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Force Distribution of a Six-Legged Walking Robot with High Constant Speed

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For a walking robot with high constant body speed, the dynamic effects of the legs on the transfer phase are dominant compared with other factors. This paper presents a new force distribution algorithm to maximize walkable terrain without slipping considering the dynamic effects of the legs on the transfer phase. Maximizing the walkable terrain means having the capability of walking on more slippery ground under the same constraint, namely constant body speed. A simple force distribution algorithm applied to the proposed walking model with a pantograph leg shows an improvement in the capability of preventing foot-slippage compared with one using a pseudo-inverse method.

Key Words : Six-Legged Walking Robot, Pantograph, Force Distribution, Friction

1. Introduction

One of the most important dynamic features to be considered for walking robot is stability; it is essential to distribute forces properly for the legs contacting the ground. But, because of the difficulty and complexity of this procedure, many walking robot researchers have solved problems under restrictive assumptions. In particular, for si x-legged walking robots, tripod gait in which three legs contact the ground was applied to (Kaneko et al., 1985; Huang et al., 1990). Here the main issue was about walking on even terrain (Choi et al., 1988; Kim et al., 1990). In this case, the closed form solution derived from the given equilibrium equation was used. To analyze dynamic features of the robot resulting from contact of the legs against the ground (Klein et al., 1987), performed simulations by treating the legs themselves as springs with compliance (Shih et al., 1987; Klein et al., 1980) and distributed forces properly, attaching sensors at the end-effectors of legs and thereby being fed back to the data for force (Klein et al., 1983). Also, force distribution algorithms which took consideration of energy efficiency and friction through constraint conditions other than equilibrium condition has been researched. (Orin et al., 1981; Klein et al., 1990). As the number of constraint conditions increased, the force distribution problem has led to the introduction of optimization (Klein et al., 1990; Sreenivasan et al., 1996).

In the case of dynamic simulation including actuator selection or force distribution problem, existing research on six-legged walking robots has been done under the assumption that dynamic effects of the legs or loads applied to the legs could be neglected as they were small compared with body effects. The work presented here is intended to analyze the dynamic effects of legs on the transfer phase for robot walking with high constant body speed. As the walking speed of the robot increases, the driving forces of the legs on the transfer phase do more and more. It means an increase in reaction force acting on the body

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against the driving force, resulting in foot-slippage because the tangential force of the leg end-effector exceeds the friction force due to body weight. Then, if normal forces of the legs on the support phase are not regulated according to these loads, the body will be unable to keep constant speed. There fore, we must consider the dynamic effects of the legs to keep high walking body speed under friction constraint.

In this paper, we will identify the relationship between friction and body speed by the dynamic effects of the legs. It will be shown that the analysis of force accompanied by walking takes a prominent role for allocating forces to maximize walkable speed without foot-slippage in the given ground, and for building a optimal force distribution algorithm. In section II, we discuss force distribution for the leg end effector on the basis of the derived dynamic formulation of a leg and entire system. In section III, the various dynamic properties are described for the case that the leg dynamics are considered as the dominant effects. A new force distribution algorithm to maximize walkable zone without slipping and its optimization are introduced in section IV. Section V summarizes the work performed in this research and discusses future work.

2. Dynamic Formulation and Pseudo Inverse Solution

2.1 Kinematic and dynamic analysis of pantograph

The entire configuration of the six-legged walking robot considered in this study is shown in Fig. 1. The pantograph, which is evaluated as having good performance in energy efficiency side, is adapted to this study. The motions in the forward and gravitational direction of the pantograph with 90° skew angle are decoupled. It means that the actuator taking charge of the forward direction need not take any care for support of system. Therefore, it exhibits relatively good energy efficiency. In addition, the pantograph has kinematic features that are easier and simpler in control side. For example, for the pantograph shown in Fig. 2, the magnifying

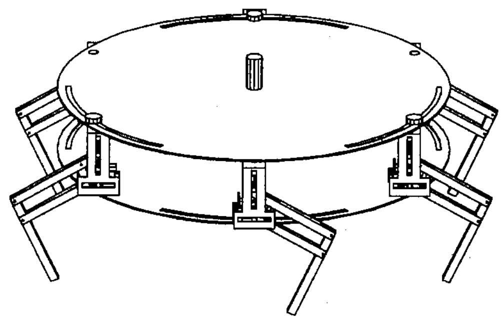


Fig. 1 Six-legged walking robot model

ratios between inputs, x, y, of actuator and outputs, which mean the variation of endpoint, are q/p and t/s. If lengths of the links in the pantograph are determined, the magnifying ratios are determined. Because the kinematic peculiarity of the end effector is very simple, the calculating loads for the kinematics and inverse kinematics taken in planning trajectory are less than those of other mechanisms.

For the leg of pantograph type, linear input motion results in linear motion of the end-effector of leg, through complex motion of coupled links. Then e.o.m can be expressed by two independent variables, i. e., x, y. And, joint variables α, β that appear during the derivation of e.o.m have to be transformed properly through geometrical consideration to be expressed in x and y. Therefore, e.o.m itself takes a complex form such as

$$A(\dot{x},x,\dot{y},\dot{y},y)\,\ddot{x}+B(x,\dot{y},y)\,\dot{x}+C(y)\,x=F_x$$
$$D(\ddot{x},\dot{x},x,\dot{y},y)\,\ddot{y}+E(\dot{x},x,y)\,\dot{y}+F(x)\,y=F_y \quad (1)$$

2.2 Dynamic formulation and Pseudo-inverse solution

We must take consideration of force allocation and gait problem to derive the full e.o.m of six-legged walking robot through e.o.m of pantograph derived in the last section. To begin with, it has to be considered that legs of the walking robot are doing closed loop or open loop motion repeatedly with the ground. And the forces of the leg end-effector have to be evaluated through a force balance equation. But the more legs that are

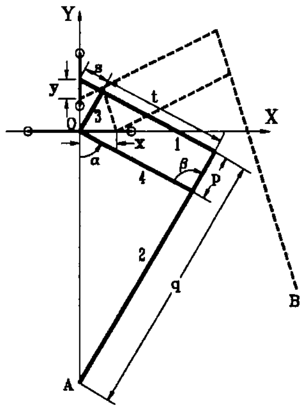


Fig. 2 Simplified configuration of pantograph leg

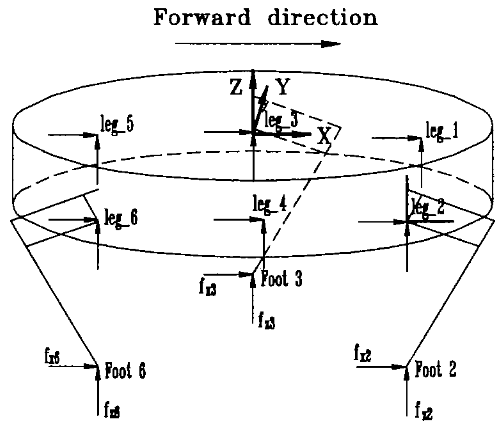


Fig. 3 Leg numbering, force index for walking robot

in contact with the ground the more difficult it is to solve the equation. Because the number of parameters to be evaluated is more than one of equations according as legs contacting with ground increase. Then we should evaluate unknown forces shown in the e. o. m through proper assumption. Primary assumptions in this study are no interaction forces between legs, no rotational motion of body, and straight constant speed locomotion.

The e.o.m for the entire system are

$$m\ddot{x}=\sum F_x,\ m\ddot{y}=\sum F_y=0,\ m\ddot{z}=\sum F_z=W$$
$$I_{xx}\ddot{\alpha}=\sum M_x=0,\ I_{yy}\ddot{\beta}=\sum M_y=0,\ I_{zz}\ddot{\gamma}=\sum M_z=0 \tag{2}$$

$$\sum F_p=\sum_{i=1}^{n}(f_p)_i+\sum_{j=1}^{6-n}(f_p)_j=0 \text{ or } W\,(p=z)$$

where

$$\sum M_p=\sum_{i=1}^{n}(r_i\times f_i)_p+\sum_{j=1}^{6-n}(r_j\times f_j)_p=0$$

Here p is x, y, z respectively, W is a body weight and n is the number of legs which are contacting with ground. Then, $(f_p)_i$, $(r_i\times f_i)_p$ are forces and moments of legs on the support phase respectively. The special feature in e.o.m given above is that all dynamic effects of legs on transfer phase are being considered. But, force terms of legs on the transfer phase are different from ones of legs on the support phase. Namely, forces of legs on the transfer phase are the reaction ones against actuator that charge the dynamic motion of pantograph, but forces of legs on the support phase are the driving ones of legs themselves divided by magnifying factor plus the reaction ones against ground. Previous works on legged locomotion neglected the dynamic terms of legs in walking. But walking with high constant body speed like this study leads to other problems.

Expressing the force balance equation of z-axis in (2) into matrix form (see Fig. 3),

$$Af_z=w \tag{3}$$

where $A=\begin{bmatrix}1 & 1 & \cdots & 1\\ x_1 & x_2 & \cdots & x_n\\ y_1 & y_2 & \cdots & y_n\end{bmatrix}$, $f_z=[f_{z1},f_{z2},\cdots,f_{zn}]^T$

$$w=[\,Wf_Tg_T]^T$$

Here $f_T=-\sum_{i=1}^{n}z_if_{xi}-\sum_{j=1}^{6-n}(x_jf_{zj}+z_jf_{xj})$, $g_T=-\sum_{i=1}^{n}z_if_{yi}-\sum_{j=1}^{6-n}(x_jf_{zj}+z_jf_{yj})$, and n is the numbers of legs contacting with ground.

But, the solutions of (3) are getting indeterminate. Therefore as the number of legs contacting with ground increases, we evaluate the normal forces by the following algebraic method:

$$f_z=A^+w \tag{4}$$

where A^+ is the pseudo-inverse of A, and is $A^T(AA^T)^{-1}$.

3. Dynamic Analysis of Legged Locomotion

3.1 Trajectory planning considering gait

The dynamic simulation for a walking robot is performed by the procedure described in Fig. 4. To begin with, a trajectory of the body is planned, and thereby ones of each leg are planned. And the inputs of each joint are calculated by the plans. Here, the trajectory planning of a legged robot is different from ones of other industrial robots. The principal differences are that the legs of the legged robot do open loop and closed loop motions with the ground repeatedly, and the number of legs consisting of closed loop varies according to the duty factor. Here, duty factor β is the time fraction of a cycle time in which a leg is in the support phase. For example, the gait diagram, which describes the sequence of placing and lifting of different feet and the duration of the support and transfer phases of each leg per one cycle, for the wave gait whose duty factor is 2/3 is shown in Fig. 5 (Song et al., 1989). Based on this type of diagram, the repeated motions of each leg should be formulated and considered in trajectory planning. Sometimes it is necessary to specify the motion in much more detail than simply stating the desired final configuration. One way to include more detail in a path description is to give a sequence of desired via points or intermediate points between the initial and final positions. But, the path points considered in locomotion are generally only the initial and final points. And,

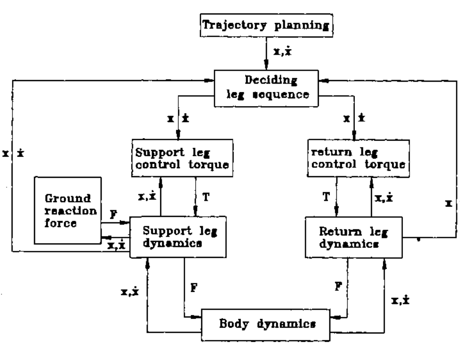


Fig. 4 Flow chart of dynamic simulation

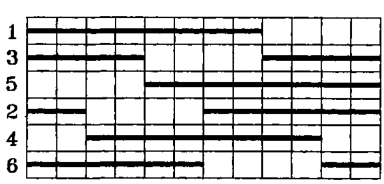


Fig. 5 Gait diagram ($\beta=2/3$)

the path shape (in space and time) is specified by various constraints. Here, we wish to be able to specify the position, velocity, and acceleration at the placing and lifting time, because it is necessary to keep the velocity of the leg end-effector in the gravitational direction at the landing instant to zero in order to minimize the impact against ground. Then, a 5th order polynomial is selected to represent the path shape.

3.2 Dynamic effect of leg on the transfer phase

We can ensure the following results through dynamic simulation of a six-legged walking robot with legs of pantograph type. Namely, having the forward direction of the body be the x-axis (referred to Fig. 3), planning trajectories of each simulation by that planning, we gain the following result. Below all simulation results are performed on the basis of numerical data given in Table 1. The data are determined with magnifying ratios of the x, y-axis to be 5, 4, which were evaluated

Table 1 Design parameters used in dynamic simulation

Link	Length (m)	Mass (kg)	Mass moment of inertia ($kg \cdot m^2$)		
			I_{xx}	I_{xx}	I_{xx}
1	0.7	0.84	10^{-6}	0.035	10^{-6}
2	1.0	1.2	10^{-6}	0.101	10^{-6}
3	0.2	0.24	10^{-6}	0.001	10^{-6}
4	0.56	0.672	10^{-6}	0.018	10^{-6}
Body	Radius (0.94)	53	0.9	1.2	0,9

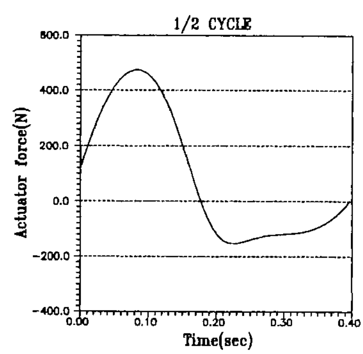
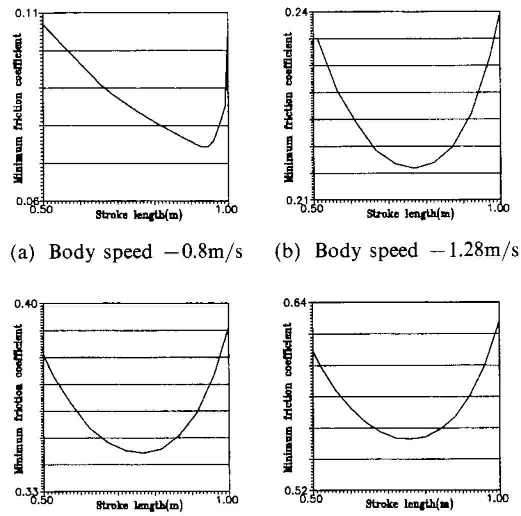


Fig. 6 Actuator force for a leg on the transfer phase

as having good energy efficiency (Song et al., 1989), and referred to the CAD data of prototype (see Fig. 1) that was designed to secure proper walking volume and structural stiffness. Figure 6 shows the needed x-axis actuator force of second leg in the case that the x-axis input on the transfer phase is given to 5th order polynomial. In this figure the driving force of the leg on the transfer phase comes up to 500 N for a body speed 2.0 m/s and is transmitted to the body. As the desired mass of the body considered in this study is 53 kg, neglecting reaction force acting on the body against the leg driving force, 500 N, has the same meaning as neglecting the body weight. Dynamic effects of the leg on the transfer phase could be ascertained by simulation through ADAMS (Dynamic Simulation Package) as well. Then, for the body to walk with constant speed, the legs on the support phase should make up for these kinds of loads. Otherwise, it is impossible for the body to keep constant speed.

3.3 Relationship between stroke and friction coefficient

If the stroke length varies under constraint of the constant body speed, the path shapes of the legs on the transfer phase also vary. Then, the reaction forces acting on the body against driving forces of the legs vary also, and the forces are transmitted to end-effector of legs on the support phase. Therefore, the minimum friction forces against the tangential forces of the end-effectors are determined so that the feet don't slip on the ground. It means that if the friction forces between the feet and given ground are less than those of the end-effectors transmitted from the legs on the transfer phase plus driving fores of the legs themselves on the support phase, the feet slip. Figure 7 shows the minimally necessary conditions of the ground to prevent the feet on the support phase from slipping according to stroke length, for the case of duty factors equal to 1/2. On the figures, the friction coefficient means the minimal one between feet and ground. The meaningful features capable of confirming from the figures are as follows. First, all the shapes of plots representing the minimum friction coefficient take



(a) Body speed −0.8m/s (b) Body speed −1.28m/s

(c) Body speed −1.6m/s (d) Body speed −2.0m/s

Fig. 7 Variation of friction coefficient according to stroke length.

convex forms. Because a decrease in stroke results in a decrease in the rising time taken for acceleration, an increase of driving force, and an increase of stroke for any supporting leg located far from the body center, results in a decrease in normal force. Then, there exists an optimal stroke to avoid feet-slipping under constant body speed. Second, as the body speed increases, the optimal stroke approaches a certain value, 0.77m. Expressing the value into a ratio wrt. body pitch, the ratio is about 0. 82 because the pitch of the body considered in this study is 0.94m. But for other duty factors these rules do not apply any more.

3.4 Relationship between duty factor and friction coefficient

The duty factor has a close connection with the body speed and stability. Usually, if legs of the legged robot contacting the ground are more than three, and the mass center of the robot is kept within polygon containing the contact point, we can say that the robot is in a statically stable state. For a six-legged robot, it is possible to sustain the stability in the case that the duty factor is more than 1/2.

For the case that the duty factor is 1/2, we already examined the relationship between duty

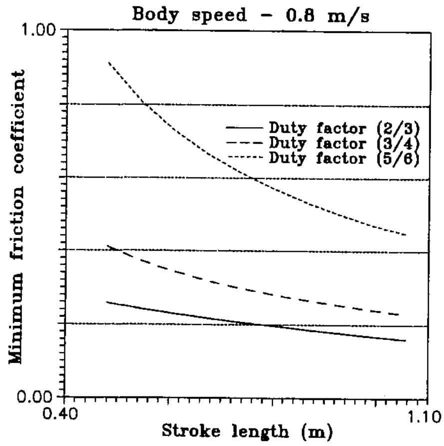


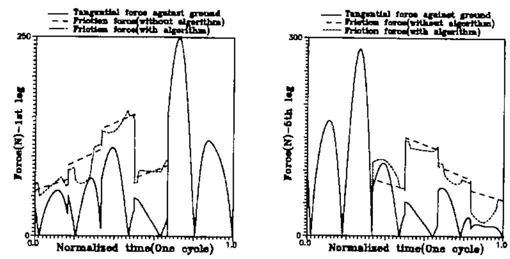
Fig. 8 Variation of friction coefficient according to duty factor

factor and friction coefficient. But, for duty facotrs other than 1/2, different results occur. Namely, for duty factor being 1/2, the shape of the minimum friction coefficient according to stroke length takes convex form. On the contrary, for other duty factors, the magnitude of coefficients decreases as the stroke length increases (see Fig. 8). In addition, the coefficients are in proportion to duty factors. This result is a reasonable one, because the normal forces per foot get much larger as the legs contacting the ground decrease, and an increase in normal force means one of static friction force. Namely, under ground condition with the same friction coefficient, the increase of normal force leads to one of static friction force.

4. Maximization of Walkable Zone Without Slipping

4.1 Introduction of weight vector

To keep the body speed constantly and endure the dynamic effects of the leg on the transfer phase, there is excessive up and down in x-axis actuator forces of legs on the support phase. Therefore, the peak forces due to a sudden increase of tangential force results in the ascent of the minimum friction coefficient. It means a reduction of the walkable zone without slipping. As explained before, in case of arbitrary duty factors the normal forces of the feet were evaluated by the pseudo-inverse method. But because of limitation of the solutions, we evaluate them as follows:



(a) First leg (b) Fifth leg

Fig. 9 Simulation results using the target vector

$$f_z = A^+ w + [I - A^T A] p \quad (5)$$

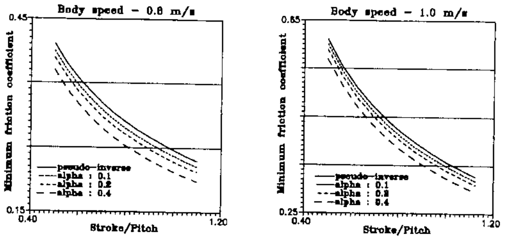
where p is a homogeneous solution of $Af_z = w$ and is the projection of any compatible vector on the null space of A. Thus the total solution of (5) is the nearest one to a given vector p. For the problem of determining a force set-point, the homogeneous solutions have been suggested to minimize discontinuities in commanded forces when the leg phase alternates between support and transfer. In this study, the target vector p is defined to trace the actuator forces of the legs during the transfer phase, with the norm of the pseudo-inverse solution kept constantly. It is as follows:

$$p = \alpha[F_{actuator}] + \beta[A^+ w] \quad (6)$$

α : weight value for actuator force

β : weight value for pseudo-inverse$(\alpha + \beta = 1)$

When α is 0.2~0.5, the effect of the above weight value is the largest. Simulating with this target vector, we can make sure that the friction coefficient is decreasing. Figure 9 shows comparative results based on the pseudo-inverse solution with one based on the target vector described in (6) for the case when the body speed is 1.6 m/s, the duty factor is 2/3, and α is 0.5. In this figure, the friction force is being regulated to the actuator force. In fact, the force regulated is not friction force but the normal force. Namely, we can manipulate the magnitude of the static friction force by regulating the normal force. Figure 10 shows the effect of the weight value for the case



(a) Body speed – 0.8m/s (b) Body speed – 1.0m/s

Fig. 10 Comparison of effect according to weight value

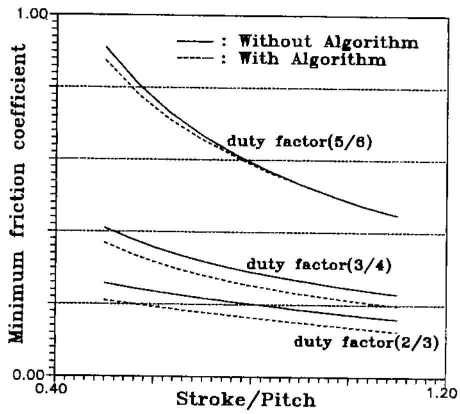


Fig. 11 Comparison of effect of target vector according to duty factor

when the duty factor is 3/4, and the body speeds are 0.8 m/s, 1.0 m/s respectively. As mentioned above, there exists a suitable weight value. And the effects of introducing target vector according to duty factor are shown in Fig. 11. In the figure, the case without algorithm means utilizing only the pseudo-inverse solution, while the one with algorithm considers the target vector. In the figure, we can confirm that the smaller the duty factor is, the lager the effects of introducing the target vector.

4.2 Optimization of force allocation

We could make sure that the introduction of the target vector is fairly effective. But because of discontinuous forces, there is a limitation in adaptation to the actual system. Then, the force solution is composed of the pseudo-inverse solution, the target vector and a perturbation in force:

$$f_z = A^+w + [I - A^TA]\{\alpha[F_{actuator}] + \beta[A^+w]\} + \Delta f \qquad (7)$$

Above Δf is the element introduced for continuity and optimization of force. This is one of the homogeneous solutions that physically means the interaction force between legs. These terms can exist in the case that the number of unknown variables is less than the number of given equations. In this case, there exist infinite solutions for $Af_z = w$, and Δf is just one solution to the equation. For optimization of the solution given like (7), the introduced objective function, equality and non-equality constraints are as follows:

— Objective function

Minimize Φ

$$\Phi = \sum_{i=1}^{n} (f_{zi}^{+} + \Delta f_i - f_{zi}^{-})^2 + (\mu(f_{zi}^{+} + \Delta f_i) - f_{xi})^2 \qquad (8)$$

where n is the number of legs contacting the ground, the first term on the right side is an element to reduce discontinuity of the force, and the second term is an element to minimize the difference between friction force and driving force.

— Equality constraints

$$A\Delta f = 0 \qquad (9)$$

where $A = \begin{bmatrix} 1 & 1 \cdots 1 \\ x_1 x_2 \cdots x_n \\ y_1 y_2 \cdots y_n \end{bmatrix}$, $\Delta f = [\Delta f_1 \Delta f_2 \cdots \Delta f_n]^T$

— Non-equality constraints

$$|f_{xi}| \leq \mu f_{zi} \qquad (10)$$

$$(f_z)_{\min} \leq f_{zi} \leq (f_z)_{\max} \qquad (11)$$

$$|f_{zi}^{+} + \Delta f_i - f_{zi}^{-}| \leq \varepsilon \qquad (12)$$

where (10) is the no slip condition, (11) is a condition to assure the definite contact of feet with the ground and to prevent the unfeasibility for z-axis forces, and (12) is a condition preventing discontinuity due to Δf. It is possible to sustain continuity taking ε to 0.3~0.5. For a duty factor of 2/3, because legs contacting with the ground is four, there are totally 15 constraints includings 3 equality constraints and 12 non-equality constraints. Since the number of constraints involved is large, it is impossible to apply the optimization algorithm for practical system in real time. Then, it is proper to make use of look-

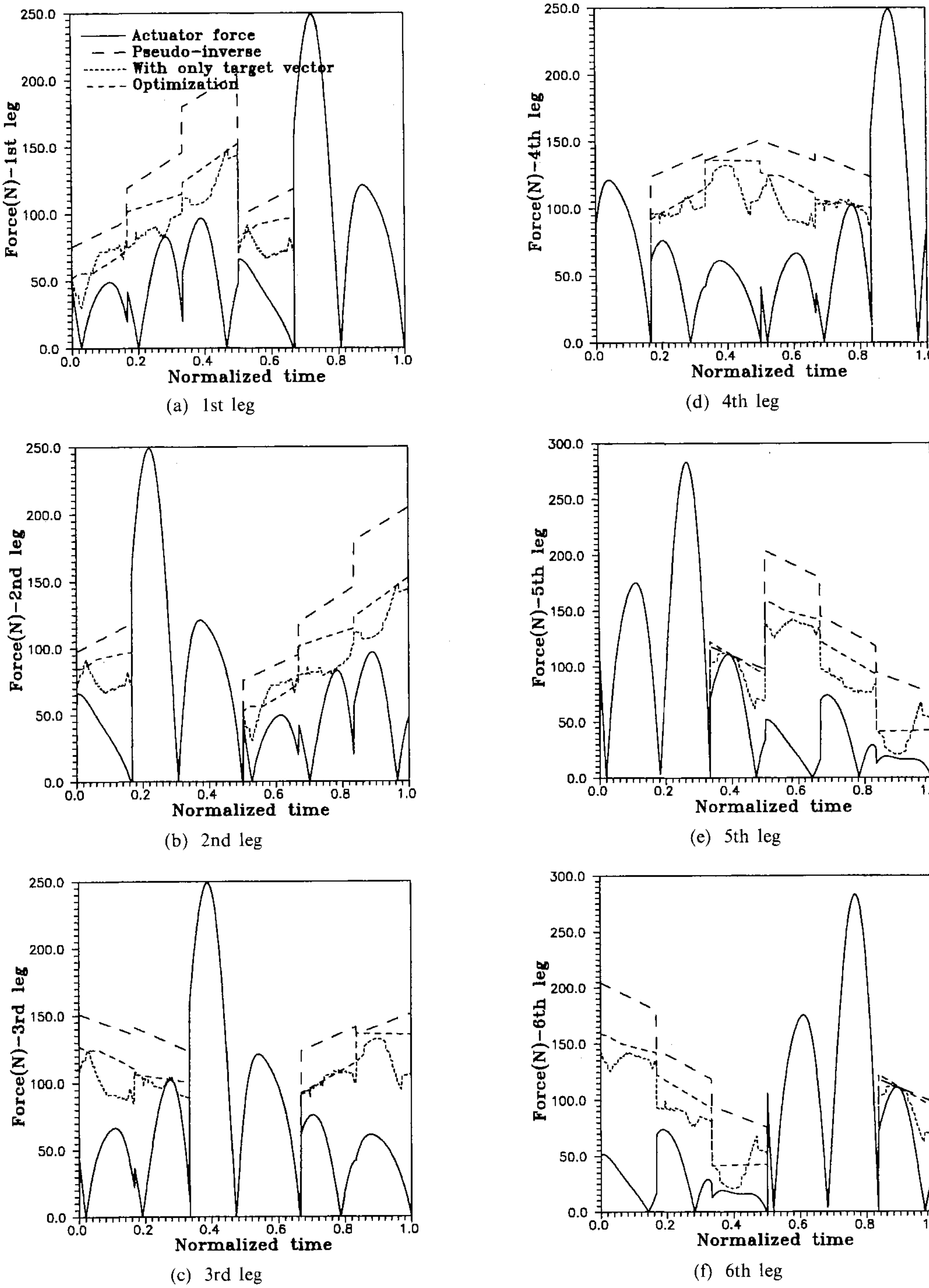


Fig. 12 Simulation result for optimized solution

up table made by optimization algorithm at the high-level control stage. But, as the walkable zone gets smaller, the solution approaches the pseudo-inverse solution, and it is possible to perform the calculation in real time.

The optimization was performed in terms of

ADS (A Fortran Program for Automated Design Synthesis) (Vanderplaats, 1985). The Sequential Quadratic Programming algorithm is used together with Method of Feasible Direction (MFD) as the optimizer, and Golden section method as the dimensional search method. Figure 12 shows the results of this optimization algorithm for the case when the body speed is 1.6 m/s and duty factor 2/3. Compared with the solution evaluated by only the pseudo-inverse, this method extends the walkable zone without slipping more and more. But, because of the constraint condition to guarantee the continuity, the walkable zone gets smaller than the method utilizing only target vector. Practically, we can know the next facts from Fig. 12. Namely, for the pseudo-inverse solution, it is possible for the robot to walk without slipping on ground condition with friction coefficient larger than 0.763. But for the optimized case, the value is 0.561, and for the case considering only the target vector, 0.537. Here the value 0.537 is difficult to accept due to the problem of continuity. Then, through proper manipulation of the normal forces, it is possible to expand the walkable zone without slipping. In the case that the dynamic terms of legs on the transfer phase are considered as dominant elements, we can maximize the walkable zone without slipping by having normal forces trace the driving forces.

5. Conclusion and Future Works

In this study, we performed the dynamic analysis for a six-legged robot with high constant body speed. Different from existing studies, the force distribution algorithm based on a new concept was realized on the basis that the dominant force for legged-locomotion with high constant body speed is not only body force but also the inertia of the legs and their driving forces. The main issues were how the normal forces of the feet should be allocated to prevent feet-slippage due to friction, and how the walkable zone without slipping could be expanded when the only constraint condition is assumed to be friction force. The principal results proved through dynamic simulations are as follows.

(1) For the legged-robot with high constant body speed, the dominant terms to be considered are the driving forces of the legs on the transfer phase, and the reaction forces acting on the body against them. Then, it is impossible for the legged-robot to keep constant speed if the dynamic effects of the legs during the transfer phase are not considered.

(2) Under a constant duty factor, there exists an optimal stroke to maximize the walkable zone without slipping. The duty factors are in proportion to the minimum friction coefficient between feet and ground. The minimum friction coefficient means the minimally necessary condition for no slip of feet.

(3) By regulating the normal forces of the feet against the ground, the walkable zone can be expanded more and more. Namely, introducing target vectors in the pseudo-inverse solution, and having the normal forces trace the driving forces of legs, the zone can be expanded.

(4) By a suitable selection of weight value in the target vector the effects can be improved even more.

In the future, we will develop this study for not only constant body speed but also deceleration (and acceleration). As mentioned before, the optimization of solution caused by introduction of the target vector results in increase in calculation time; the next study will be focused on the realization of real time calculation.

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