Robust Control of Multi-Echelon Production-Distribution Systems with Limited Decision Policy (Ⅱ)
— Numerical Simulation —

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A typical production-distribution system consist of three main echelons representing the retailer, distributors, and a factory each with an on-site warehouse. The system is sufficiently general and realistic to represent many industrial situations. However, decision functions and parameters have been selected to apply particularly to the production and distribution of consumer durables. The flows included in the model are materials, orders, and those information flows needed to support the material and order-rate decisions.

In this work, a realistic production-distribution system has been used as a basic model, which consists of three sectors : retailer, distributor, and factory. That system is a nonlinear 25th-order continuous system interconnected between the echelons. Using a modern control algorithm, a typical multi-echelon production-distribution system using a dynamic controller is numerically simulated in the nominal plant and in the perturbed plant when the piecewise constant manufacturing decision is limited by a factory manufacturing upper-limit due to capital equipment, manpower, and factory lotsize.

Key Words : Supervisory Control, Dynamic Controller, Saturation Control, Multi-Echelon Production-Distribution Systems, Limited Decision Policy

1. Introduction

A production-distribution system embodies three main sectors representing the retailer, distributors, and a factory each with an on-site warehouse. The system is sufficiently general and realistic to represent many industrial situations. However, decision functions and parameters have been selected to apply particularly to the manufacture and distribution of consumer durables such as automobiles. The flows included in the model are materials and orders and those information flows needed to support the material and order-rate decisions.

A mathematical continuous-time model of the dynamic production and distribution process has been developed by Forrester (1961) and Roberts (1978). Meyer and Groover (1972) developed the simple 3-echelon inventory system, modeled as a production-distribution system with a 2-echelon system consisting of a third-order plant and a reference model of the factory. Most control schemes in production-distribution systems use output feedback. This means that purchasing rate decisions (in the retailer or distributor) and manufacturing rate decisions are based on local differences between actual and desires values of inventory levels and other variables (pipeline orders and unfilled orders).

In this work, a realistic production-distribution system described by Forrester (1961) has

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been used as a basic model, which consists of three sectors: retailer, distributor, and factory.

That system is a nonlinear 25th-order continuous system interconnected between the three echelons. The complete system of equations is given in Appendix B. Using a modern control algorithm, a typical multi-echelon production-distribution system using a dynamic controller is numerically simulated in the nominal plant and in the perturbed plant when piecewise constant manufacturing decision is limited by a factory manufacturing upper-limit due to capital equipment, manpower, and factory lotsize.

2. Dynamic Control Strategy

We use the nonlinear highly coupled model of a production-distribution system (Jeong, 1993b). The Integrated Error with State Feedback and Filtering (ISFF) control scheme is applied to the multi-echelon production-distribution system to control information flow effectively. It has been shown previously that the dynamic control scheme is superior to output feedback for performance robustness and stability robustness (Jeong, 1992a, 1992b). Especially with ISFF control, it is easy to implement an algorithm, called the supervisor, to handle actuator saturation. After linearization of the production-distribution system with three echelons, each echelon is described by an independent ISFF control. Then, the echelons are cascaded for the coupled production-distribution system. The terms k_1 through k_{22} are ISFF controller gains, and a_1 through a_{20} are constants given in Appendix C. The distributor sector includes the 7th-order factory sector, while the retail sector includes the 7th-order distributor sector in its plant. Here the outgoing materials flow at each echelon is given by

$$SSF(t) = a_1 UOF(t) + a_2 IAF(t) - a_6 RSF(t) \quad (1)$$

$$SSD(t) = a_7 UOF(t) + a_8 IAF(t) - a_{13} RSF(t) \quad (2)$$

$$SSR(t) = a_{14} UOF(t) + a_{15} IAF(t) - a_{20} RSF(t) \quad (3)$$

The previous echelon block with high-order dynamics in the current echelon plant can be approximated by an equivalent first-order delay by using Matsubara's Equivalent Delay Theorem to obtain the ISFF controller gains at each echelon. If the previous echelon is exponentially stable, the following lemma can be applied.

Lemma 1 (Matsubara's Equivalent Delay Theorem, 1965) If a transfer function $G(s)$ in $|s|<\infty$ has no poles in the right-half plane and on the imaginary axis and has no zeros at the origin, there exists an equivalent delay τ_e with respect to the indicial response $z(t)$ of $G(s)$ and the unit step input $r(t)$ such that

$$\tau_e = \int_0^\infty \left\{ r(t) - \frac{z(t)}{K_t} \right\} dt \quad (4)$$

where $\lim_{t\to\infty} z(t) = K_t$

From the above lemma, the equivalent delay of the previous echelon can be obtained and can be approximated by a first-order delay as shown in Fig. 1. Applying Lemma 1 to the distributor and retail sector, the resulted state-space representation of each echelon can be written as follows: For the factory sector,

$$\dot{x}_{fa}(t) = A_{fa} x_{fa}(t) + B_{fa} r_{fa}(t) \quad (5)$$

where

$$x_{fa}(t) = [x_{r1}\ x_{r2}\ UOF\ IAF\ SRF\ MOF\ RSF]^T$$

$$A_{fa} = \begin{bmatrix} 0 & 0 & k_1 & 0 & 0 & 0 & 0 \\ 1 & -k_6 & -k_2 & -k_3 & -k_4 & -k_5 & 0 \\ 0 & 0 & -a_1 & -a_2 & 0 & 0 & a_6 \\ 0 & 0 & -a_1 & -a_2 & 1 & 0 & a_6 \\ 0 & 0 & 0 & 0 & -a_3 & a_3 & 0 \\ 0 & a_4 & 0 & 0 & 0 & -a_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_5 \end{bmatrix}$$

$$B_{fa} = \left[-\frac{k_1}{a_1}\ 0\ 1\ 0\ 0\ 0\ a_5 \right]^T$$

$$r_{fa}(t) = RRF(t)$$

x_{r1}, x_{r2} = auxiliary state vector of ISFF controller

For the distributor sector,

$$\dot{x}_{di} = A_{di} x_{di}(t) + B_{di} r_{di}(t) \quad (6)$$

where

$$x_{di}(t) = [x_{r3}\ x_{r4}\ UOD\ IAD\ SRD\ SSF\ RRF\ PSD\ RSD]^T$$

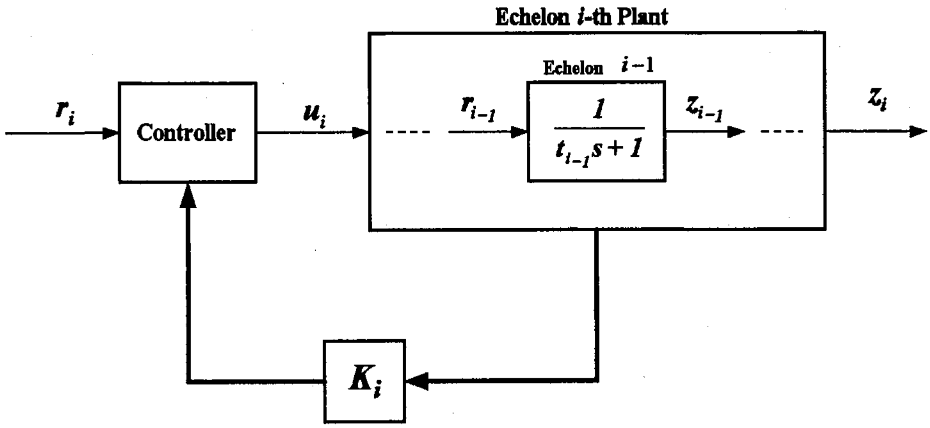


Fig. 1 Reduced-order model with equivalent delay at the previous echelon

$$A_{di}=\begin{bmatrix} 0 & 0 & k_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -k_{14} & -k_8 & -k_9 & -k_{10} & -k_{11} & -k_{12} & -k_{13} & 0 \\ 0 & 0 & -a_h & -a_8 & 0 & 0 & 0 & 0 & a_{13} \\ 0 & 0 & -a_7 & -a_8 & 1 & 0 & 0 & 0 & a_{13} \\ 0 & 0 & 0 & 0 & -a_9 & a_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_f} & \frac{1}{\tau_f} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_{10} & a_{10} & 0 \\ 0 & a_{11} & 0 & 0 & 0 & 0 & 0 & -a_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{12} \end{bmatrix}$$

$$B_{di}=\left[-\frac{k_7}{a_7}\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ a_{12}\right]^T$$

$r_{di}(t)=RRD(t)$

x_{r3}, x_{r4}=auxiliary state vector of ISFF controller

τ_f=equivalent delay for the factory sector

For the retail sector,

$$\dot{x}_{re}(t)=A_{re}x_{re}(t)+B_{re}r_{re}(t) \tag{7}$$

where

$$x_{re}(t)=[x_{r5}\ x_{r6}\ UOR\ IAR\ SRR\ SSD\ RRD\ PSR\ RSR]^T$$

$$A_{re}=\begin{bmatrix} 0 & 0 & k_{15} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -k_{22} & -k_{16} & -k_{17} & -k_{18} & -k_{19} & -k_{20} & -k_{21} & 0 \\ 0 & 0 & -a_{14} & -a_{15} & 0 & 0 & 0 & 0 & a_{20} \\ 0 & 0 & -a_{14} & -a_{15} & 1 & 0 & 0 & 0 & a_{20} \\ 0 & 0 & 0 & 0 & -a_{16} & a_{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_d} & \frac{1}{\tau_d} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_{17} & a_{17} & 0 \\ 0 & a_{18} & 0 & 0 & 0 & 0 & 0 & -a_{18} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_{19} \end{bmatrix}$$

$$B_{re}=\left[-\frac{k_{15}}{a_{14}}\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ a_{19}\right]^T$$

$r_{re}(t)=RRR(t)$

x_{r5}, x_{r6}=auxiliary state vector of ISFF controller

τ_d=equivalent delay for the distributor sector

The ISFF controller gains k_1 through k_{22} can be determined by pole-placement at each echelon. In general, multiple-order poles are assigned at each echelon and the ISFF controller gains for each sector can be obtained numerically using the Jordan-Canonical form (Jeong, 1994c).

To get the piece-wise constant polices for the manufacturing decision in the factory sector and for the purchasing decision in the distributor and the retail sector, the ISFF discrete-time controller can be implemented.

3. Saturation Control

In the more realistic situation of limited factory capacity, some surprising new effects appear. One assumes retail sales never exceed production capacity. However, because of inventory and pipeline effects, distributor orders can exceed its capacity. In this case, based on the approach in Part I, the supervisor can be applied to the saturated system in the factory sector, since the factory sector has a saturated control input which is a limited manufacturing capacity. The supervisor error dynamics can be written by

$$\dot{e}(t) = A_e e(t) + B_e u_e(t) \qquad (8)$$

where

$$e(t) = [e_1(t)\ e_2(t)\ e_3(t)\ e_4(t)]^T$$
$$e_1(t) = UOF(t) - UOFs(t),$$
$$e_2(t) = IAF(t) - IAFs(t),$$
$$e_3(t) = SRF(t) - SRFs(t),$$
$$e_4(t) = MOF(t) - MOFs(t),$$
$$A_e = \begin{bmatrix} -a_1 & -a_2 & 0 & 0 \\ -a_1 & -a_2 & 1 & 0 \\ 0 & 0 & -a_3 & a_3 \\ a_4 f_1 & a_4 f_2 & a_4 f_3 & a_4(f_4 - 1) \end{bmatrix}$$

f_i=supervisor error gains, for i=1, 2, 3, 4

$$B_e = [0\ 0\ 0\ a_4]^T$$
$$u_e(t) = sat\ MDF(t) - MDF(t)$$

The subscript s denotes the supervisor state.

The supervisor error gains can be determined by pole-placement.

4. Numerical Simulations

Using the previous formulation, numerical simulations for multi-echelon production-distribution system are made. In business, simulation means setting up in a digital computer the conditions which describe company operation. On the basis of the descriptions and assumptions about the company, the computer generates the resulting information concerning finance, manpower, product movement, etc. Different management policies and market assumptions can be tested to determine their effects on company profit.

In this illustration, two simulation comparisons are shown: the saturated vs. unsaturated system in discrete-time ISFF control, and the nominal saturated system vs. the uncertain saturated system. A very informative, and one of the simplest, test inputs for the study of system dynamics is the step function. Here, after the beginning of the run, the value of the retail sale RRR is increased by 100 units per week, giving a 10% upward step in retail sales at t=1 week from the initial steady-state value given as 1000 units/week. The system parameters and initial conditions are given in Appendix D.

4.1 Saturated vs. unsaturated system with ISFF control

The effect of limited capital equipment and manpower is approximated by simply restricting permissible factory production. This can be done by imposing an upper limit on the rate at which manufacturing orders can be sent to the factory, which is an arbitrary upper limit that is not a function of the actual variable representing the

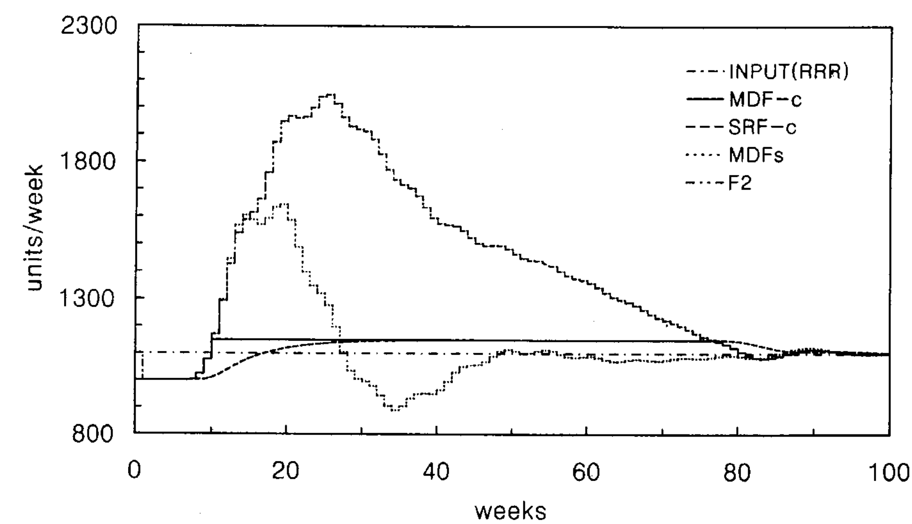


Fig. 2 Step response of MDFc, SRFc, and ISFF controller output F_2 for saturated system with decision variable constraint of 1150 units/week : Subscript-c stands for saturated system and subscript-s stands for supervisor

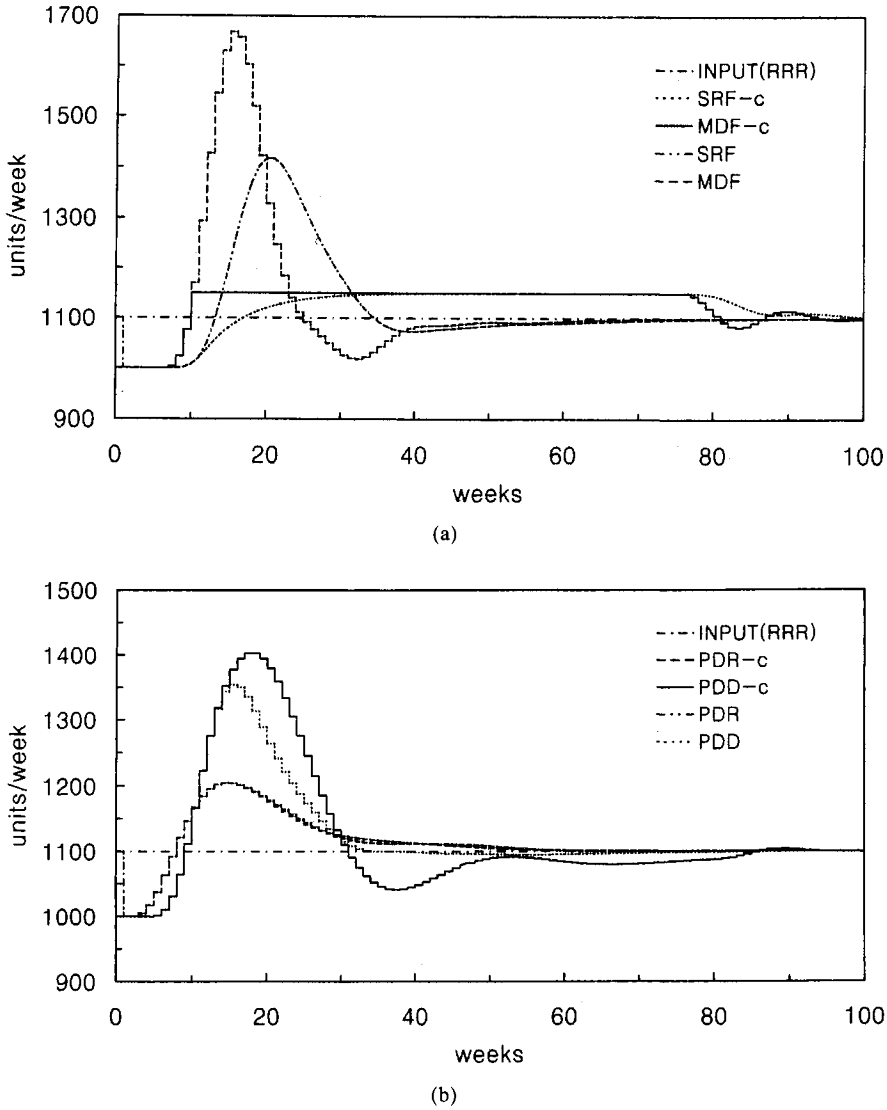
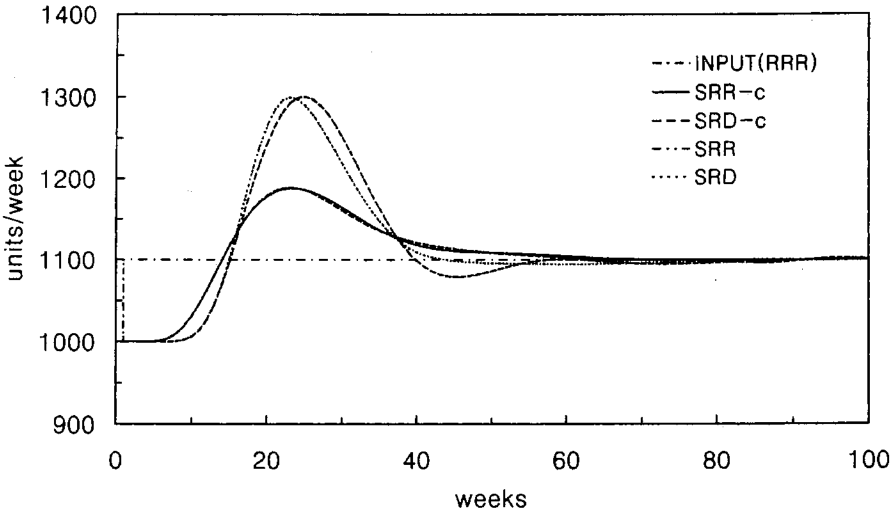


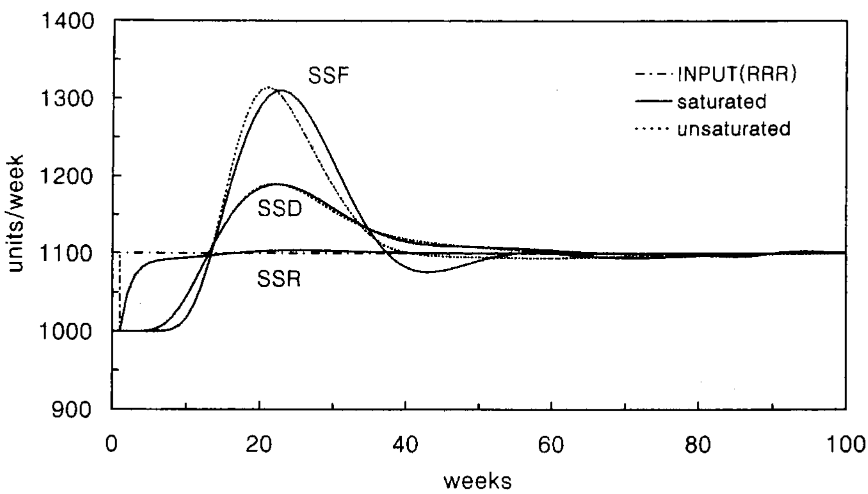
Fig. 3 Step response of SRF, MDF, PDR, PDD for saturated system and unsaturated system : Subscript -c stands for saturated system

availability of capital equipment, labor, and materials. A factory manufacturing limit is assumed to be 15% above initial steady-state retail consumption, which is 1150 units/week. The supervisor is implemented in the factory to control the saturated factory sector effectively. The repeated poles of the supervisor error dynamics are assigned at $s=-0.37$ to determine error gains f_1 through f_4. Discrete-time ISFF control is utilized at each echelon to achieve the piecewise constant manufacturing decision and purchasing decision, where the sampling time $T=1$ week and assigned multiple-order poles are $z_f=0.368$ for the factory, $z_d=0.497$ for the distributor, and $z_r=0.684$ for the retailer, while the equivalent delay is given by $\tau_d=2.7$ weeks for the distributor, and $\tau_f=2.53$ weeks for the factory.

The time responses of the (nominal) saturated system with a constrained manufacturing decision and (nominal) unsaturated system are illustrated in Fig. 3, Fig. 4, and Fig. 5. It is seen that the saturated system is stabilizable by implementing a



(a)



(b)

Fig. 4 Step response of SRD, SRR, SSF, SSD, SSR for saturated system and unsaturated system

supervisor.

The details of the saturated manufacturing decision rate (MDFc), shipments received at factory inventory SRF (manufacturing output), supervisor control input MDFs, and ISFF controller output F2 are shown in Fig. 2. SRF does not exceed the manufacturing limit of 1150 units/week and the very large values of the ISFF controller output F2 that follow the supervisor control input MDFs is dependent on the location of the supervisor-error poles.

In the saturated system, the manufacturing limit does not affect the response of the retail sector and IAD, UOD, and SSD of distributor. The large backlog of UOF and the large decrease in IAF result in an increasing delay in the filling of orders due to the production limit and in causing the distributor to order still further ahead of needs because of the growing delay of SSF. During a period of 10 weeks to 80 weeks of constrained factory production, this effect is usually called regenerative (more orders cause a large backlog of UOF, which causes more delay and more ordering ahead), resulting in a

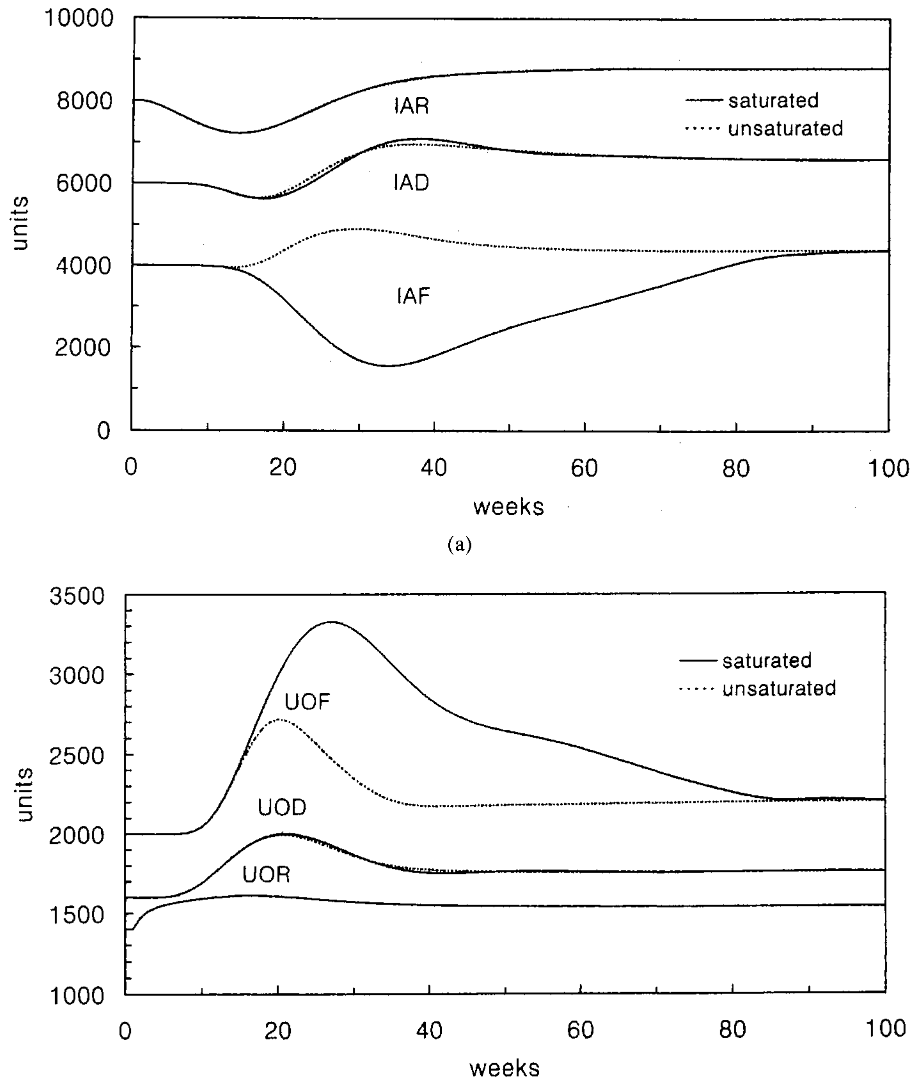


Fig. 5 Step response of inventory levels and unfilled orders at each echelon for saturated system and unsaturated system

maintained upper limit of production rate for 10 weeks to 80 weeks as shown in Fig. 3. In this illustration, the inventory at the factory IAF does not reach zero, even in the presence of a large backlog of unfilled orders UOF.

4.2 Uncertain saturated system vs. nominal saturated system

Suppose that each echelon has 50% increased perturbations of nominal values with DTR$\doteq$1 week, DTD=2 weeks, and DPF=6 weeks, while perturbations are ΔDTR=0.5 week for the retail sector, ΔDTD=1 week for the distributor sector, and ΔDPF=3 weeks for the factory. Here DTR and DTD are the averaged transportation delays at the retail and the distributor sector, respectively.

According to the simulation results in Fig. 6, Fig. 7, and Fig. 8 with perturbations ΔDTR=0.5 week, ΔDTD=1 week, and ΔDPF=3 weeks, the large overshoots in PDD, PDR, SRD, SRR, SSF, SSD, UOF and UOD are produced by

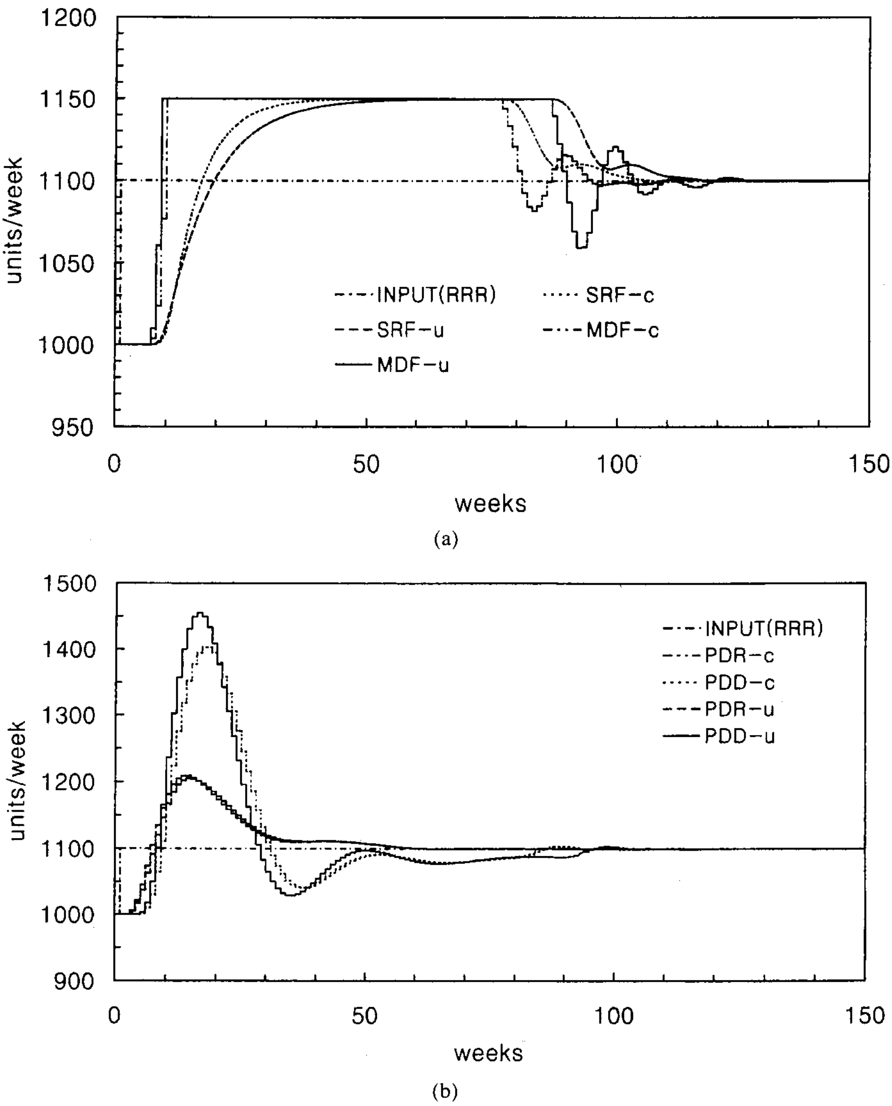


Fig. 6 Step response of SRF, MDF, PDR, PDD for nominal saturated system and uncertain saturated system with 50% increased pertubations ΔDPF=3 weeks, ΔDTD=1 week, and ΔDTR=0.5 week : Subscript -c stands for nominal saturated system and subscript -u stands for uncertain saturated system

uncertainties as well as a larger saturating region in MDF and SRF. On the other hand, the outgoing material flow at the retailer SSR is not changed by those uncertainties. From the above simulation results, it is clear that the saturated multi-echelon production-distribution system, with perturbations of 50% increases in DTR, DTD, DPF, has stability robustness and performance robustness, when the saturated system is controlled by a cascaded ISFF control with a supervisor.

5. Conclusions

A typical multi-echelon production-distribution system is simulated by using the ISFF dynamic control law with the supervisor after cascading the interconnected system, when the piecewise constant manufacturing decision is saturated by a factory manufacturing upper limit

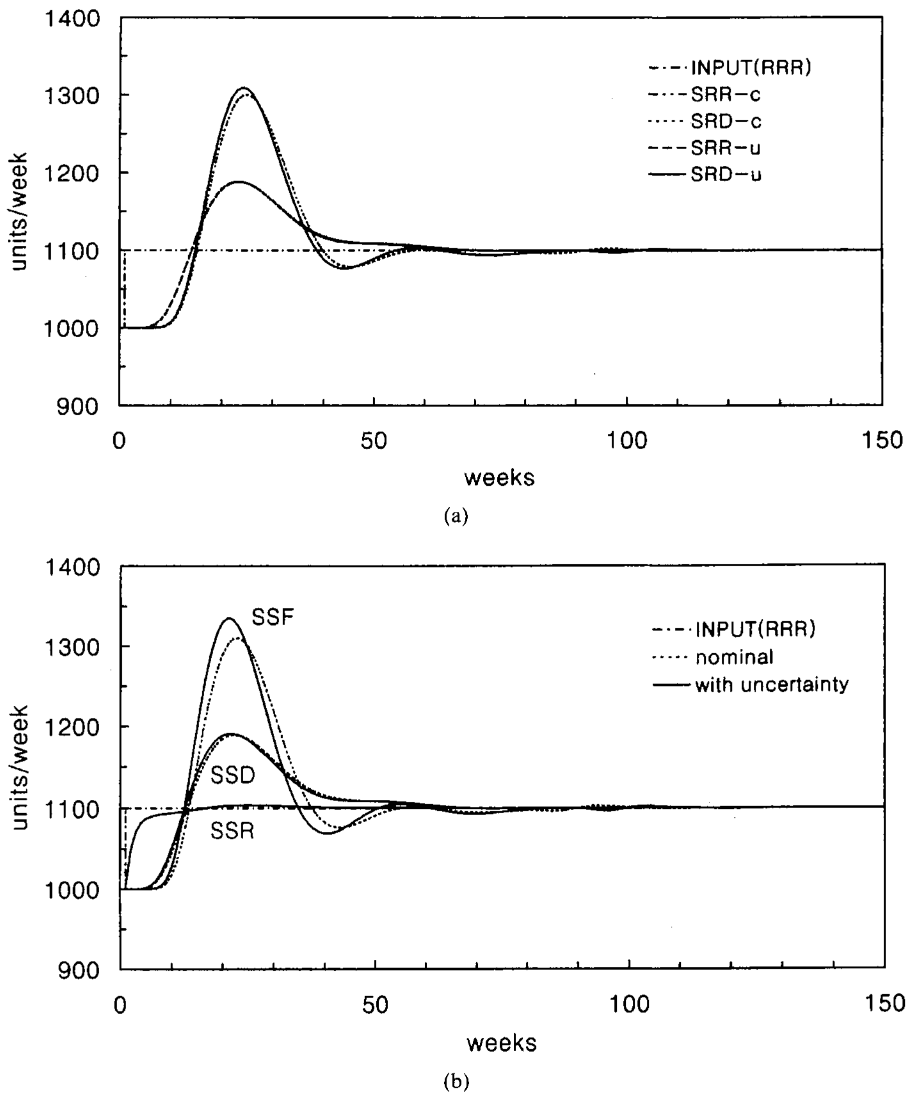


Fig. 7 Step response of SRD, SRR, SSF, SSD, SSR for nominal saturated system and uncertain saturated system with 50% increased perturbations ΔDPF=3 weeks, ΔDTD=1 week, and ΔDTR=o.5 week

due to capital equipment, manpower, factory space, etc. Numerical illustrations show that the large backlog of unfilled orders and the large decrease of actual inventory at the factory result from the production limit. Hence, the factory sector becomes regenerative, resulting in a maintained upper-limit of the production rate and manufacturing decision rate.

The perturbed multi-echelon system has stability robustness and performance robustness under perturbations of a 50% increase in nominal transportation delay-time and production lead-time.

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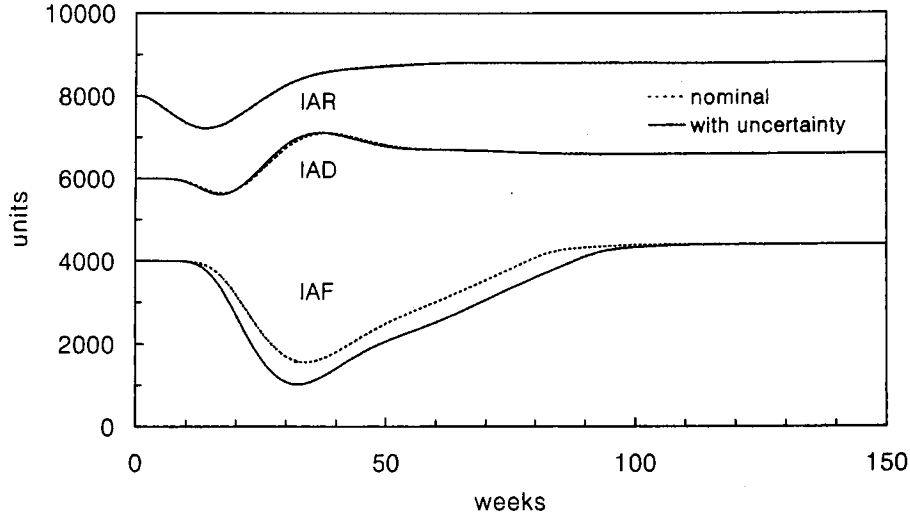
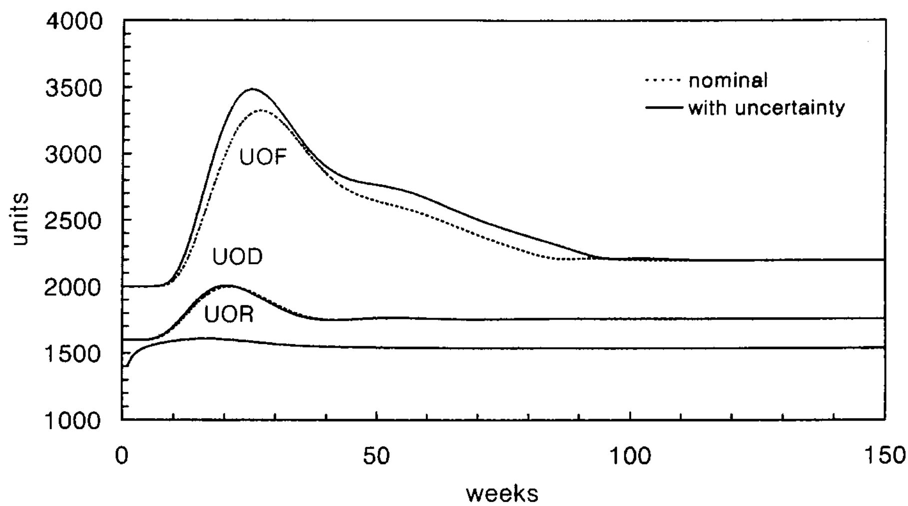


(a)



(b)

Fig. 8 Step response of inventory levels and unfilled orders at each echeolon for nominal saturated system and uncertain saturated system with 50% increased perturbations ΔDPF=3 weeks, ΔDTD=1 week, and ΔDTR=0.5 week

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Appendix A : Nomenclature

ISFF	: Integrated-errorr with state-feedback and filtering
sat	: Saturation function
Δ	: Perturbation
subscript s	: Supervisor system
subscript c	: Nominal saturated system
subscript u	: Uncertain saturated system
F_2	: Controller output at factory
MDF	: Manufacturing rate decision at factory
MDFs	: Supervisor control input at factory
MDFc	: Saturated manufacturing rate decision at factory
MDFu	: Saturated manufacturing rate decision at uncertain factory
PDR, PDD	: Purchasing rate decision at retailer and distributor
IAR, IAD, IAF	: Actual Inventory at each echelon (retailer, distributor, or factory)
UOR, UOD, UOF	: Unfilled orders at each echelon
SRR, SRD, SRF	: Shipments received at the inventory of each echelon
SSR, SSD, SSF	: Shipments sent from each echelon
RRR, RRD, RRF	: Requisitions (Orders) received at each echelon
RSR, RSD, RSF	: Smoothed (Averaged) sales at each level
DPF	: Delay in production lead-time at factory
DMR, DMD	: Delay in order mailing at retailer and distributor
DTR, DTD	: Delay in transportation to each echelon
DCR, DCD, DCF	: Clerical delay at each echelon
DHR, DHD, DHF	: Delay due to minimum handling time at each echelon
DRR, DRD, DRF	: Delay in smoothing requisitions at each echelon
AIR, AID, AIF	: Proportionality con-

	stant for inventory
DUR, DUD, DUF	: Delay due to unfilled orders at each echlon caused by out-of -stock items when inventory is normal

Appendix B : Equations of Model

Equations for the retailer

$$U\dot{O}R(t)=RRR(t)-SSR(t)$$
$$I\dot{A}R(t)=SRR(t)-SSR(t)$$
$$SSR(t)=\frac{UOR(t)}{DFR(t)}$$
$$DFR(t)=DHR+DUR\frac{IDR(t)}{IAR(t)}$$
$$IDR(t)=AIR\ RSR(t)$$
$$R\dot{S}R(t)=\frac{1}{DRR}(RRR(t)-RSR(t))$$
$$PDR(t)=RRR(t)+\frac{1}{DIR}[(IDR(t)-IAR(t))+(LDR(t)-LAR(t))+(UOR(t)-UNR(t))]$$
$$LDR(t)=RSR(t)(DCR+DMR+DFD(t)+DTR)$$
$$LAR(t)=CPR(t)+PMR(t)+UOD(t)+MTR(t)$$
$$UNR(t)=RSR(t)(DHR+DUR)$$
$$C\dot{P}R(t)=PDR(t)-PSR(t)$$
$$P\dot{S}R(t)=\frac{1}{DCR}(PDR(t)-PSR(t))$$
$$P\dot{M}R(t)=PSR(t)-RRD(t)$$
$$R\dot{R}D(t)=\frac{1}{DMR}(PSR(t)-RRD(t))$$
$$M\dot{T}R(t)=SSD(t)-SRR(t)$$
$$S\dot{R}R(t)=\frac{1}{DTR}(SSD(t)-SRR(t))$$

Equations for the distributor

$$U\dot{O}D(t)=RRD(t)-SSD(t)$$
$$I\dot{A}D(t)=SRD(t)-SSD(t)$$
$$SSD(t)=\frac{UOD(t)}{DFD(t)}$$
$$DFD(t)=DHD+DUD\frac{IDD(t)}{IAD(t)}$$
$$IDD(t)=AID-RSD(t)$$
$$R\dot{S}D(t)=\frac{1}{DRD}(RRD(t)+RSD(t))$$
$$PDD(t)=RRD(t)+\frac{1}{DID}[(IDD(t)-IAD(t))+(LDD(t)-LAD(t))+(UOD(t)-UND(t))]$$
$$LDD(t)=RSD(t)(DCD+DMD+DFF(t)+DTD)$$
$$LAD(t)=CPD(t)+PMD(t)+UOF(t)+MTD(t)$$
$$UND(t)=RSD(t)(DHD+DHD)$$
$$C\dot{P}D(t)=PDD(t)+PSD(t)$$
$$P\dot{S}D(t)=\frac{1}{DCD}(PDD(t)-PSD(t))$$
$$P\dot{M}D(t)=PSD(t)-RRF(t)$$
$$R\dot{R}F(t)=\frac{1}{DMD}(PSD(t)-RRF(t))$$
$$M\dot{T}D(t)=SSF(t)-SRD(t)$$
$$S\dot{R}D(t)=\frac{1}{DTD}(SSF(t)-SRD(t))$$

Equations for the factory

$$U\dot{O}F(t)=RRF(t)-SSF(t)$$
$$I\dot{A}F(t)=SRF(t)-SSF(t)$$
$$SSF(t)=\frac{UOF(t)}{DFF(t)}$$
$$DFF(t)=DHF+DUF\frac{IDF(t)}{IAF(t)}$$
$$IDF(t)=AIF\ RSF(t)$$
$$R\dot{S}F(t)=\frac{1}{DRF}(RRF(t)-RSF(t))$$
$$MDF(t)=RRF(t)+\frac{1}{DIF}[(IDF(t)-IAF(t))+(LDF(t)-LAF(t))+(UOF(t)-UNF(t))]$$
$$LDF(t)=RSF(t)(DCF+DPF)$$
$$LAF(t)=CPF(t)+MOF(t)$$
$$UNF(t)=RSF(t)(DHF+DUF)$$
$$C\dot{P}F(t)=MDF(t)-MOF(t))$$
$$M\dot{O}F(t)=\frac{1}{DCF}(MDF(t)-MOF(t))$$
$$O\dot{P}E(t)=MOF(t)-SRF(t)$$
$$S\dot{R}F(t)=\frac{1}{DPF}(MOF(t)-SRF(t))$$

APPENDIX C : Parameters of ISFF Control

Factory

$$a_1=\frac{1}{DHF+DUF}$$

$$a_2=\frac{DUF}{AIF(DHF+DUF)}$$

$$a_3=\frac{1}{DPF}$$

$$a_4=\frac{1}{DCF}$$

$$a_5=\frac{1}{DCF}$$

$$a_6=\frac{DUF}{DHF+DUD}$$

Distributor

$$a_7=\frac{1}{DHD+DUD}$$

$$a_8=\frac{DUD}{AID(DHD+DUD)}$$

$$a_9=\frac{1}{DTD}$$

$$a_{10}=\frac{1}{DMD}$$

$$a_{11}=\frac{1}{DCD}$$

$$a_{12}=\frac{1}{DRD}$$

$$a_{13}=\frac{DUD}{DHD+DUD}$$

Retailer

$$a_{14}=\frac{1}{DHR+DUR}$$

$$a_{15}=\frac{DUR}{AIR(DHR+DUR)}$$

$$a_{16}=\frac{1}{DTR}$$

$$a_{17}=\frac{1}{DMR}$$

$$a_{18}=\frac{1}{DCR}$$

$$a_{19}=\frac{1}{DRR}$$

$$a_{20}=\frac{DUR}{DHR+DUR}$$

Appendix D : Parameters and Initial Conditions

Parameters of the system

Retailer	Distributor	Factory
$DHR=1$ week	$DHD=1$ week	$DHF=1$ week
$DUR=0.4$ week	$DUD=0.6$ week	$DUF=1$ week
$AIR=8$ weeks	$AID=6$ weeks	$AIF=4$ weeks
$DRR=8$ weeks	$DRD=8$ weeks	$DRF=8$ weeks
$DIR=4$ weeks	$DID=4$ weeks	$DIF=4$ weeks
$DCR=3$ weeks	$DCD=2$ weeks	$DCF=1$ week
$DMR=0.5$ week	$DMD=0.5$ weeks	$DPF=6$ weeks
$DTR=1$ week	$DTD=2$ week	

Initial conditions

Retailer	Distributor
$RRR=RRI=1000$	$RRD=RRR$
$RSR=RRR$	$RSD=RRD$
$UOR=RSR(DHR+DUR)$	$UOD=RSD(DHD+DUD)$
$IAR=AIR\cdot RSR$	$IAD=AID\cdot RSD$
$CPR=DCR\cdot RRR$	$CPD=DCD\cdot RRD$
$PMR=DMR\cdot RRR$	$PMD=DMD\cdot RRD$
$MTR=DTR\cdot RRR$	$MTD=DTD\cdot RRD$
$SRR=RRR$	$SRD=RRD$

Factory

$RRF=RRR$

$RSF=RRF$

$UOF=RSF(DHF+DUF)$

$IAF=AIF\cdot RSF$

$CPF=DCF\cdot RRF$

$DPF=DPF\cdot RRF$

$MOF=SRF=RRF$