On the Approximate Solution of Aircraft Landing Gear Under Nonstationary Random Excitations

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The motion of an aircraft landing gear over a rough runway can be modeled by a nonclassically damped system subject to nonstationary random excitations. In this paper, the approximate analysis methods based on either the real or complex normal modes for the computation of nonstationary response covariances are proposed. It has been found by simulation involving a realistic example that, for the nonclassically damped random vibrational systems, the approximate solution method based on the complex normal mode is superior to other approaches with respect to the accuracy and computation time.

Key Words : Nonstationary Random Excitation, Aircraft Landing Gear, Surface Roughness, Complex Normal Modes, Real Normal Modes

1. Introduction

During take-off and landing, an aircraft landing gear is subjected to random excitations arising from the surface roughness of a landing strip (Yadav and Nigam, 1978 ; Soong and Mircea, 1993 ; Sobczyk et al., 1977 ; Virchis and Robson, 1978). The level of aircraft vibration due to such random excitations needs to be taken into account during structural design not only to guarantee the structural integrity but also to protect the cargo and on-board instrumentation. The analysis and design of an aircraft landing gear by applying the probabilistic approach can often yield better solution compared with the traditional factor-of-safety approach based on the deterministic design.

The surface roughness of a landing strip is transformed into a time varying random excitation by appropriate modeling of the aircraft motion and the assumed point contact between the aircraft wheel and the landing strip. Generally, the randomness of the surface roughness can be treated as a homogeneous random process. Under this condition, the surface excitation of the landing surface becomes a stationary random process for the case of a constant aircraft velocity, and a nonstationary random process for the case of the variable aircraft velocity (Caughey, 1963 ; Hammond and Harrison, 1981 ; Newland, 1975). Except for the initial landing stage during which a large impulsive load is acting, the aircraft landing gear can be modeled as a linear system. Therefore, the present study is concerned with the dynamic analysis of the landing gear in which a nonstationary random excitation is applied to the linearized model of the landing gear system.

When the excitation is a random process, the response covariance is usually obtained by using either the impulse response function or the frequency response function (Caughey, 1963 ; Hammond and Harrison, 1981 ; Newland, 1975 ; Crandall and Mark, 1963). For the case of nonstationary random process, however, these methods tend to be computationally inefficient.

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To overcome this deficiency, analytical methods based on the state-space approach have been proposed, and the methods have been applied to the dynamic analysis of the aircraft landing gear (Hammond and Harrison, 1981 ; Hwang et al., 1997a ; 1997b). The state-space approach is computationally more efficient than solving the integrations involved in the response function methods, and especially suited for obtaining the nonstationary random responses due to the variable aircraft velocity. The state-space approach can be divided into two steps : The first step involves deriving the covariance propagation equation corresponding to the state equations in the physical coordinate space. The second step involves solving the propagation equation through numerical analysis. The main advantage of the state-space approach is that the exact covariances can be obtained by directly solving the covariance propagation equations. Since these equations are in the matrix form, however, the amount of the computation rapidly increases as the degrees-of-freedom of the random system is increased. To address this problem, approximate analysis methods based on the modal analysis technique are proposed.

The methods for obtaining an approximate solution of nonstationary random vibration problems by applying the modal analysis technique can be divided into two main categories. The first method uses the real normal modes. The advantages of the method are that the computation is relatively simple and that the real coordinate space is involved. For the case of nonclassically damped vibrational systems, however, large computational errors may be generated. The reason is that to use the real normal modes, non-diagonal elements of the transformed modal damping matrix must be neglected. In addition, statistical correlations between the modal coordinate have been ignored. For proportionally damped systems, however, the error may be quite acceptable, as there is no non-diagonal element to be neglected in the first place, and the errors are due to ignoring statistical correlations only. The second method uses the complex normal modes. Since the off-diagonal elements of the transformed modal damping matrix are accounted for, the accuracy of the solution will be improved. The error generated here is solely due to ignoring the correlation between the modal coordinates. The additional computation due to applying the complex normal modes can be minimized by noting that the modes occur in the complex conjugates. For these reasons, the complex normal mode method is particularly suited for solving the nonclassically damped systems. In the present study, two approximation methods for solving the nonstationary random vibration problems are proposed. The methods are then applied to an example involving the aircraft landing gear. The approximate solutions are compared with the exact one. The proposed methodology can be applied to any dynamical systems with finite degrees-of-freedom subjected to random base excitations.

2. Modeling the Landing Gear and Landing Strip Surface

For dynamic analysis purpose, the aircraft landing gear can be modeled as a two degrees-of-freedom vibrational system composed of the upper spring-mass-damper corresponding to the aircraft structure and the shock absorber and the lower spring-mass-damper corresponding the wheel/tire assembly. The shock absorber for absorbing the landing impact can be modeled as a spring-damper. Depending on the range of motion, the shock absorber can either be modeled as a linear spring-damper or a nonlinear spring-damper. In the present study, the absorber is modeled as a linear system shown in Fig. 1 by assuming a small range of motion. The linearized spring and damping constants are derived from the specification for the landing gear adapted from KTX-1 aircraft used for the pilot training. The contact between the landing gear and the landing strip surface is modeled as a point contact. Neglecting all external forces except for the excitation due to the surface roughness, the equations of motion are given as follows:

$$M\ddot{y}+C\dot{y}+Ky=C_h\dot{h}+K_h h \tag{1}$$

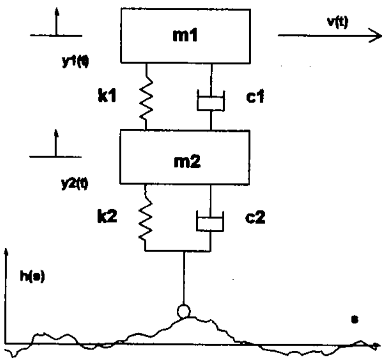

Fig. 1 Simplified model of aircraft landing gear

where $M=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $C=\begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1+c_2 \end{bmatrix}$,

$$K=\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix},$$

$$C_h=\begin{bmatrix} 0 \\ c_2 \end{bmatrix},\ K_h=\begin{bmatrix} 0 \\ k_2 \end{bmatrix},\ y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The variables in Eq. (1) are shown in Fig. 1. The lumped mass elements m_1 and m_2 denote the mass of the aircraft and wheel/tire assembly, respectively. The elements k_1 and c_1 denote linearized spring and damping coefficients of the shock absorber, while k_2 and c_2 denote linearized spring and damping coefficients of the tire. The surface profile of the landing strip is given by the variable $h(s)$.

The randomness of the landing strip surface can usually be assumed to be a homogeneous random process. Under this assumption, if the aircraft velocity is constant, the excitation from the rough surface becomes a stationary random process, whereas for the variable aircraft velocity the surface excitation becomes a nonstationary random process. From among the various statistical models of the aircraft landing strip, the most widely used model can be expressed by the equation (Virchis and Robson, 1978)

$$R_{hh}(\xi)=\sigma^2 e^{-a|\xi|},\ (\xi=s_1-s_2) \tag{2}$$

The surface roughness expressed by Eq. (2) can be attained by passing the white noise through a first-order shape filter

$$\frac{dh}{ds}+ah=k_w w(s) \tag{3}$$

where

$w(s)$ = white noise with zero mean

$E[w(s_1)w^T(s_2)]=Q\delta(s_1-s_2)$

$k_w=\sigma\sqrt{2a}$ (magnitude of the white noise)

σ = standard deviation of h

a = correlation parameter

s = space variable

The height $h(s)$ given by Eq. (3) is a homogeneous random process. To express a nonstationary random process $h(t)$ in the case of the variable aircraft velocity, Eq. (3) can be transformed by applying the chain rule

$$\dot{h}=-\dot{s}ah+\dot{s}k_w w(s(t)) \tag{4}$$

3. Analysis using the State-Space Method

The response analysis of the aircraft landing gear subjected to a nonstationary random excitation by the application of the state-space approach (Hwang et at., 1997a) is briefly described here. Expressing the landing gear and the surface of the landing strip in the state-space, the following equations are obtained. For convenience, all state variables have been normalized by σ, which is the standard deviation of h

$$\frac{d}{dt}\begin{bmatrix} \frac{y}{\sigma} \\ \frac{\dot{y}}{\sigma} \\ \frac{h}{\sigma} \end{bmatrix}=\begin{bmatrix} 0 & I & 0 \\ -M^{-1}K & -M^{-1}C & (-M^{-1}C_h\dot{s}a+M^{-1}K_h) \\ 0 & 0 & -\dot{s}a \end{bmatrix}\begin{bmatrix} \frac{y}{\sigma} \\ \frac{\dot{y}}{\sigma} \\ \frac{h}{\sigma} \end{bmatrix}$$

$$+\dot{s}\begin{bmatrix} 0 \\ M^{-1}C_h\sqrt{2a} \\ \sqrt{2a} \end{bmatrix} w[s(t)] \tag{5}$$

where the functional form of $s(t)$ is restricted so that it satisfies the condition $v(t)=\dot{s}(t)\geq 0$. Equation (5) can be expressed in the form of

$$\dot{x}(t)=Ax(t)+\dot{s}(t)Bw[s(t)] \tag{6}$$

where

$$x=\begin{bmatrix} \frac{y}{\sigma} & \frac{\dot{y}}{\sigma} & \frac{h}{\sigma} \end{bmatrix}^T$$

$$A=\begin{bmatrix} 0 & I & 0 \\ -M^{-1}K & -M^{-1}C & (-M^{-1}C_h\dot{s}a+M^{-1}K_h) \\ 0 & 0 & -\dot{s}a \end{bmatrix},$$

$$B=\begin{bmatrix} 0 \\ M^{-1}C_h\sqrt{2a} \\ \sqrt{2a} \end{bmatrix}$$

$$E[w(s_1)w^T(s_2)]=Q\delta\ (s_1-s_2) \tag{7}$$

The covariance propagation equation corresponding to equation (6) can be derived as follows (Hwang et al., 1997a)

$$\dot{P}=AP+PA^T+\dot{s}BQB^T \tag{8}$$

where the matrix P expresses covariance matrix for 5 state variables. The initial conditions applied in the present study come in two types. The first type corresponds to the case of the aircraft taking off from rest. The second type corresponds to the case of the aircraft moving with a constant velocity before coming in contact with the landing strip. For the purpose of analyzing the dynamic response of the system subjected to a nonstationary random input, the state-space method described here is more effective than the usual methods involving the impulsive response function or the frequency response function (Hwang et al., 1997a). Since the covariance propagation Eq. (8) is a matrix equation, however, the amount of computation rapidly increases for increased degrees-of-freedom of the system. For n state variables, the number of equations that needs to be solved is given by $n(n+1)/2$. To overcome this problem, an approximate method based on the modal analysis is proposed in the next section.

4. Approximate Method Based on the Real Normal Modes

In this section, a method for obtaining an approximate solution of the covariance for the random vibrational system (1) by using the real normal modes is presented. The method presented here combines the modal analysis technique and the state-space approach described in Sec. 3. By the orthogonality of the vibrational modes of the system, the equations of motion are decoupled in the modal coordinates. On each decoupled modal equation the state-space method can then be applied. In comparison with the method of Sec. 3 in which the covariance propagation Eqs. (8) are solved in the physical coordinates, the method proposed here achieves substantial savings in the computational load. However, after the covariances of the modal coordinates are computed, an inverse transformation needs to be performed to obtain the covariance for the physical coordinates. During the inverse transformation, the statistical correlations between the modal coordinates are usually neglected since they are not known. In this sense, the present method is an approximate method.

For the case of the proportionally damped random vibrational systems, the error generated by the approximation scheme is due to neglecting the statistical correlation between the modes only. For nonclassically damped system, however, the modal equations of motion cannot be decoupled. For the present method to be applicable, the off-diagonal elements of the transformed modal damping matrix need to be neglected. In this way, the additional approximation error is introduced by neglecting the off-diagonal terms on top of the error due to ignoring the statistical correlation the modal coordinates. Hence, the method expounded here is suitable for the proportionally damped systems. In the next section, a method based on the complex normal modes will be introduced for tackling the nonclassically damped vibrational systems.

Upon obtaining the real eigensolution of M, K matrices in the system Eq. (1), the modal matrix S can be constructed from the eigenvectors. The following coordinate transformation can then be performed

$$y=Sx. \tag{9}$$

Substituting Eq. (9) into Eq. (1) and premultiplying by S^T, the following equations in the modal coordinates can be obtained

$$\ddot{x}+\tilde{C}\dot{x}+\Lambda x=C_s\dot{h}+K_s h \tag{10}$$

where

$$S^TMS=I,\ S^TCS=\tilde{C}$$

$$S^TKS=\Lambda=\begin{bmatrix} w_1^2 & 0 & 0 \\ 0 & \vdots & 0 \\ 0 & 0 & w_n^2 \end{bmatrix}$$

$$C_s=\begin{bmatrix} C_{s1} \\ \vdots \\ C_{sn} \end{bmatrix}=S^TC_h,\ K_s=\begin{bmatrix} k_{s1} \\ \vdots \\ k_{sn} \end{bmatrix}=S^TK_h$$

If the system is proportionally damped, the modal damping matrix is given by

$$\tilde{C}=\begin{bmatrix} 2\zeta_1 w_1 & 0 & 0 \\ 0 & \vdots & 0 \\ 0 & 0 & 2\zeta_n w_n \end{bmatrix}$$

For the nonclassically damped system, however, the off-diagonal elements of the transformed modal matrix must be neglected for the application of the approximation method based on the real normal modes to be feasible. Applying the state-space approach on each of the modal coordinates in Eq. (10), the state equations can be written as follows

$$\begin{bmatrix} \frac{\dot{x}_i}{\sigma} \\ \frac{\ddot{x}_i}{\sigma} \\ \frac{\dot{h}}{\sigma} \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ -w_i^2 & -2\varsigma_i w_i & k_{si}-c_{si}\dot{s}\alpha \\ 0 & 0 & -\dot{s}\alpha \end{bmatrix}\begin{bmatrix} \frac{x_i}{\sigma} \\ \frac{\dot{x}_i}{\sigma} \\ \frac{h}{\sigma} \end{bmatrix} + \dot{s}\begin{bmatrix} 0 \\ c_{si}\sqrt{2\alpha} \\ \sqrt{2\alpha} \end{bmatrix} w(s(t)), \ (i=1,\ 2,\ \cdots,\ n) \quad (11)$$

By deriving the covariance propagation equation from the above equation and performing numerical integration, the covariance for i-th mode can be obtained. From the covariances for the modal coordinates, the covariance for the physical coordinates can be obtained though a suitable coordinate transformation

$$\frac{y}{\sigma}=S\frac{x}{\sigma}$$

$$\begin{bmatrix} \frac{y_1}{\sigma} \\ \vdots \\ \frac{y_n}{\sigma} \end{bmatrix}=\begin{bmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & \vdots & \vdots \\ s_{n1} & \cdots & s_{nn} \end{bmatrix}\begin{bmatrix} \frac{x_1}{\sigma} \\ \vdots \\ \frac{x_n}{\sigma} \end{bmatrix} \quad (12)$$

By taking the second moment of Eq. (12), the variance of $\frac{y}{\sigma}$ can be derived as follow

$$E\left\{\left[\frac{y_i}{\sigma}\right]^2\right\}=E\left\{\left[s_{i1}\frac{x_1}{\sigma}+s_{i2}\frac{x_2}{\sigma}+\cdots+s_{in}\frac{x_n}{\sigma}\right]^2\right\} \ (i=1,\ 2,\ \cdots,\ n)$$

By neglecting the correlation between the modal coordinates, the following equation can be obtained

$$E\left\{\left[\frac{y_i}{\sigma}\right]^2\right\}=s_{i1}{}^2E\left\{\left[\frac{x_1}{\sigma}\right]^2\right\}+s_{i2}{}^2E\left\{\left[\frac{x_2}{\sigma}\right]^2\right\}+\cdots+s_{in}{}^2E\left\{\left[\frac{x_n}{\sigma}\right]^2\right\} \quad (13)$$

The approximation method based on the real normal modes presented in this section is computationally very efficient. For the nonclassically damped systems, however, the modal coordinates are still coupled by the off-diagonal elements of the modal damping matrix. If the present method is applied to the nonclassically damped systems, the error due to neglecting the off-diagonal elements of the modal damping matrix on top of the error due to neglecting the statistical correlation between the modal coordinates means the total error could be quite substantial.

5. An Approximation Method Based on the Complex Normal Modes

In this section, an approximation method based on the complex normal modes that can be used to solve the nonclassically damped vibrational systems is introduced. The equations of motion given by Eq. (1) can be rewritten in the form of the following state equations, with $x_1=\dot{y}$, $x_2=y$

$$\begin{bmatrix} 0 & M \\ M & C \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ C_h \end{bmatrix}\dot{h}+\begin{bmatrix} 0 \\ K_h \end{bmatrix}h \quad (14)$$

Rewriting Eq. (14) in the expanded vectors and matrices,

$$M_s\dot{X}+K_sX=C_{hs}\dot{h}+K_{hs}h \quad (15)$$

where M_s, K_s, X, C_{hs}, K_{hs} can readily be ascertained by comparing Eqs. (14) and (15). In the proposed complex modal analysis, the complex eigensolutions of M_s and K_s in Eq. (15) are first computed. By then applying the modal matrix V composed of the complex eigenvectors, the following coordinate transformation can be performed

$$X=VQ \quad (16)$$

where Q denotes the complex modal coordinate vector. Applying Eq. (16) in Eq. (15) and rearranging,

$$V^TM_sV\dot{Q}+V^TK_sVQ=V^TC_{hs}\dot{h}+V^TK_{hs}h \quad (17)$$

where

$$V^T M_s V = I,\ V^T K_s V = \Omega = diag[w_1,\ w_2,\ \cdots,\ w_{2n}],$$
$$V^T C_{hs} = \Phi,\ V^T K_{hs} = \Psi$$

Equation (17) can be written in the simplified form of

$$\dot{Q} + \Omega Q = \Phi \dot{h} + \Psi h \qquad (18)$$

Since Ω is a diagonal matrix, Eq. (18) represents a set of decoupled equations in the modal coordinates. The state-space approach described in Sec. 3 can be individually applied on each complex modal coordinate, substantially reducing the amount of computation. Since Q, Ω, Φ, Ψ are all composed of the complex conjugate pairs, the number of equations that needs to be solved is only half of the total number of the modes

$$q_r = \bar{q}_{r+1},\ w_r = \bar{w}_{r+1},\ \phi_r = \bar{\phi}_{r+1},\ \Psi_r = \bar{\Psi}_{r+1} \quad (r = 1,\ 3,\ \cdots,\ 2n-1)$$

Reducing the number of Eq. in Eq. (18) in half and expanding the real and imaginary parts separately,

$$(\dot{q}_{rR} + i\dot{q}_{rI}) + (w_{rR} + iw_{rI})(q_{rR} + iq_{rI}) = (\phi_{rR} + i\phi_{rI})\dot{h} + (\Psi_{rR} + i\Psi_{rI})h \qquad (19)$$

$$\dot{q}_{rR} + w_{rR}q_{rR} - w_{rI}q_{rI} = \phi_{rR}\dot{h} + \Psi_{rR}h \quad \text{Real parts} \qquad (20)$$

$$\dot{q}_{rI} + w_{rI}q_{rR} + w_{rR}q_{rI} = \phi_{rI}\dot{h} + \Psi_{rI}h \quad \text{Imaginary parts} \quad (r = 1,\ 3,\ \cdots,\ 2n-1)$$

In Eqs. (19) and (20), subscripts R and I denote the real and imaginary parts, respectively. By applying the state-space approach, the following set of state equations for the individual modes can be obtained.

$$\begin{bmatrix} \frac{\dot{q}_{rR}}{\sigma} \\ \frac{\dot{q}_{rI}}{\sigma} \\ \frac{\dot{h}}{\sigma} \end{bmatrix} = \begin{bmatrix} -w_{rR} & w_{rI} & \Psi_{rR} - \phi_{rR}\dot{s}\alpha \\ -w_{rI} & -w_{rR} & \Psi_{rI} - \phi_{rI}\dot{s}\alpha \\ 0 & 0 & -\dot{s}\alpha \end{bmatrix} \begin{bmatrix} \frac{q_{rR}}{\sigma} \\ \frac{q_{rI}}{\sigma} \\ \frac{h}{\sigma} \end{bmatrix} + \dot{s} \begin{bmatrix} \phi_{rR}\sqrt{2\alpha} \\ \phi_{rI}\sqrt{2\alpha} \\ \sqrt{2\alpha} \end{bmatrix} w(s(t)) \qquad (21)$$
$$(r = 1,\ 3,\ \cdots,\ 2n-1).$$

By first deriving and then solving the covariance propagation equations, the covariances on r-th modal coordinate can be obtained, i. e., the following quantities can be computed

$$E\left[\frac{q^2_{rR}}{\sigma^2}\right],\ E\left[\frac{q_{rR}q_{rI}}{\sigma^2}\right],\ E\left[\frac{q_{rR}h}{\sigma^2}\right]$$
$$E\left[\frac{q^2_{rI}}{\sigma^2}\right],\ E\left[\frac{q_{rI}h}{\sigma^2}\right],\ E\left[\frac{h^2}{\sigma^2}\right]$$

Utilizing the fact that q_r, w_r, ϕ_r, Ψ_r $(r = 1,\ 3,\ \cdots,\ 2n-1)$ possess the complex conjugates, the covariances for complex modal coordinates can be used to compute the covariances for the physical coordinates by the following transformation

$$X = W_{RI}Q_{RI} \qquad (22)$$

where

$$Q_{RI} = [q_{1R}\ q_{1I}\ q_{3R}\ q_{3I}\ \cdots\ q_{(2n-1)R}\ q_{(2n-1)I}]^T$$
$$W_{RI} = [V_{1RI}\ V_{3RI}\ \cdots\ V_{(2n-1)RI}]$$
$$V_{rRI} = 2[\mathrm{Re}\,V_r - \mathrm{Im}\,V_r] \quad (r = 1,\ 3,\ \cdots,\ 2n-1)$$

V_r denotes r-th complex eigenvector. If the statistical correlations between the complex modal coordinates are neglected, the covariances for the physical coordinates can be expressed in a form similar to Eq. (13). Since the approximation based on the complex normal modes presented in this section does not entail any error other than that due to neglecting the statistical correlation between the modes, it can be particularly suitable for obtaining the solutions of the nonclassically damped systems. In comparison with the method based on the real normal modes, the present method is clearly superior with respect to the accuracy without any increase in the computational effort required.

6. Results and Discussion

To demonstrate the analytical results developed in the previous sections, the approximation methods are applied to an example problem involving an aircraft landing gear subjected to a nonstationary random excitation. The parameters given for the landing gear are based on KTX-1 trainer aircraft. The parameters are summarized in Table 1. The initial condition applied here corresponds to the case of the aircraft taking off from rest.

During the execution of the numerical algorithm, the coefficient α representing the surface roughness of the runway has been set at 0.5. The greater the value of, the closer the randomness of the surface to the white noise. Assuming the constant acceleration, the velocity $v(t)$ is given by the function $10t$.

To evaluate the effectiveness of the proposed methods, the covariances of the response have been computed for each of the three solution methods. They are : the exact analysis in the physical coordinates by applying the state-space approach ; the approximate method based on the real normal modes ; and the approximate method based on the complex normal modes. For each case, the covariances of the response of the aircraft landing gear subjected to a nonstationary random excitation are computed. The comparison of the results for the three cases is also provided. Since the first method in the physical coordinates yields the exact solution for linear random vibrational systems, it can be used to evaluate the errors due to the two approximation methods. The landing gear considered in this section is readily seen to be a nonclassically damped system. Based on the previous analysis, the method using the real normal modes can be expected to yield greater error than the method using the complex normal modes.

Table 1 Parameters of an aircraft landing gear

Parameter Name	Numerical Value(SI)
Sprung Mass	1189.2(kg)
Unsprung Mass	19.1(kg)
Stiffness Coefficient of Absorber	57831.6(N/m)
Stiffness Coefficient of Tire	900000(N/m)
Damping Coefficient of Absorber	6455.9(Ns/m)
Damping Coefficient of Tire	0(Ns/m)

The aircraft displacement covariances are computed and shown in Fig. 2. A substantial discrepancy exists between the result of the approximate method based on the real normal modes and the exact solution. In contrast, the solution based on the complex normal modes almost coincides with the exact solution. The wheel/tire vertical displacement covariances are computed and shown in Fig. 3. Again, the complex normal mode solution lies between the normal mode solution and the exact solution. The reason is, as mentioned before, that the landing gear is a nonclassically damped system. The com-

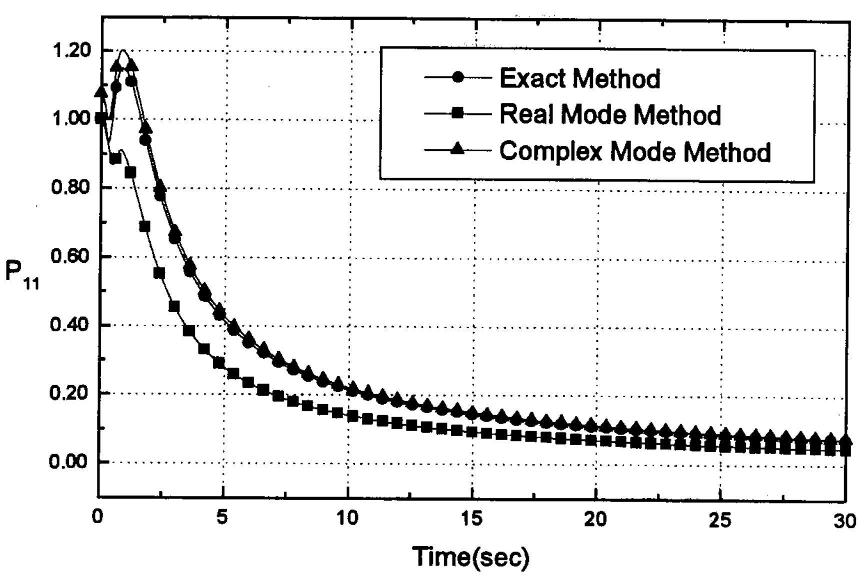

Fig. 2 Comparison of covariance of aircraft displacement (y_1)

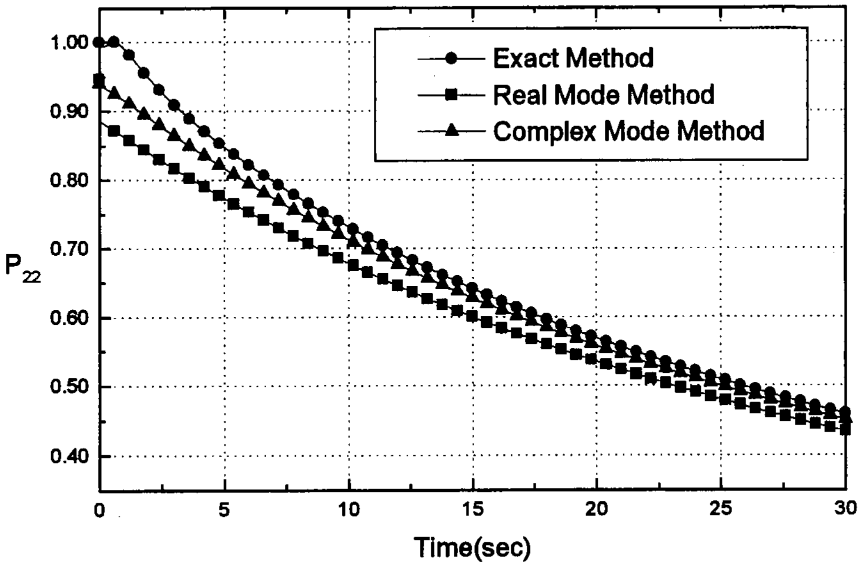

Fig. 3 Comparison of covariance of wheel/tire displacement(y_2)

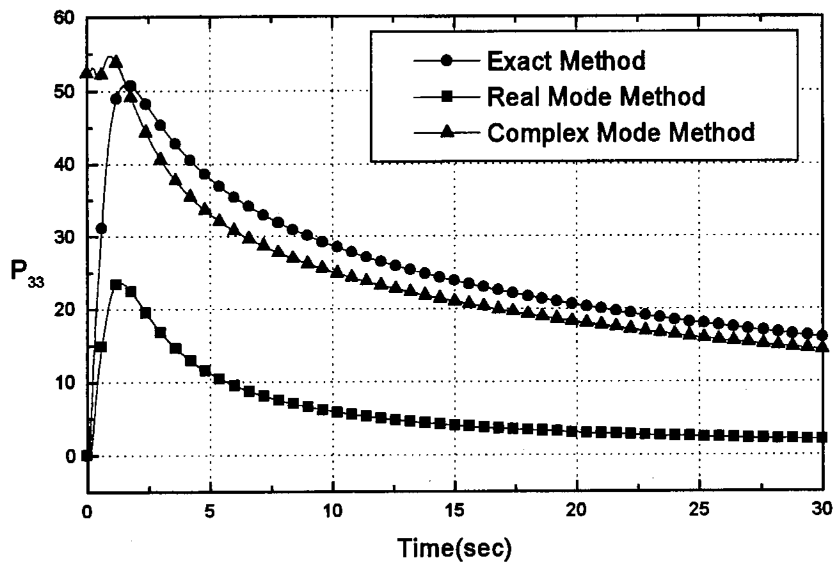

Fig. 4 Comparison of covariance of aircraft velocity($\dot{y}_1$)

parison of the aircraft body velocity covariance is provided in Fig. 4. The real normal mode solution shows a large deviation from the exact solution. In marked contrast, the complex normal mode solution is much closer to the exact solution. To obtain the real normal mode solution of the nonclassically damped landing gear system, the off-diagonal elements of the transformed modal damping matrix had to be neglected. The resulting error is especially large for P_{33} (t). Indeed, this particular case reserved to motivate the authors to develop an approximate analysis based on the complex normal mode. Finally, the wheel/tire assembly velocity covariances are presented in Fig. 5. Although the errors are small, the results qualitatively similar to the previous cases are observed.

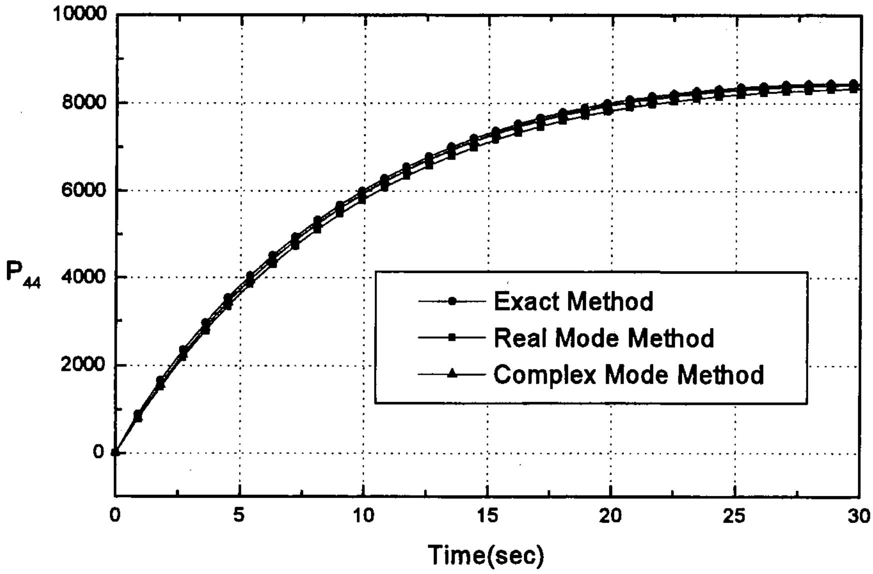

Fig. 5 Comparison of covariance of wheel/tire velocity ($\dot{y}_2$)

7. Conclusions

The methods for obtaining approximate solutions of the aircraft landing gear dynamics are proposed. The exact analysis method in the physical coordinates based on the application of the state-space approach entails rapidly increasing computational load as the degrees of freedom of the system is increased. The approximate solution methods proposed in the present study are developed in the modal coordinate space and are computationally efficient for systems possessing high degrees of freedom. The exact and approximate analyses are applied to an example involving KTX-1 trainer aircraft, and their results compared. Although the approximate analysis based on the real normal modes can be quite effective for solving the proportionally damped systems, large approximation error may develop for the nonclassically damped systems. For such systems, the approximate analysis based on the complex normal modes is more adequate. This method improves on the approximation error and yet entails essentially the same computational load as the method based on the real normal modes. The improved accuracy stems from the fact that the off-diagonal elements of the transformed modal damping matrix are accounted for. The amount of computation is similar to the real mode analysis since the number of the complex modal coordinates that need to be solved is cut in half by utilizing the property that the modes occur in the complex conjugate pairs. By a realistic example, the approximate method based on the complex normal modes has shown to be highly effective for solving the nonclassically damped random vibrational systems.

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