Supervisory Control of Multi-Echelon Production-Distribution Systems with Limited Decision Policy(I)

— Control Algorithm —

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In industrial production-distribution systems, production and purchasing rates, associated inventories, and sales are very critical for the profits of each component in the system. The objective of this study is to investigate an effective information control scheme for a production-distribution system by automatic feedback control techniques. In this work, a dynamic control scheme that has an integrated-error with state-feedback and filtering(ISFF) is proposed as a new algorithm for a dynamic controller. Generalized formulations of the dynamic control are proposed in the continuous-time and discrete-time cases. A methodology for an evaluation of ISFF controller gains using the eigen structure property is presented. When an upper-limit is imposed on the production capability by available factory space and capital equipment, supervisory control is provided to avoid integrator-windup and deterioration of system performance.

Key Words : Supervisory Control, Dynamic Controller, Actuator Saturation, Reset Windup, Multi-Echelon Production-Distribution Systems

1. Introduction

Of central significance in many industrial companies is the information management of production and distribution systems. A recurring problem is to match the production rates to the rate of final consumer sales. It is well known that the factory production rate often fluctuates more widely than does the actual consumer purchasing rate. It has frequently been observed that a distribution system of cascaded inventories and ordering procedures amplify small disturbances which occur at the retail level.

A schematic structure of a multi-echelon production-distribution system which consists of three echelons — retailer, distributor, and factory — is shown in Fig. 1. Demand from the customer market results in orders which enter the system at the lowest echelon. The demand is satisfied from inventory located at the retail sector. As the inventory is out of stock, orders for goods are placed to the distributor. This process of ordering from the above stocking point continues up the line to the top echelon which is supplied from the source or factory. The circled lines show the upward flow of orders for goods, while the solid lines represent the shipment of goods flowing downward. One notes that three levels of inventory exist : factory, distributor, and retailer. One component of the orders being processed is necessarily proportional to the average level of business activity and to the length of lead-time required to fill an order. Both an increased sales volume and an increased delivery lead-time give rise to increased total orders in the supply line (Forrester, 1961).

Production-distribution systems require con-

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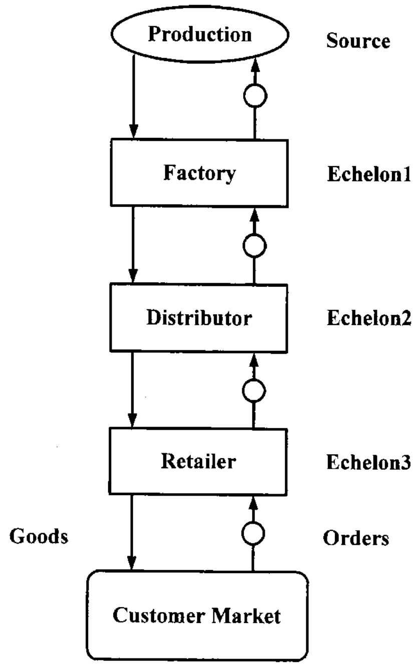

Fig. 1 Schematic structure of multi-echelon production-distribution system

sideration of the flow of goods. The material flow is controlled by orders representing decisions based on information about inventories and sales. Because of the critical importance of purchasing or manufacturing decision to total procurement costs and inventory costs, it is desired to maintain a relatively stable production policy rather than going through hiring and lay-off cycles as sales fluctuate. At the same time unnecessary fluctuations in inventory levels are to be avoided (Christensen 1971).

Conventional studies in the analysis of production-distribution systems is to conceptualize the system as one in which three types of inventory costs are subject to control : carrying cost, shortage cost, and procurement cost. The objective of production-distribution system analysis is to minimize the sum of each cost component (Meyer 1972). However, for certain kinds of complicated production-distribution systems, an additional purpose of analysis is to investigate the behavior of the system as a function of time, as well as to minimize the aggregate of inventory costs. In studying complex production-distribution systems, one is frequently interested as much in the dynamic stability and frequency response of the system as in optimality when the system is subjected to parametric uncertainties. Most of the research associated with this kind of production-distribution system using continuous linear system analysis has been accomplished by utilizing an output feedback control law, which is a proportional control based upon the difference between the desired inventory and the actual inventory, to decide a purchasing policy or manufacturing policy.

The motivation of this work is to demonstrate in a comprehensive manner the application of modern control techniques to a class of production-distribution system problems. Generally, this kind of dynamic control gives stability robustness and good performance in the perturbed system as well as the nominal plant (Jeong 1994c, 1996).

On the other hand, in many practical situations, production-distribution systems are in fact controlled by piecewise-constant policies with an upper-limit, which is one of the critical nonlinear characteristics, imposed on production capability by available factory space and capital equipment. The effect of limited capital equipment can be demonstrated by simply restricting permissible factory production. This could be done by imposing an upper-limit on the rate at which manufacturing orders can be sent to the factory (Forrester 1961). Saturation of decision policies not only deteriorates control system performance but also can lead to a large overshoot, inducing a limit cycle or an unstable oscillation in the output response. In the presence of unusually large initial conditions, plant disturbances, or parametric system uncertainties, it is well known that reset-windup appears when controllers with integral action are used with saturating actuators and may lead to a large overshoot in the system response (Krikelis 1984). The major motivation of this study is focused on the investigation of a dynamic information control scheme of multi-echelon production-distribution systems with saturated decision policy to guarantee anti-reset-windup and stability robustness as well as good performance for disturbance rejection.

2. Literature Survey

2.1 Control of production-distribution systems

Generally, it is known that an efficient production-distribution system can only be designed and operated if the dynamic behavior of the constituent parts is properly understood. Two approaches to the deeper understanding of the dynamic behavior of production-distribution systems have appeared in the literature. The older established approach uses linear control theory to single or multiple systems. There are many variants to this analysis approach, first introduced by Simon (1952). Simon applied Laplace transform concepts to a single loop continuous time system of the production control problem. The recent control theory based on the state-space representation is also applicable. An early contribution was by Christensen and Brogan (1971), who optimized a simulation model describing the manufacturing process of automobile gearboxes. Meyer and Groover (1972) have developed continuous systems analysis and simulation in simple multi-echelon inventory systems using output feedback control, although general results were not described. Since then, a wide number of related techniques have been introduced, chiefly by Brian Porter and his associates at the University of Salford (1974, 1975, 1976). Using full-state feedback control in production-distribution systems, Tao and Zunde (1986) proposed a stochastic optimal control approach to a class of production and inventory problems with a known disturbance. Recently, Towill (1982) accomplished a dynamic analysis of an inventory and order-based production control system based on Laplace transfer function analysis. A few years later, Vecchio and Towill (1990) suggested a knowledge-based simulation framework for the simple two-echelon production-distribution system and implemented a reference model at the factory level using a output feedback scheme. The alternative approach is to use a special methodology originally called Industrial Dynamics by the originator, Jay W. Forrester (1961), but now often referred to as System Dynamics (Roberts 1978; Legasto et al. 1980), Management Dynamics (Coyle 1977), or Urban Dynamics (Alfeld and Graham 1976). Essentially, industrial dynamics involves the solution of the dynamical equations of the model including nonlinear effects and time-varying systems (Forrester 1961, 1975). However, in those approaches, there is not sufficient information about the system behavior 1) when the production-distribution systems have uncertainties and 2) when upper-limits are imposed on productive capacity.

The objective of this work is 1) to describe a control scheme to improve performance robustness of uncertain systems and 2) to show how to handle systems with decision variable constraints in a feedback control framework.

2.2 Dynamic control

In the design of control systems, it is necessary to eliminate the effect of offset errors caused by bounded disturbances. Integral action on the dynamic controllers results in a closed-loop system in which the outputs follow step commands and reject unmeasurable arbitrary disturbances with bounded constant values. The stabilizing effect of the integral control can be supplemented by appropriate state-feedback action so that one can achieve a satisfactory transient response as well as the desired zero steady-state error. The pseudo-derivative feedback (PDF) control, a dynamic control with integral action, was introduced by Phelan (1977), who suggested that the PDF controller constitutes an optimum scheme for all types of plants. The PDF control scheme of n-th order plant consists of one integrator in the feed-forward loop with (n-1)-th order derivatives in the feedback loop. PDF demonstrates very good performance when utilized with certain low order systems but encounters serious noise effects in higher order systems. Seraji (1979) has applied PI-type controllers for multivariable systems, and Krikelis (1982) has developed the PDF control scheme for 4th-order tracking problems with two PDF controllers in series included in one derivative term in the feedback loop. In that system, controller parameters have to be tuned by trial and error procedures. Maday (1987) formalized the Integrated-Error with State-Feedback (IESF) control scheme by a closed-loop pole-placement technique in hybrid control systems. It is an extension of PDF without the derivative term in the feedback loop. Maday and Johnson (1989) applied IESF control for a decentralized noncollocated active vibration control systems. Recently, Aida and Kidamori (1990) designed an optimal servo-system by a classical PI-type state-feedback control. They adopted an identity transfer function between the PI-type state-feedback control and the conventional full-state feedback to optimize a servo system.

In this work, generalized formulations about dynamic controllers are provided in the continuous-time and discrete-time case. Moreover, a new methodology for an evaluation of ISFF controller gains using the Jordan-Canonical form is employed to simplify the calculations.

2.3 Saturated system control

In practical control systems, the dynamic range of actuators is limited (or saturated) when the actuators are driven by sufficiently large signals. This gives rise to a nonlinearity as a result of actuator saturation. For example, the upper limit is imposed on productive capability by available factory space and capital equipment. A saturating actuator may lead to not only a large overshoot but also deterioration of performance. It is well-known that reset-windup appears when regulators (with integral action) are used with saturating actuators, and the reset-windup may lead to a large overshoot in the system response (Wang and Chen 1988; Glattfelder and Schaufelberger 1983; Astrom and Wittenmark 1984). Eventually, a large overshoot results in a limit cycle or an unstable oscillation in the output response (Su 1989, 1990). Glattfelder and Schaufelberger (1983) have proposed an anti-reset windup circuit to avoid the windup problem by adding a signal limiter on nonlinear feedback to the regulator. Another attempt to a limiting the feedback-output signal of integrators, called the *intelligent* integrator, has been proposed by Krikelis (1980). Maday and Jeong (1994) developed a supervisory controller to avoid reset windup and improve system performance by implementing a nominal reference model for a simple production model with saturated purchasing rate.

In this work, supervisory control is extended to a discrete-time dynamic control. Moreover, the characteristics of robustness in the supervisory control for saturated systems with uncertainties is investigated.

3. Problem Formulation of Dynamic Control

Let us consider a linear time invariant (LTI) dynamic system as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad x(0) = x_0 \quad (1)$$

$$y(t) = Dx(t) \quad (2)$$

where $x(t) \in R^n$ is the state of the plant, $u(t) \in R^n$ is the control input to the plant, and $y(t) \in R^l$ is the output of the plant. It is assumed that (A, B) is controllable, and (A, D) is observable. A, B, and D are real matrices whose sizes are appropriate to each system, matrix B being of rank m and D of rank l. If the plant is controlled by a continuous-time controller with q-th order error dynamic, a generalized feedback and feedforward control law is described by

$$\dot{x}_r(t) = Fx_r(t) + Gx(t) + Pr(t) \quad (3)$$

$$u(t) = Rx_r(t) - Qx(t) + Cr(t) \quad (4)$$

where $x_r(t) \in R^q$ is the state vector of the dynamic controller of order q, $\alpha(t) \in R^v$ is a reference input, and F, G, P, R, Q, and C are matrices of appropriate dimensions. Pure integrators or filters can be included in Eq. (3). The system Eq. (1) augmented by Eq. (3) and Eq. (4) is

$$\begin{bmatrix} \dot{x}_r(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} F & G \\ BR & A-BQ \end{bmatrix} \begin{bmatrix} x_r(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} P \\ BC \end{bmatrix} r(t) \quad (5)$$

Next, consider a continuous-time plant controlled by a discrete-time controller with sampling time T. The plant can be discretized by

$$x(kT+T) = \Phi(T)x(kT) + \Theta(T)u(kT), \quad x(0) = x_o \quad (6)$$

where

$$\Phi(T) = e^{AT}$$

$$\Theta(T) = \int_0^T \Phi(T-\tau) B \, d\tau \quad k = 0, \cdots, \infty$$

and $u(kT)$ is piecewise constant.

A generalized discrete-time controller is represented by

$$x_r(kT+T) = x_r(kT) + T\{Fx_r(kT) + Gx(kT) + \Pr(kT)\} \quad (7)$$

$$u(kT) = Rx_r(kT) - Qx(kT) + Cr(kT) \quad (8)$$

The closed-loop system combining the controller dynamics can be written as

$$\begin{bmatrix} x_r(kT+T) \\ x(kT+T) \end{bmatrix} = \begin{bmatrix} I+TF & TG \\ \Theta R & \Phi - \Theta Q \end{bmatrix} \begin{bmatrix} x_r(kT) \\ x(kT) \end{bmatrix} + \begin{bmatrix} TP \\ \Theta C \end{bmatrix} r(kT) \quad (9)$$

where $x(kT) \in R^n$ is the state vector of the plant, and $x_r(kT) \in R^q$ is the state vector of the dynamic controller.

Dynamic systems including a dynamic controller as well as any conventional controllers, full-state feedback or output feedback, can be described by Eq. (5) and Eq. (9). The ISFF dynamic control scheme is introduced in the next section, which consists of at least one PDF controller, with a full-state feedback, and a first-order filter.

4. Integrated Error with State-Feedback and Filtering (ISFF)

We are concerned with a class of control problems for which the control system should satisfy the following conditions (Krikelis 1982):

a) the steady-state error for a desired constant reference input should be zero ;

b) the effect of constant but unknown disturbances should be completely removed from the steady-state output;

c) the output should not be susceptible to system parametric uncertainties, while conditions (a) and (b) continue to be valid.

One of the design methodologies to meet the specifications (a) through (c) is ISFF control. It is constructed by a serial connection of at least one PDF controller, with full-state feedback, and a first-order filter. If the closed-loop system is stable, ISFF control rejects step disturbance (input or output) due to the integrator in the feed-forward loop. This is seen readily in the construction of total transfer function. Figure 2 shows a block diagram of a typical single-input single-output (SISO) ISFF control in the n-th order plant.

Selecting F, G, R, and Q corresponding to the continue-time ISFF control, the total system becomes

$$\dot{x}_c^s(t) = A_c^s x_c^s(t) + B_c^s r(t), \quad x_c^s(0) = x_{c0}^s \quad (10)$$

where

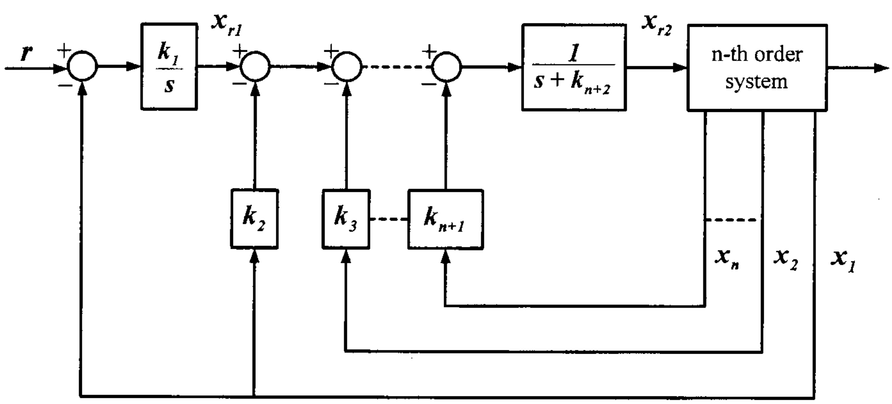

Fig. 2 ISFF control of n-th order plant.

$$A_c^s=\begin{bmatrix} 0 & 0 & -k_1 & 0 & \cdots & 0 \\ 1 & -k_{n+2} & -k_2 & -k_3 & \cdots & -k_{n+1} \\ 0 & b_1 & a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & b_n & a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$B_c^s=[k_1 \ \ 0 \ \cdots \ 0]^T_{1\times(n+2)}$$

$$x_c^s(t)=[x_{r1} \ x_{r2} \ x_1 \cdots x_n]^T$$

and x_{ri} is the auxiliary state vector of ISFF, for $i=1$, 2 and k_i are ISFF controller gains, for $i=1$, $\cdots$, $n+2$. The order of the total system controlled by ISFF control is increased by two for the n-th order subsystem. The controller gains k_1, $\cdots$ k_{n+2} can be obtained by pole-placement at the desired location in the s-plane. In this system, we can easily determine controller gains by using techniques described in the next section. Symbolic manipulation packages need not be used in these higher-order systems to calculate the gains.

If the subsystem is controlled by discrete-time ISFF control, the total system can be expressed by

$$x_d^s(kT+T)=A_d^s x_d^s(kT)+B_d^s r(kT), \quad x_d^s(0)=x_{d0}^s \tag{11}$$

where

$$A_d^s=\begin{bmatrix} 1 & 0 & -k_1T & 0 & \cdots & 0 \\ 1 & 1-k_{n+2}T & -k_2T & -k_3T & \cdots & -k_{n+1}T \\ 0 & \theta_1 & \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \theta_n & \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}$$

$$B_d^s=[k_1T0 \ \cdots \ 0]^T_{1\times(n+2)}$$

$$x_d^s(kT)=[x_{r1} \ x_{r2} \ x_1 \cdots x_n]^T$$

The ISFF controller gains can be obtained by pole-placement at the desired location in the z-plane. The general formulation for obtaining ISFF controller gains is derived in the next sections.

5. Evaluation of ISFF Controller Gains

The ISFF controller gains in Eq. (10) and Eq. (11) can be determined with the aid of the Jordan-Canonical form. If the total system matrix M_t has real multiple-order poles at one point of n-th order, the Jordan Canoncal form is given by the following transformation:

$$\Lambda=P^{-1}M_tP \tag{12}$$

where

$$\Lambda=\begin{bmatrix} \lambda & 1 & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \lambda & 1 \\ 0 & \cdots & \cdots & \cdots & 0 & \lambda \end{bmatrix}$$

$$P=[P_1 \ \cdots \ P_n]$$

with λ a multiple-order pole in the s-domain.

Rewriting Eq. (12),

$$(\lambda I-M_t)P_k=-P_{k-1}, \text{ for } k=0, \cdots, n \tag{13}$$

where $P_o=0$

Applying Eq. (13) for ISFF control, which increases the total order of the closed-loop system by 2, the following equation with (n+2)-th order is obtained:

$$\begin{bmatrix} \lambda & 0 & k_1 & \cdots & 0 \\ -1 & \lambda+k_{n+2} & k_2 & \cdots & k_{n+1} \\ 0 & -b_1 & \lambda-a_{11} & \cdots & -a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -b_n & -a_{n1} & \cdots & \lambda-a_{nn} \end{bmatrix}\begin{bmatrix} P_{1j} \\ P_{2j} \\ \vdots \\ \vdots \\ P_{(n+2)j} \end{bmatrix}=\begin{bmatrix} -P_{1(j-1)} \\ -P_{2(j-1)} \\ \vdots \\ \vdots \\ -P_{(n+2)(j-1)} \end{bmatrix} \tag{14}$$

where $P_{m0}=0$, for $m=1, \cdots, n+2, j=1, \cdots, n+2$.

We use the following submatrix to determine P_{1j}, P_{2j}, $\cdots$, P_{n+2j}:

$$E_cP_j^c=P_{j-1}^c \tag{15}$$

where

$$E_c=\begin{bmatrix} -b_1 & \lambda-a_{11} & \cdots & -a_{1(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n1} & -a_{(n-1)1} & \cdots & \lambda-a_{(n-1)(n-1)} \\ -b_n & -a_{n1} & \cdots & -a_{n(n-1)} \end{bmatrix}$$

$$P_j^c=\begin{bmatrix} P_{2j} \\ P_{3j} \\ \vdots \\ \vdots \\ P_{(n+1)j} \end{bmatrix}$$

$$P_{j-1}^c=\begin{bmatrix} -P_{3(j-1)}+a_{1n}P_{(n+2)j} \\ -P_{4(j-1)}+a_{2n}P_{(n+2)j} \\ \vdots \\ \vdots \\ -P_{(n+2)(j-1)}-\lambda+a_{nn}P_{(n+2)j} \end{bmatrix}$$

Setting one of the eigenvector elements to be equal to unity and choosing $P_{(n+2)j}=1$, then P_{1j}, P_{2j}, $\cdots$, $P_{(n+2)j}$ can be obtained easily by solving Eq. (15), assuming E_c is not singular after a multiple-order eigenvalue is placed in the desired location. From the first row of Eq. (14), we obtain

$$\lambda P_{1j}+k_1P_{3j}=-P_{1(j-1)}$$

Rearranging the above equation

$$P_{1j}=-k_1\alpha_j \tag{16}$$

where

$$\alpha_j=\frac{1}{\lambda}(-\alpha_{j-1}+P_{3j}),\ \text{for } j=1,\ \cdots,\ n+2,\ \alpha_0=0$$

Since P_{3j} is already known from Eq. (15), P_{1j} can be determined by Eq. (16).

Next, consider the second row of Eq. (14):

$$-P_{1j}+(\lambda+k_{n+2})P_{2j}+k_2P_{3j}+\cdots$$
$$+k_{n+1}P_{(n+2)j}=-P_{2(j-1)} \tag{17}$$

Substituting Eq. (16) into Eq. (17)

$$\alpha_jk_1+P_{3j}k_2+P_{4j}k_3+\cdots+P_{(n+2)j}k_{n+1}$$
$$+P_{2j}k_{n+2}=-\lambda P_{2j}-P_{2(j-1)} \tag{18}$$

Rewriting Eq. (18), the resulting linear equation becomes

$$[\alpha_j\ P_{3j}\ P_{4j}\ \cdots\ P_{(n+2)j}\ P_{2j}]\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n+2} \end{bmatrix}$$
$$=[-\lambda P_{2j}-P_{2(j-1)}] \tag{19}$$

This process is done for $j=1,\ \cdots,\ n+2$. Finally, the ISFF controller gains $k_1,\ \cdots,\ k_{n+2}$ can be determined by a simple inversion of the matrix obtained from $j=1,\ \cdots,\ n+2$.

If the system is controlled by a discrete-time ISFF controller, the discrete-time controller gains $k_1,\ \cdots,\ k_{n+2}$ can be determined by a similar manner. When multiple-order poles are placed at the desired location in the z-domain, the generalized expression for a discrete-time ISFF control is denoted as follows:

$$\begin{bmatrix} z_p-1 & 0 & k_1T & \cdots & 0 \\ -T & z_p-1+k_{n+2}T & k_2T & \cdots & k_{n+1}T \\ 0 & -\theta_{11} & z_p-\phi_{11} & \cdots & -\phi_{1(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\theta_{n1} & -\phi_{n1} & \cdots & z_p-\phi_{nn} \end{bmatrix}\begin{bmatrix} P_{1j} \\ P_{2j} \\ \vdots \\ \vdots \\ P_{(n+2)j} \end{bmatrix}=\begin{bmatrix} -P_{1(j-1)} \\ -P_{2(j-1)} \\ \vdots \\ \vdots \\ -P_{(n+2)(j-1)} \end{bmatrix} \tag{20}$$

where

$P_{m0}=0,\ m=1,\ \cdots,\ n+2,\ j=1,\ \cdots,\ n+2$

z_p=multiple-order pole in the z-domain.

The submatrix of the above equation can be expressed by

$$E_dP_j^d=P_{j-1}^d \tag{21}$$

where

$$E_d=\begin{bmatrix} -\theta_{11} & z_p-\phi_{11} & \cdots & -\phi_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ -\theta_{(n-1)1} & -\phi_{(n-1)1} & \cdots & -\phi_{(n-1)(n-1)} \\ -\theta_n & -\phi_{n1} & \cdots & -\phi_{n(n-1)} \end{bmatrix}$$

$$P_j^d=\begin{bmatrix} P_{2j} \\ P_{3j} \\ \vdots \\ \vdots \\ P_{(n+1)j} \end{bmatrix}$$

$$P_{j-1}^d=\begin{bmatrix} -P_{3(j-1)}+\phi_{1n}P_{(n+2)j} \\ -P_{4(j-1)}+\phi_{2n}P_{(n+2)j} \\ \vdots \\ \vdots \\ -P_{(n+2)(j-1)}-z_p+\phi_{nn}P_{(n+2)j} \end{bmatrix}$$

In a similar manner, setting $P_{(n+2)j}$ to be equal to 1, for $j=1,\ \cdots,\ n+2$, Eq. (21) gives $P_{2j},\ \cdots,\ P_{(n+2)j}$. From the first and second rows of Eq. (20), the consequent expression is

$$T\cdot[\beta_j\ P_{3j}\ \cdots\ P_{(n+2)j}\ P_{2j}]\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n+2} \end{bmatrix}$$
$$=[-(z_p-1)P_{2j}-P_{2(j-1)}] \tag{22}$$

where $P_{20}=0,\ \beta_0=0$, for $j=1,\ \cdots,\ n+2$

$$\beta_j=(-\beta_{j-1}+T\cdot P_{3j})/(z_p-1)$$

By inverting the coefficient matrix obtained from $T\cdot[\beta_j\ P_{3j}\ \cdots\ P_{(n+2)j}\ P_{2j}]$ of Eq. (22) for $j=1, \cdots, n+2$, the discrete-time ISFF controller gains $k_1, \cdots, k_{n+2}$ can be determined easily.

6. Supervisory Control Scheme for Saturated System

A supervisor implementation is introduced to guarantee good performance in the saturation region and to prevent reset windup. The basic idea is to construct a supervisor of the nominal plant such that it is an unconstrained model of the plant. The supervisor technique to control the system with actuator saturations assures good performance as well as a less conservative stability even when the saturating system has large changes in a error. Bounded-input bounded-output stability condition for supervisory control is derived by Jeong(1992).

Let the n-th order LTI system with actuator saturations be

$$\dot{x}(t)=Ax(t)+B\ sat\ u(t),\ x(0)=x_0 \quad (23)$$

where $x(t)\in R^n$ is the state of the plant, $u(t)\in R^m$ is the control input to the actuators, and $sat\ u(t)\in R^m$ is the saturated control input to the plant. It is assumed that (A, B) is controllable, and A and B are real matrices whose size is appropriate to each system.

The saturation function is defined as follows (Fig. 3):

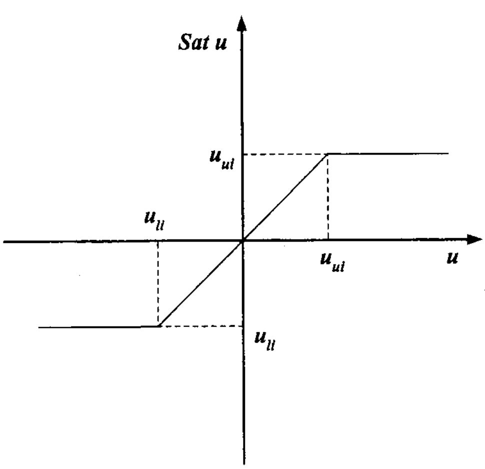

Fig. 3 Actuator saturation function

$$sat\ u(t)=[sat\ u_1(t)\ sat\ u_2(t)\ \cdots\ sat\ u_m(t)]^T$$

where

$$sat\ u_i(t)=\begin{cases} u_{ui} & \text{for } u_i>u_{ui} \\ u_i(t) & \text{for } u_{li}\le u_i\le u_{ui} \\ u_{li} & \text{for } u_i<u_{li} \end{cases} \quad (24)$$

for $i=1, \cdots, m$

u_{ui} : constant maximum limit

u_{li} : constant minimum limit

The supervisor dynamics are given by

$$\dot{\hat{x}}(t)=A\hat{x}(t)+B\hat{u}(t) \quad (25)$$

where

$$\hat{u}(t)=u(t)-f(x(t)-\hat{x}(t)) \quad (26)$$

and $\hat{x}(t)\in R^n$ is the supervisor state vector, $\hat{u}(t)\in R^m$ is the control input vector to the supervisor, and $f\in R^{m\times n}$ is the error gain matrix between the plant and the supervisor.

The dynamic control law is of the form

$$\dot{x}_r(t)=Fx_r(t)+G\hat{x}(t)+Pr(t), \quad (27)$$

$$u(t)=Rx_r(t)-Q\hat{x}(t), \quad (28)$$

where $x_r(t)\in R^q$ is the state vector of the dynamic controller of order q, $r(t)\in R^v$ is a reference input, and F, G, P, R, Q, and C are matrices of appropriate dimensions dependent on the dynamic controller type.

Setting the error between the plant and supervisor as $e_s(t)=x(t)-\hat{x}(t)$, the resulting error -dynamics can be expressed by

$$\dot{e}_s(t)=(A+Bf)e_s(t)+B(sat\ u(t)-u(t)) \quad (29)$$

Combining Eq. (25), Eq. (26), Eq. (27), Eq. (28) and Eq. (29), we obtain the following system :

$$\dot{\tilde{x}}(t)=\tilde{A}_c\tilde{x}(t)+\tilde{U}_c(t)+\tilde{P}r(t) \quad (30)$$

where

$$\tilde{x}(t)=[x_r(t)\ \hat{x}(t)\ e_s(t)]^T$$

$$\tilde{A}_c=\begin{bmatrix} F & G & 0 \\ BR & A-BQ & -Bf \\ 0 & 0 & A+Bf \end{bmatrix}$$

$$\tilde{U}_c(t)=\begin{bmatrix} 0 \\ 0 \\ B\{sat\ u(t)-u(t)\} \end{bmatrix} \quad \tilde{P}=\begin{bmatrix} P \\ 0 \\ 0 \end{bmatrix}$$

Equation (30) describes the closed-loop system with a possibly saturated actuator and its

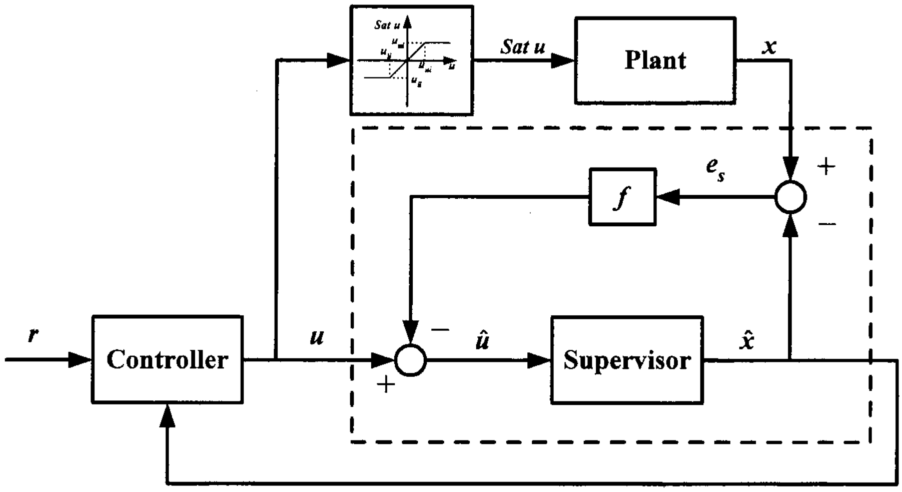

Fig. 4 Supervisory control with saturation

associated supervisor in the continuous-time case. Figure 4 shows a typical block diagram of supervisory control with actuator saturation. F, G, R, and Q can be determined by pole-placement for a submatrix of $\tilde{A}_c$ at the desired stable region in the s-plane for linear operation, while the supervisor error-gain matrix f can be chosen separately due to the structure of the closed-loop system matrix $\tilde{A}_c$.

If the continuous plant is controlled by a discrete-time controller, the plant equation can be discretized by

$$x(kT+T)=\Phi(T)x(kT)+\Theta(T)sat\ u(kT) \quad (31)$$

The discrete-time dynamic control law, including IESF or ISFF, is written by

$$\begin{aligned} x_r(kT+T) &= x_r(kT)+T\{Fx_r(kT) \\ &\quad + G\hat{x}(kT)+\Pr(kT)\} \quad (32) \\ u(kT) &= Rx_r(kT)-Q\hat{x}(kT) \quad (33) \end{aligned}$$

The discrete-time supervisor dynamics are described by

$$\hat{x}(kT+T)=\Phi(T)\hat{x}(kT)+\Theta(T)\hat{u}(kT) \quad (34)$$

where

$$\hat{u}(kT)=u(kT)-f(x(kT)-\hat{x}(kT)) \quad (35)$$

Setting the error between the plant and supervisor as $e_s(kT)=x(kT)-\hat{x}(kT)$, the resulting error dynamics becomes

$$\begin{aligned} e_s(kT) &= (\Phi(T)+\Theta(T)f)e_s(kT) \\ &\quad +\Theta(T)(sat\ u(kT)-u(kT)) \quad (36) \end{aligned}$$

From Eq. (32), Eq. (33), Eq. (34), Eq. (35) and Eq. (36), the closed-loop system can be expressed by

$$\tilde{x}(kT+T)=\tilde{A}_d\tilde{x}(kT)+\tilde{U}_d(kT)+\tilde{P}r(kT) \quad (37)$$

where

$$\tilde{x}(kT)=[x_r(kT)\quad \hat{x}(kT)\quad e_s(kT)]^T,$$

$$\tilde{A}_d=\begin{bmatrix} I+TF & TG & 0 \\ \Theta R & \Phi-\Theta Q & -\Theta f \\ 0 & 0 & \Phi+\Theta f \end{bmatrix},$$

$$\tilde{U}_d(kT)=\begin{bmatrix} 0 \\ 0 \\ \Theta\{sat\ u(kT)-u(kT)\} \end{bmatrix},$$

$$\tilde{P}=\begin{bmatrix} TP \\ 0 \\ 0 \end{bmatrix}.$$

Equation (37) implies a discrete-time closed-loop system with the actuator saturations and its associated supervisor. F, G, R, and Q can be determined by pole-placement of the submatrix of $\tilde{A}_c$ at the desired location in the z-plane, while the supervisor error-gain matrix f can be chosen independently from the error dynamics.

7. Conclusions

A design method for a dynamic controller (ISFF) is provided for continuous-time and discrete-time systems. ISFF controller gains k_1, $\cdots$, k_{n+2} can be obtained by a simple matrix inversion using the Jordan-Canonical form. This procedure does not require any symbolic mathematical package to get k_1, $\cdots$, k_{n+2}. When actuators are saturated, a newly developed algorithm to prevent integrator wind-up and performance deterioration is proposed by implementing a supervisor as a nominal plant.

Based on the modern control algorithm, a typical multi-echelon production-distribution system can be numerically simulated by using the ISFF dynamic control law when the piecewise constant manufacturing decisions is upper-limited due to capital equipment, limited by a factory manufacturing manpower, and factory lotsize. Information management scheme and numerical illustrations for perturbed multi-echelon systems will be shown in the part Ⅱ.

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