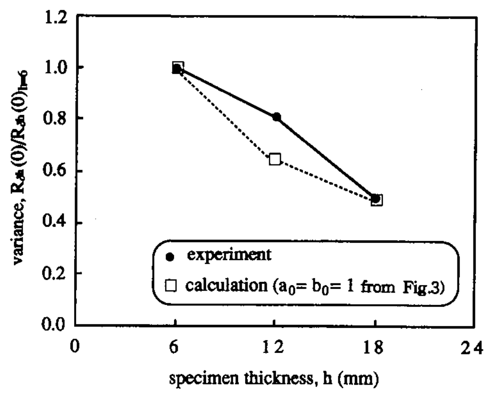

Fig. 11 Effect of the specimen thickness on the variance of $S_{\delta h}(x)$

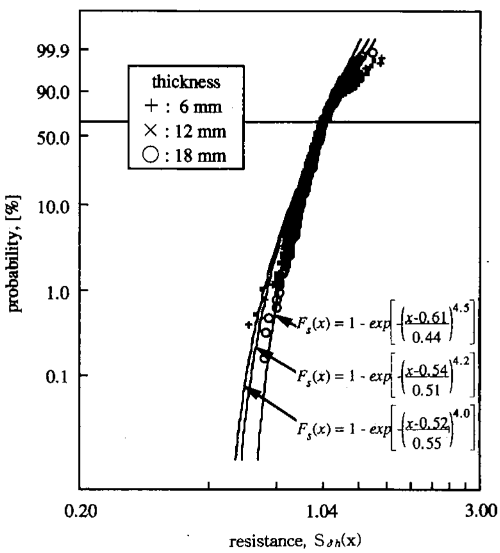

Fig. 12 Weibull plots of the material resistance

there was scatter in the fatigue crack growth rate data. It is evident that specimen thickness has the influence on fatigue crack growth rate of BS 4360 steel in the thickness range investigated in this study. The mean crack growth rate is increased with increasing the specimen thickness. Some workers have reported that a higher crack growth rate occurs in specimens with greater thicknesses because of the presence of a plane strain state (Broek, 1988).

Figure 9 shows the examples of the residuals versus crack length. As is seen in the figure, the residual shows remarkable fluctuations. And also, the data show that the high fluctuations is smaller with increasing specimen thickness because of the averaging effect.

The ensemble autocorrelation functions obtained from tests for each specimen thickness are shown in Fig. 10. The autocorrelation function reflects the correlation between observations of the stochastic process. It is clear from these figures that the autocorrelation functions are almost independent of the specimen thickness of the BS 4360 steel investigated in this study, except for the orgin, $R(0)$. The rate of decay is very rapid. The exponential function seems to be a reasonable shape. The function is also shown in the figure with arbitrary chosen coefficients. The area under correlation function is called correlation length or the scale of correlation. The value of the correlation length for BS 4360 steel specimens used in the present study is about 1.0 mm. The advantage of this approach is an understanding of the scale of variation for the spatial stochastic process.

Figure 11 shows the effect of the specimen thickness on the variance of fatigue crack growth resistance, $S_{\delta h}(x)$. The variance increases with decreasing specimen thickness. This is a good agreement with the theoretical results.

Next, in order to investigate the effect of the specimen thickness on the probability distribution of fatigue crack growth resistance, the material resistance coefficients obtained from the experiments are plotted on Weibull probability paper, as shown in Fig. 12.

As shown in Fig. 12, the experimental data tend to deviate from a straight line. It is thought that the location parameter, γ in the Weibull distribution function must be used. Therefore, 3-parameter Weibull distribution function is used to fit the data. The parameters α, β and γ are estimated by the direct search of optimization method (Buto, Sugie and Okazaki, 1977). The estimated functions are also shown in the figure. It may be said that $S_{\theta h}(x)$ follows 3-parameter Weibull distribution function. The probability distribution functions of $S_{\theta h}(x)$ show a slight dependence on specimen thickness. The shape parameter, α and location parameter, γ are increased by increasing the specimen thickness, but the scale parameter, β is decreased.

As far as the present investigation is concerned, it is possible to estimate the parameters of the statistical properties of fatigue crack growth resistance for a thin specimen, and hence, the relation between the crack length, a and the number of cycles, N could be determined by the stochastic process simulation method using the obtained parameters. And also, the reliability assessment with respect to fatigue crack growth is also possible using the method proposed by Itagaki (Itagaki, et al., 1993).

7. Conclusions

Our results clearly indicate that the effect of autocorrelation function of fatigue crack growth resistance on the specimen thickness has no significant influence, but the variance increases with decreasing specimen thickness. And, the probability distribution functions of the fatigue crack growth resistance obtained from the experimental data are 3-parameter Weibull distribution and show a slight dependence on the specimen thickness of BS 4360 steel. Therefore, one might expect this approach to be useful in problems for which fatigue crack growth of a very thin specimen is estimated by using the statistical data obtained from fatigue crack growth tests for a rather thick specimen.

References

Broek, D., 1988, "The Practical Use of Fracture Mechanics," KAP.

Buto, S., Sugie, H. and Okazaki, A., 1977, "Fortran and Numerical Analysis," *Baifukan.*

Itagaki, H., Ishizuka, T. and Huang, P. Y., 1993, "Experimental Estimation of the Probability Distribution of Fatigue Crack Growth Lives," *Prob. Eng. Mech.*, Vol. 8, pp. 25~34.

Kim, S. J., 1999, "An Analysis of Crack Growth Rate Due to Variation of Fatigue Crack Growth Resistance," *Trans. KSME*, Vol. 23, No. 7, pp. 1139~1146.

Kozin, F. and Bogdanoff, J. L., 1989, "Recents Thoughts on Probabilistic Fatigue Crack Growth," Part 2. *Appl. Mech. Rev.*, Vo. 42, No. 11, S121-S127.

Lapetra, C., Mayo, J. and Dominguez, J., 1996, "Randomness of Fatigue Crack Growth under Constant Amplitude Loads," *Fat. Fract. Eng. Mater. Struct.*, Vol. 19, pp. 589~600.

Mcmaster, F. J., Tabrett, C. P. and Smith, D. J., 1998, "Fatigue Crack Growth Rates in AL-Li Alloy, 2090. Influence of Orientation, Sheet Thickness and Specimen Geometry," *Fat. Fract. Eng. Mater. Struct.*, Vol. 21, pp. 139~150.

Ortiz, K. and Kiremidjian, A. S., 1986, "Time Series Analysis of Fatigue Crack Growth Rate Data," *Eng. Fract. Mech.*, Vol. 24, pp. 657~676.

Paris, P. C. and Erdogan, F., 1963, "A Critical Analysis of Crack Growth Propagation Law," *J. Bas. Eng.* (*Trans. ASTM D*) Vol. 85, pp. 528~536.

Putatunda, S. K. and Rigsbee, J. M., 1985, "Effect of Specimen Size on Fatigue Crack Growth Rate in AISI4340 Steel," *Eng. Fract. Mech.*, Vol. 22, pp. 335~345.

Shim, D. S. and Kim, J. K., "A Stochastic Analysis on Variation of Fatigue Crack Propagation due to Thickness Effect," *Trans. KSME*, Vol. 22, No. 8, pp. 1523~1532.

Sobczyk, K., 1993, "Stochastic Approach to Fatigue : Experiments, Modelling and Reliability Estimation," *Springer Verlag Wein-New York.*

Tanaka, S., Ichikawa, M. and Akita, S., 1981, "Variability of m and C in the Fatigue Crack Propagation Law," *Int. J. Fract.*, Vol. 17, R121.

Virker, D. A., Hillberry, B. M. and Goel, P. K., 1979, "The statistical nature of fatigue crack propagation," *J of Eng. Mat. and Tech.*, *ASME*, Vol. 101, pp. 148~153.

Yamazaki, F. and Shinozuka, M., 1986, "Digital Generation of non-Gaussian Stochastic Fields," Technical Report, *Columbia Univ. Press*, pp. 211~235.