Reverse Engineering of Compound Surfaces Using Boundary Detection Method

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This paper proposes an efficient reverse engineering technique for compound surfaces using a boundary detection method. This approach consists in extracting geometric edge information using a vision system, which can be used in order to drastically reduce geometric errors in the vicinity of compound surface boundaries. Through the image-processing technique and the interpolation process, boundaries are reconstructed by either analytic curves (e. g. circle, ellipse, line) or parametric curves (B-spline curve). In other regions, except boundaries, geometric data are acquired on CMM as points inspected using a touch type probe, and then they are interpolated on several surfaces using a B-spline skinning method. Finally, the boundary edge and the skinned surfaces are combined to reconstruct the final compound surface. Through simulations and experimental works, the effectiveness of the proposed method is confirmed.

Key Words : Reverse Engineering, Free-formed Surface, Computer Vision System, Edge Detection, Image processing, Segmentation, Surface Skinning

1. Introductioin

In order to represent the geometric shapes of industrial products, analytical surfaces such as planes, spheres and cylinders as well as 3-dimensional free-formed surfaces (sculptured surfaces) are widely being used. In the past, simple shapes have been mainly used emphasizing functional aspects because they can be easily analyzed and represented. However, the relative importance of free-formed surfaces is rapidly on the increase because the trend today is for the aesthetical functions of a product's external appearance to be emphasized. In general, it is difficult to analytically represent such complex shapes using mathematical formulations. It is not easy to model required shapes in a CAD (Computer-Aided Design) System. For the problems of free-formed surface CAD modeling, reverse engineering technology, based on applications of the 3-dimensional inspection technique, has been recently focused on (Peng & Loftus, 1998; Varady et al., 1997). In fact, when new products comprising free-formed surfaces are developed through the conventional CAD modeling technique, it is necessary to take much time and to make several modifications because of geometric complexity. To improve these drawbacks, Woodward and Cho et al. (Woodward, 1988; Cho et al., 1995) proposed reverse engineering technologies, which consist in acquiring geometric data, required in

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CAD systems, through the 3-dimensional inspection process from manually made mock-ups or clay models. On the other hand, reproducing, modifying and analyzing existing products have often led to the development of new products.

In order to generate geometric data of a 3-dimensional shape using the reverse engineering technique without CAD data, the following steps are required (Varady et al., 1997) :

(1) Point data acquisition using the 3-dimensional inspection technique;

(2) Region subdivision and surface fitting based on acquired point data;

(3) CAD model generation through numerical transformation of surface data.

To reconstruct an entire surface based on acquired point data, B-spline interpolation and NURBS interpolation methods are generally used (Cho & Seo, 1999). However, general shapes of products correspond to compound surfaces composed of several analytic surfaces and sculptured surface segments and patches. In the conventional inspection process, the entire surface is reconstructed without taking into account boundaries between each surface. Hence, in order to reduce the inspection errors in the vicinity of the boundary regions between different surfaces, it is necessary to excessively increase the number of measuring points. In spite of numerous measuring points, it is actually difficult to accurately represent the edge shape of boundary regions.

The purpose of our study is to suggest an improved reverse engineering technique, which allows for accurately and effectively reconstructed geometric shapes taking into account boundary detection process. To extract boundary data of compound surfaces, this study uses a vision camera system and an image-processing technique.

Based on extracted boundary data, we divide the compound surface into elemental surface regions (Medioni & Yasumoto, 1987). Then, we inspect each elemental surface region on a CMM (Coordinate Measuring Machine). Finally, we reconstruct the sculptured surface from the acquired point data through a surface skinning

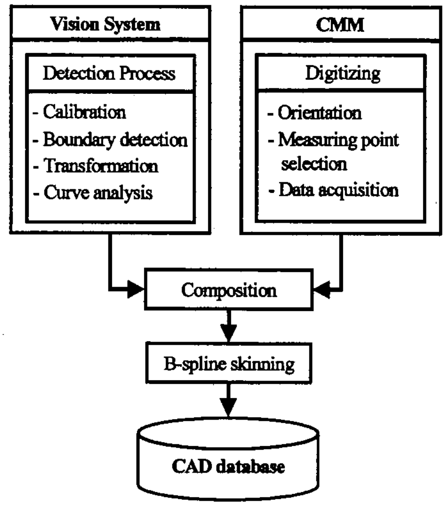


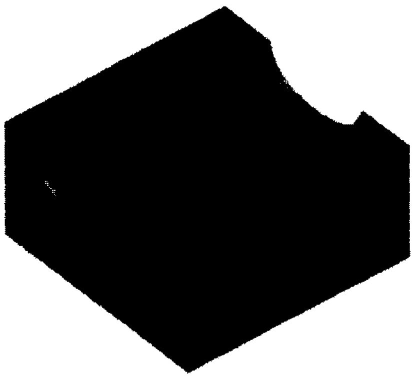
Fig. 1 Proposed reverse engineering process

process (Piegl & Tiller, 1995; Werner et al., 1997; Woodward, 1988). Figure 1 illustrates the process of the approach presented.

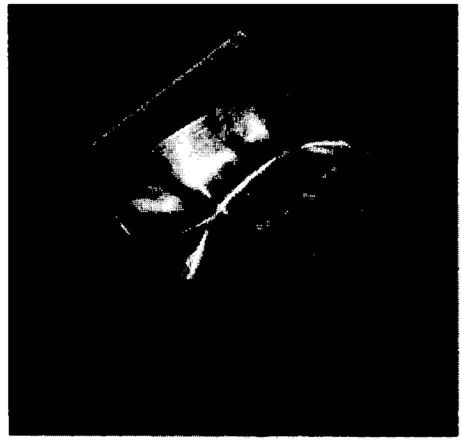
The validity of the proposed approach was demonstrated by experimentation. Using our improved reverse engineering technique significantly reduces interpolation errors occurring in the conventional reverse engineering.

2. Boundary Detectioin and Surface Segmentation

The first step of our approach is to obtain geometric sharp edge information within a compound surface. The edges correspond to the boundaries between differently defined elemental surfaces in a compound surface. Detecting these boundaries allows us to recover the accurate geometric shape of compound surface through a decreased number of measuring points. By using a computer vision system, we acquire the image data of the compound surface before the inspection process on CMM. It is possible to extract sudden geometric changes in the image data, which correspond to the boundaries. In this paper, an image-processing technique is used as a boundary detection method. In order to detect sudden geometrical changes as edges, edge operators (i. e. Sobel operator and Laplacian of Gaussian operator) are applied to original image data

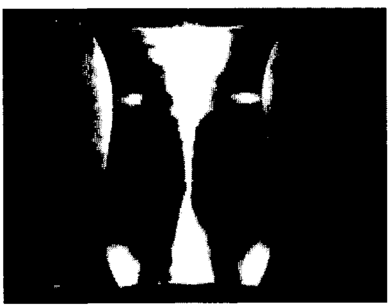


(a) Solid modeling

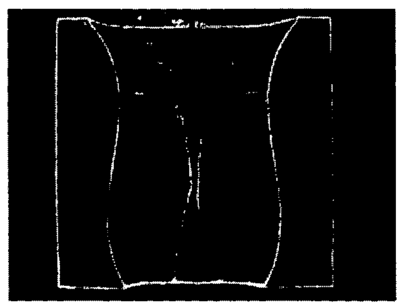


(b) Machined Part

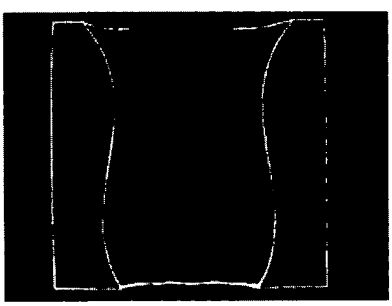
Fig. 2 Compound surface model



(a) Original Image



(b) Edge detection



(c) Noise trimming

(d) Thinning

Fig. 3 Boundary detection process

(Mortenson, 1997). Then, we interpolate the detected edge data to geometrically define the boundaries in the form of analytic curves (circle, ellipse, line) and parametric curves (B-spline curves). Subsequently, the compound surface is divided into several elemental surfaces on the basis of detected boundaries in order to carry out an independent inspection process for each surface.

2.1 Boundary detection using computer vision system

First, the computer vision system is used in order to acquire original image data represented by brightness distribution. From the original image data, it is possible to extract geometric data of sharp edges (i. e. boundary) by emphasizing discontinuity due to abrupt brightness changes. This can be accomplished through the image-processing technique. In this paper we deal with a concrete example to illustrate this boundary detection process.

Figure 2 depicts this compound surface model : Fig. 2(a) shows a designed compound surface model through solid modeling in a CAD system and Fig. 2(b) shows a machined part (aluminum) through rough and finish cutting.

Using a vision camera system, we capture an original image data in gray-level from the machined part as shown in Fig. 3(a). In fact, an edge is the boundary between two regions with relatively distinct gray-level properties. Therefore, applying Sobel operator (Gonzalez & Woods, 1992) to this monochrome image data, it is possible to extract the discontinuity corresponding to transition from dark to bright. Figure 3(b) shows the detected edges from the original image data. Figure 3(c) shows the binary image of edge geometric data with improved accuracy through the noise-trimming phase. To geometrically analyze and recognize the accurate edge shape, it is necessary to reduce the thickness of detected edges. This reduction can be accomplished by obtaining the skeleton of the detected thick edges via a thinning (also called skeletonizing) algorithm (Gonzalez & Woods, 1992). This thinning process consists in deleting unnecessary neighbor pixels to obtain continuous central pixels. Finally, we obtain the thinned edge image as shown in Fig. 3(d).

Generally, the captured image by vision camera and real edge shape are not coincidental because the image coordinates and the real coordinates (i. e. CMM coordinates) are not coincidental. By comparing positions of the same point in two different coordinates (image and real coordinates), a transformation matrix can be derived. The dimension of a captured image is not the same as the real dimension. Hence, the scale factor is required to distinguish between the real dimension and the dimension of the captured image. Comparing the distance between two known points with their distance in image plane, it is possible to derive the scale factor. On the other hand, it is actually difficult to set the vision camera in an accurate position. In order to derive

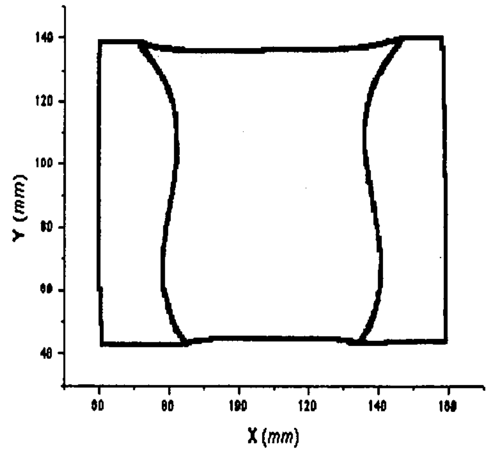


Fig. 4 Extracted boundary shape

the accurate geometric data from the captured image, a camera calibration procedure is necessary. The compensation process, which reduces image data errors due to inaccurate camera set-up, requires a set of image points whose real coordinates are known. The computational procedure used to obtain the camera parameters using these known points often is referred to as camera calibration (Gonzalez & Woods, 1992). In this research, we use an error compensation matrix (i. e. transformation matrix), which is obtained by comparing image data of known points with their real positions on CMM. Through a series of the processes previously mentioned, we obtain the final boundary image in real dimension, as shown in Fig. 4, from the thinned boundary shape (see Fig. 3(d)).

2.2 Interpolation of extracted boundary

In order to divide the compound surface into elemental surfaces, it is necessary to interpolate the extracted boundary data with analytic and parametric curves. By analyzing the detected image data, the geometric boundary data, extracted through the image-processing technique presented above, can be defined by either analytic curves (circle, ellipse, line) or parametric curves. In this research, based on acquired image data, the analytic curves are derived by the least square method. For boundary data that cannot be interpolated by the analytic curves, B-spline interpola-

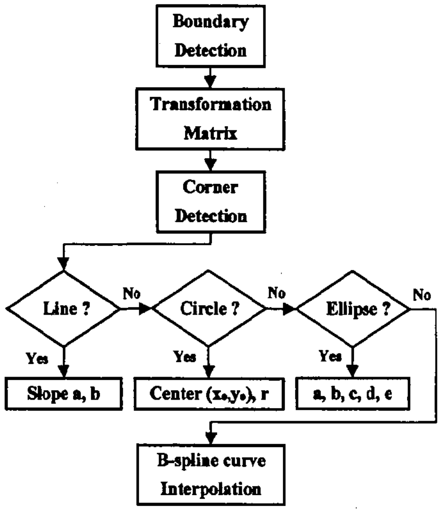


Fig. 5 Schematic diagram for boundary recognition

tion is applied to fit the edge shape.

Figure 5 presents a schematic diagram of a boundary recognition method. The interpolation method of detected boundary data is presented in the following sections.

2.2.1 Analytic curve interpolation

In general, analytic curves (circle, ellipse, line) can be represented by a conic section curve (Mortenson, 1997), whose standard form is given by :

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0 \quad (1)$$

The conic section curve represents several analytic curves (circle, ellipse, line) according to values of coefficients and is invariant with respect to rotation and translation. The extracted boundary data correspond to coordinate values projected on xy-plane. In this case, if ith coordinate value is presented by (x_i, y_i), this does not always satisfy Eq. (1); hence, error ei between conic section curve and (x_i, y_i) is given by:

$$e_i=Ax_i^2+2Bx_iy_i+Cy_i^2+2Dx_i+2Ey_i+F \quad (2)$$

If the edge is composed of n points, total error ER is given by :

$$ER=\sum_{i=1}^{n}e_i^2 \quad (3)$$

Therefore, if through Eq. (4) the coefficients are determined to minimize the total error ER using least square method, it is possible to derive conic section curve from the set of point.

$$\frac{\partial ER}{\partial A_i}=0 \quad (4)$$

Ai corresponds to each coefficient (A, B, ⋯ F) of Eq. (1), which varies according to interpolating conic section curves.

In this study, using the approach presented above, we approximate acquired boundary data as conic section curve and check error amount between the acquired boundary data and interpolated curve. If checked error amount does not fulfill definite tolerance, we try to interpolate the boundary data by repeating a comparison with other types of conic section curves.

2.2.2 B-spline curve interpolation

For boundary shapes which cannot be represented by analytic curves as mentioned above, B-spline interpolation method (Medioni & Yasumoto, 1987) is used in this paper.

Suppose we are given a set of acquired points $Q_k(k=0, 1, \ldots n)$ and we want to interpolate these points with a p^{th}-degree B-spline curve in order to obtain the boundary data (see Eq. (5)).

$$Q_k=C(\bar{u}_k)=\sum_{i=0}^{n}N_{i,p}(\bar{u}_k)P_i \quad (5)$$

The control points, P_i, are the $(n+1)$ unknowns. If we assign a parameter value, $\bar{u}_k$, to each Q_k using chord length method, and select an appropriate knot vector $U=\{u_0, \cdots u_m\}$ by averaging the parameters, we can set up the $(n+1)$ $(n+1)$ system of linear equations (Piegl & Tiller, 1995; Mortenson, 1997). This system of linear equations can be solved by LU decomposition. Consequently, the control points P_i are determined and acquired boundary data are interpolated by B-spline curve.

2.3 Elemental surface segmentation

When Interpolating the boundary data by analytic curves or B-spline curves as mentioned above, the compound surface could be subdivided into several regions. Figure 6 presents our subdivided compound surface model. This surface is composed of two plane regions and one sculp-

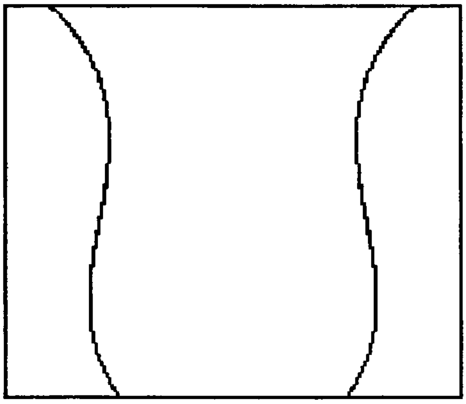


Fig. 6 Extracted boundaries and elemental surface segmentation

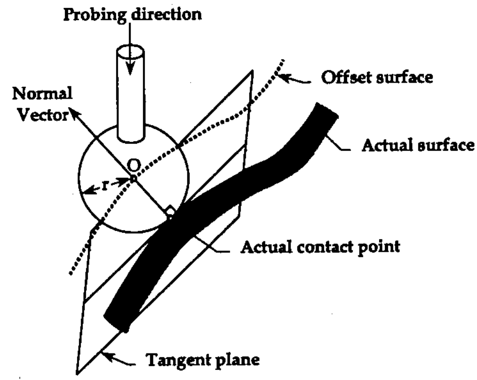


Fig. 7 Offset surface generation from measured data

ured surface region.

3. Sculptured Surface Generation Based on Inspection Data

3.1 Inspection process on CMM

To minimize the measuring errors induced by a ball-type probe in a general inspection process, he probe should move along the normal direction of the actual surface (Duffie, 1984). The nspection errors can occur in the vicinity of the ransition surface edges. Using the approach proposed in this paper can effectively reduce these errors, and consequently, the inspection errors due to unknown normal direction of target surface will be reduced.

3.2 Probe radius compensation

When a ball-type probe is used approaching and touching the unknown target surface, the nspected point and actual contact point are not coincidental. To accurately inspect the unknown sculptured surface, a compensation process is required, often called probe radius compensation. If the target surface is mathematically unknown, as in reverse engineering processes, the probe approaching direction and compensation amount cannot be determined easily. To determine the normal direction, the offset surface concept is employed in this study.

In a simple geometric interpretation of the offset surface, the surface locus is swept out by the center of a sphere of radius r as it rolls over an entire surface. As shown in Fig. 7, if a ball-type probe is used to measure an unknown surface, the center of the probe $O(x, y, z)$ will be placed on the offset surface whenever the probe touches any point $A(x, y, z)$ on the actual surface along any direction. Since the coordinates of the probe center $O(x, y, z)$ can be known from the CMM, a mathematical representation of the offset surface can be obtained by applying a B-spline interpolation method as discussed in a previous section.

Mathematically, an offset surface $S_{off}(u, v)$, to given parametric surface $S(u, v)$, can be defined as an envelope of the continuum of vectors $d \cdot n(u, v)$, where d is the offset magnitude along the unit normal vector $n(u, v)$. An explicit representation of the offset surface can be written as follows (Faux, 1979):

$$S_{off}(u, v) = S(u, v) + r \cdot n(u, v) \quad (6)$$

where the surface normal vector at each surface point can be defined as:

$$n(u, v) = \frac{S_u(u, v) \times S_v(u, v)}{|S_u(u, v) \times S_v(u, v)|} \quad (7)$$

An unique unit surface normal vector exists if the surface patch has continuous, non-zero first order derivatives and $|S_u(u, v) \times S_v(u, v)|$.

After $S_{off}(u, v)$ is determined by using B-spline interpolation, corresponding points on the actual surface, to the measured points, can be determined from the following equation.

$$S(u, v) = S_{off}(u, v) - r \cdot n(u, v) \quad (8)$$

Thus, applying B-spline interpolation again to the actual contact points, an exact mathematical representation of the target surface can be obtained. Above procedures can be summarized as following steps.

Step 1 : Digitize the unknown freedom surface by equi-interval grid pattern.

Step 2 : Apply B-spline interpolation to obtain offset surface points which passe through all the center coordinates of a ball-type probe.

Step 3 : Calculate normal vectors from the offset surface.

Step 4 : Move the measured points to the actual surface.

Step 5 : Again, apply B-spline interpolation to determine the mathematical representation of the surface.

3.3 Surface skinning based on sectional curves

If the compound surface is inspected through an equally spaced grid pattern, the number of measuring points in sculptured surface regions varies according to measuring path. Therefore, in this paper we use a surface skinning method to generate a sculptured surface. This method consists in blending several sectional curves interpolated from measured point data (Piegl & Tiller, 1995). Subsequently, composing the skinned surfaces, we obtain a final reconstructed compound surface. Details are presented in following sections.

3.3.1 B-Spline skinning surface

Generally, given sectional curves $C_k(u)$ can be defined with different degrees and knot vectors. Applying degree elevation algorithm and knot insertion algorithm (Piegl & Tiller, 1995), it is possible to transform all sectional curves into compatible B-spline curves defined with respect to the same degree and knot vector (Piegl & Tiller, 1996; Yau, 1999). The p^{th}-degree k^{th} sectional curve $C_k(u)$ defined for the same knot vector U is given by:

$$C_k(u)=\sum_{i=0}^{n}N_{i,p}(u)\,Q_{i,k} \quad (k=0,\ 1,\ \ldots,\ K) \tag{9}$$

where $Q_{i,k}$ k is i^{th} control point of kth curve. Let $S(u, v)$ be final surface skinned along v-direction. If the sectional curve $C_k(u)$ is the iso-parametric curve determined with constant value $\bar{v}_k$ as v parameter, this iso-parametric curve $S(u, \bar{v}_k)$ is given by:

$$\begin{aligned} S(u,\ \bar{v}_k)&=C_k(u)=\sum_{i=0}^{n}N_{i,p}(u)\,Q_{i,k} \\ &=\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)\,N_{j,p}(\bar{v}_k)\,P_{i,j} \\ &=\sum_{i=0}^{n}N_{i,p}(u)\left\{\sum_{j=0}^{m}N_{j,p}(\bar{v}_k)\,P_{i,j}\right\} \\ &(k=0,\ 1,\ \ldots\ K) \end{aligned} \tag{10}$$

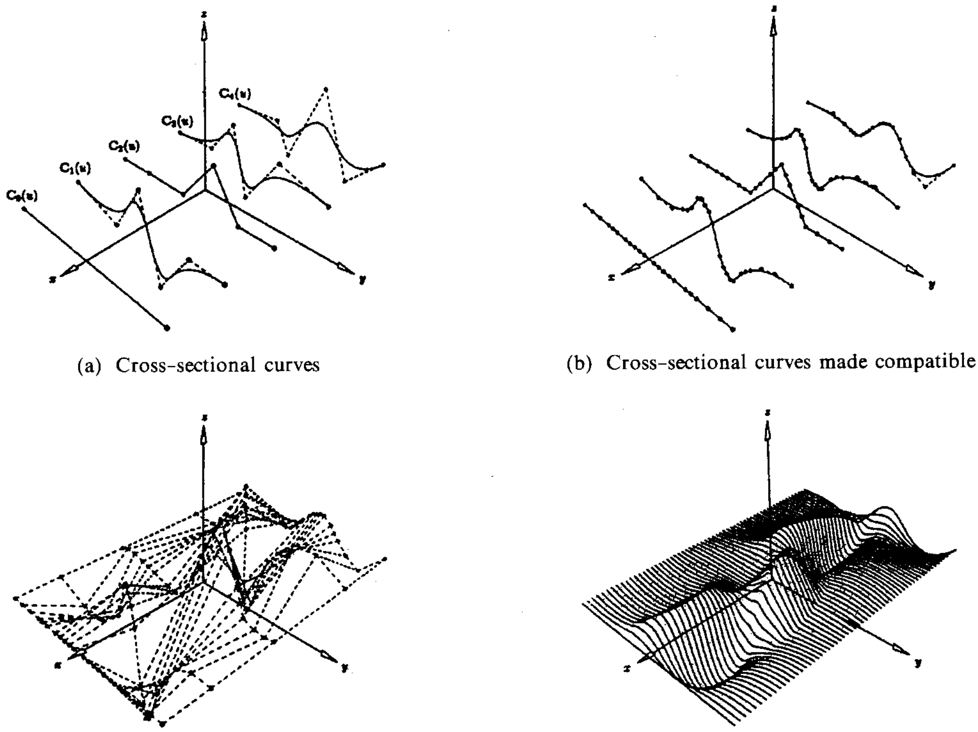
As $N_{i,p}(u)$ is independent of all u in the first and the third equation of Eq. (10), we obtain the following relation (cf. Eq. (11)) by rearranging Eq. (10) for each i.

$$Q_{i,k}=\sum_{j=0}^{m}N_{j,p}(\bar{v}_k)\,P_{i,j} \quad (k=0,\ 1,\ \ldots\ K) \tag{11}$$

Considering that i is fixed, Eq. (11) is an identical case compared to B-spline curve interpolation problem with given point data Q_k. Consequently, the control points of skinning surface correspond to control points of q^{th}-degree B-spline curves, which are interpolated from vertical directionally collected control points of given sectional curves with the same parameter $\bar{v}_k$ and knot vector V. Hence, Eq. (11) can be represented in matrix form as following:

$$[M_{k,j}][P_{i,j}]^T=[Q_{i,k}]^T,\ (i=0,\ \ldots,\ n,\ j=0,\ \ldots,\ m,\ k=0,\ \ldots,\ K) \tag{12}$$

where $M_{k,j}$ is $(K+1)(m+1)$ matrix, $P_{i,j}$ is $(n+1)(m+1)$ matrix and $Q_{i,k}$ is $(n+1)(K+1)$ matrix. If $m = K$, $N_{k,j}$ is a square matrix, it exists a unique solution of skinning surface, control point matrix $P_{i,j}$. Equally to the curve interpolation problem, v-directional parameter $\bar{v}_k$ and knot vector V of each sectional curve are significant factors to determine the shape of skinning surface. For the case that sectional curves are mutually parallel, it is possible to determine v-directional parameter $\bar{v}_k$ and knot vector V using the chord length method and averaging the parameters in order to obtain an appropriate final skinned surface. Fig. 8 illustrates the process of surface skinning. Figure 8(a) shows a set of



(a) Cross-sectional curves (b) Cross-sectional curves made compatible

(c) Control points (d) Skinned surface

Fig. 8 Process of surface skinning

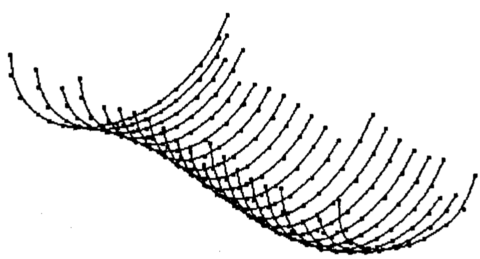


Fig. 9 B-spline skinning surface

parallel cross sectional curves mutually parallel, Fig. 8 (b) shows that the cross sectional curves are made compatible through degree raising and knot refining, Fig. 8 (c) shows the control polygon of a skinned surface and Fig. 8 (d) shows the final skinned surface.

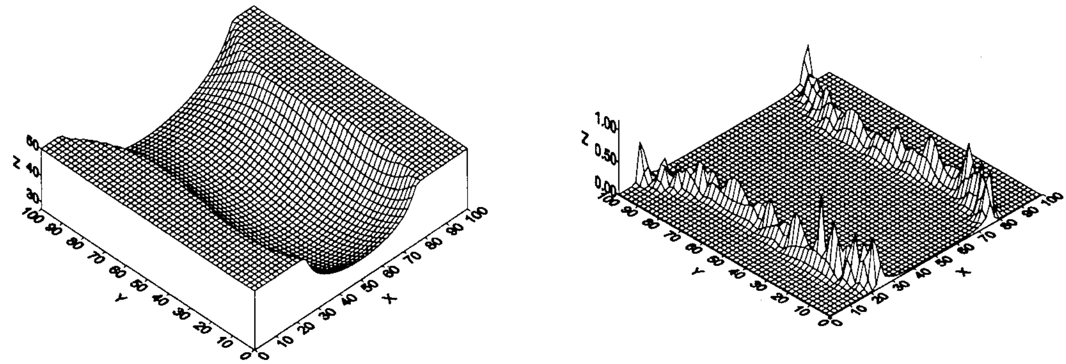
Based on the inspection data in the sculptured surface region, the surface skinning process is carried out for our compound surface model. Figure 9 shows the skinned surface.

3.3.2 Composition of reconstructed surfaces

Based on the inspection process independently accomplished for analytic and sculptured surfaces as well as the boundary edges, it is necessary to combine the inspected data. The Boolean operation is used to join these inspected data. Finally, we obtain the final compound surface.

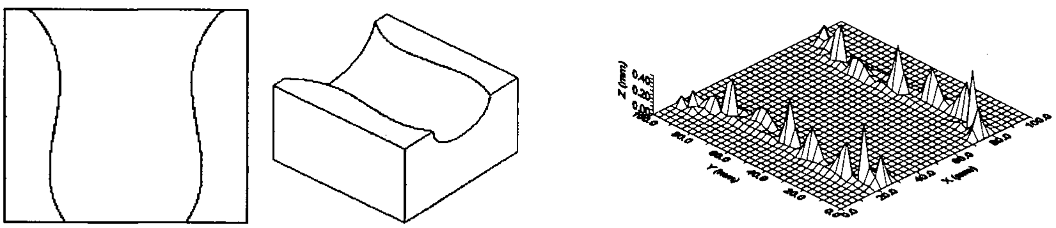
4. Simulations and Experimental Works

In this section, we present an application example to show the effectiveness of our approach. For our compound surface model (see Fig. 2), we apply both the conventional reverse engineering process and our improved reverse engineering process, and then we compare reconstructed surfaces with CAD modeling data. First, we reconstruct our compound surface model in the conventional reverse engineering process. Figure 10 (a) shows the surface interpolated by B-spline surface from inspected point data on CMM without



(a) Interpolated surface (b) Error map (RMS error=0.256 mm)

Fig. 10 Reverse engineering without boundary detection



(a) Reconstructed surface (b) Error map (RMS error=0.026 mm)

Fig. 11 Reverse engineering with boundary detection

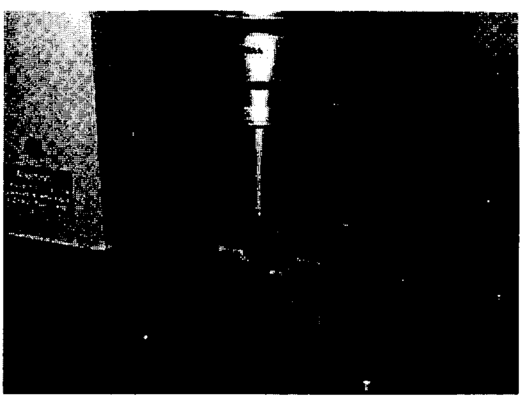


Fig. 12 Digitization process using CMM (Zeiss)

boundary detection. As mentioned in the introduction, indistinct edge parts appear on the reconstructed surface. Therefore, it can be known that the main interpolation errors are concentrated on the boundary parts of a compound surface. In this case, the RMS error of the entire surface is 0. 256mm. Figure 10(b) shows the interpolation error distribution.

Figure 11 shows the results of the improved reverse engineering based on the boundary detection process proposed in this paper. As shown in Fig. 11(b), some interpolation errors remain on the boundary parts; however, the error amount is significantly reduced. In this case, the RMS error is 0.026mm, which corresponds to a 90% of reduction with respect to the RMS error of the reconstructed surface without boundary detection. Figure 12 depicts the inspection process of our compound surface model on CMM (Zeiss).

5. Discussion and Conclusion

This paper proposed an improved reverse engineering technique, which minimizes the interpolation errors occurring in the vicinity of the boundary edge on compound surfaces. This approach consists of (1) detecting the boundary edges of compound surfaces using a vision camera system, (2) improving the accuracy of detected boundary data using an image-processing technique, (3) interpolating the boundary data by analytic and parametric curves, (4) dividing the compound surface into several elemental surfaces according to interpolated boundaries and (5) reconstructing sculptured surfaces from measured point data on the elemental surfaces.

To interpolate detected boundary data, we take into account analytic and parametric curves, which can be discriminated by the least square method. Hence, conic section curves and B-spline curves have been used in order to interpolate the detected boundary data by analytic and parametric curves. To reconstruct the sculptured surfaces from measured point data, we have used the B-spline surface skinning method.

Through experimentations, we have demonstrated the effectiveness of the reverse engineering technique proposed in this paper. Concretely, we have achieved about a 90% reduction of the RMS error of reconstructed surface through our approach.

This study is confined to the reconstruction of a unitary analytic curve or free-formed curve as a sharp edge. If the edges were a compound curve consisted of differently defined elemental curves, it would be very difficult to recognize exact geometric shapes by segmenting the compound curve. Theoretically, using the corner detection method (Medioni & Yasumoto; 1987), it seems possible to detect the corner between different segmented curves. However, unnecessary corners may be detected because the acquired image data include some noises, which can be treated like a corner. It is obvious that this problem will have to be resolved in the aspect of image-processing.

6. Acknowledgement

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