Active Vibration Control of a Structure with Output Feedback Based on Simultaneous Optimization Design Method

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Recent advances in the field of control theory have enabled us to design active vibration control systems for various structures. In many studies, the controller used to suppress vibration has been synthesized for the given mathematical model of structure. In these cases, the designer has not been able to utilize the degree of freedom to adjust the structural parameters of the control object. To overcome this problem, so called "Structure/Control Simultaneous Optimization Method" is used. In this context of view, this paper is concerned with the active vibration control of bridge towers, platforms and ocean vehicles etc. Simultaneous design method is used to achieve optimal system performance. Here, a general framework for the simultaneous design problem of output feedback case is introduced based on LMI (Linear Matrix Inequality). The simulation results show that the proposed design method achieves desirable control performance.

Key Words : Active Vibration Control, Structural Parameter, Simultaneous Optimization, Output Feedback, Linear Matrix Inequality, Control Performance

1. Introduction

System design practice has become more interdisciplinary. This has been caused by increasingly demanding performance criteria and design specifications of all types of machines and structures in various fields. Passive control alone may not meet the high specifications. On the other hand, pure active control which has been applied to various control problems (Shin et al., 1996 ; Oh et al., 1998 ; Park et al., 1998) may be very expensive to realize. This has led researchers to integrate the passive and active control design in a certain optimal sense to satisfy the high demanding performance requirements (Iwatsube et al. 1993 ; Onoda, 1995 ; Obinata, 1997 ; Shi and Skelton, 1996 ; Tanaka and Sugie, 1998).

The modeling and control problems are not independent. The structure design and control design are not separable and necessarily are iterative. This paper introduces an iterative algorithm to integrate structure and control design. Specifically, the algorithm simultaneously finds: i) the optimal values of the stiffness, damping ratios, and actuator location parameters, and ii) an optimal stabilizing state feedback and output feedback controller such that the active control energy is minimized subject to: a) the prespecified RMS constraints on the outputs, and b) the constraints on the structure parameters. The algorithm provides a systematic approach to tune the structure parameters and design an active controller. This algorithm is applied to vibration control system design for a structure. It can be easily extended to anti-roll control system design problems of ocean vehicles, platforms, etc.

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2. Problem Formulation

Consider a linear time-invariant dynamic model for a structure illustrated in Fig. 1 with the following representation:

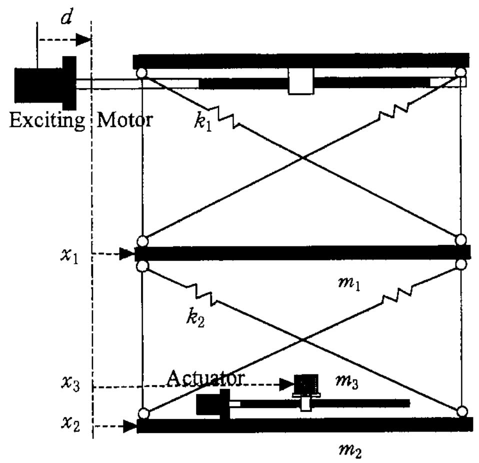


Fig. 1 Schematic diagram of a controlled structure

$$M\ddot{q}+D(\alpha)\dot{q}+K(\beta)q=b_1w+b_2(\delta)u$$
$$z_1=u,\ z_2=C[q\ \dot{q}]^T,\ y=z_2 \tag{1}$$

where, $q\in R^{n_q}$ is the displacement vector, $\dot{q}$ and $\ddot{q}$ are the velocity and acceleration vectors, respectively. And, $u\in R^{n_u}$ is the control input vector, $z_1\in R^{n_{z1}}$ and $z_2\in R^{n_{z2}}$ are the output vectors to be regulated, $y\in R^{n_y}$ is the vector of measurements, $w\in R^{n_w}$ is the disturbance input.

$$D(\alpha)=D+\Delta D(\alpha)=D+\sum_{i=1}^{d}\alpha_iD_i$$
$$K(\beta)=K+\Delta K(\beta)=K+\sum_{j=1}^{k}\beta_jK_j \tag{2}$$
$$b_2(\delta)=b_2+\Delta b_2(\delta)=b_2+\sum_{l=1}^{b}\delta_lb_l$$

where, $\alpha=[\alpha_1\ \alpha_2\ \cdots\ \alpha_d]$ is the set of system parameters that can be designed to adjust the system damping, $\beta=[\beta_1\ \beta_2\ \cdots\ \beta_k]$ is the set of system parameters that can be designed to adjust the system stiffness, $\delta=[\delta_1\ \delta_2\ \cdots\ \delta_l]$ is the vector of actuator and location parameters, matrices D_i, K_j and b_l are the corresponding basis matrices. The matrices M, D, K, b_1, b_2 and C are constant matrices with appropriate dimensions that represent the nominal structure design. Also

$$\alpha\in[\underline{\alpha},\ \bar{\alpha}],\ \beta\in[\underline{\beta},\ \bar{\beta}]$$

where, $\underline{\alpha}$, $\bar{\alpha}$, $\underline{\beta}$ and $\bar{\beta}$ are specified constants that represent the structure design constraints. Define $p=[\alpha\ \ \beta]$. The state space representation for the dynamic system (1) is given by

$$\dot{x}=A(p)x+b_1w+b_2(\delta)u$$
$$z_1=u \tag{3}$$
$$z_2=Cx$$

where,

$$x=\begin{bmatrix} q \\ \dot{q}\end{bmatrix},\ A(p)=A+\Delta A(p)$$
$$A=\begin{bmatrix} 0 & I_{n_q} \\ -M^{-1}K & -M^{-1}D\end{bmatrix}$$
$$\Delta A(p)=\begin{bmatrix} 0 & 0 \\ -M^{-1}\Delta K & -M^{-1}\Delta D\end{bmatrix}$$
$$B_2(\delta)=B_2+\Delta B_2(\delta)$$
$$B_2=\begin{bmatrix} 0 \\ M^{-1}b_2\end{bmatrix},\ \Delta B_2(\delta)=\begin{bmatrix} 0 \\ M^{-1}\Delta b_2(\delta)\end{bmatrix}$$
$$B_1=\begin{bmatrix} 0 \\ M^{-1}b_1\end{bmatrix}$$
$$p=[\alpha_1,\ \cdots,\ \alpha_d,\ \beta_1,\ \cdots,\ \beta_k]$$
$$\underline{p}=[\underline{\alpha}_1,\ \cdots,\ \underline{\alpha}_d,\ \underline{\beta}_1,\ \cdots,\ \underline{\beta}_k]$$
$$\bar{p}=[\bar{\alpha}_1,\ \cdots,\ \bar{\alpha}_d,\ \bar{\beta}_1,\ \cdots,\ \bar{\beta}_k].$$

Here, the system matrix $A(p)$ is a linear function of p. The matrices A, B_1, B_2, C are real constant with appropriate dimensions, and Δ means the variation around the nominal value. We shall make the following assumptions for the system (3).

Assumptions : For any $p\in[\underline{p}\ \bar{p}]$ and $\delta\in[\underline{\delta}\ \bar{\delta}]$,

(i) The pair $[A(p),\ B_2(\delta)]$ is stabilizable,

(ii) The pair $[A(p),\ B_1]$ is stabilizable.

3. H_∞ Performance Objective of State Feedback Case

Consider a LTI system described by

$$\dot{x}=Ax+B_1w+B_2u$$
$$z_1=C_1x+D_{11}w+D_{12}u \tag{4}$$
$$z_2=C_2x+D_{21}w+D_{22}u$$

where, x, w, u, z_1 and z_2 take values in finite dimensional vector spaces : $x\in R^n$, $w\in R^r$, $u\in R^m$, $z_1\in R^p$ and $z_2\in R^q$. The system parameters A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21} and D_{22} are matrices of appropriate dimensions. Based on these assumptions, the H_∞ performance objective of the state feedback case is defined as follows. For the system (4), given a performance

$$\|T_{zw}\|_\infty<\gamma,\ \gamma>0$$

find a state feedback controller $u=Gx$ such that the closed-loop system is asymptotically stable, where $z=[z_1^T\ z_2^T]$ and $\|T_{zw}\|_\infty$ denotes ∞-norm of T_{zw} which is the transfer function from w to z. The closed loop system is represented by

$$\begin{aligned}\dot{x}&=(A+B_2G)x+B_1w\\ z_1&=(C_1+D_{12}G)x+D_{11}w\\ z_2&=(C_2+D_{22}G)x+D_{21}w.\end{aligned}\tag{5}$$

Lemma 1 (Gahinet and Apkarian, 1994; Goh and Papavassilopoulos, 1994; Boyd and Ghaoui, 1993): Using Schur complement, the following inequality can be obtained as H_∞ constraint of the closed-loop system (5), for $X_\infty>0$,

$$\begin{bmatrix}(A+B_2G)X_\infty+X_\infty(A+B_2G)^T & B_1 & X_\infty(C_1+D_{12}G)^T\\ B_1^T & -\gamma I & D_{11}^T\\ (C_1+D_{12}G)X_\infty & D_{11} & -\gamma I\end{bmatrix}<0\tag{6}$$

Then the H_∞ constraint control problem is to find a controller $u=Gx$ such that it minimizes $\|T_{zw}\|_\infty<\gamma(>0)$.

4. LMI Based H_∞ Control with Output Feedback

In this section, we give the formulation of LMI based output feedback control in detail. So that the result to be introduced in this section is natural extension of the state feedback case. Here, the objective is to make clear this formulation for practical use.

Let us consider the system (4) and a proper real rational controller represented by

$$\begin{aligned}\dot{x}_c&=A_cx_c+B_cy\\ u&=C_cx_c+D_cy.\end{aligned}\tag{7}$$

Lemma 2 (Gahinet and Apkarian, 1994 ; Goh and Papavassilopoulos, 1994 ; Boyd and Ghaoui, 1993) : The system (4) is stabilizable with H_∞ disturbance attenuation γ via output feedback (7) if and only if there exist symmetric matrices R and S satisfying the following LMI system:

$$\left[\begin{array}{c|c}N_R & 0\\ \hline 0 & I\end{array}\right]^T\left[\begin{array}{cc|c}AR+RA^T & RC_1^T & B_1\\ C_1R & -\gamma I & D_{11}\\ \hline B_1^T & D_{11}^T & -\gamma I\end{array}\right]\left[\begin{array}{c|c}N_R & 0\\ \hline 0 & I\end{array}\right]<0$$

$$\left[\begin{array}{c|c}N_S & 0\\ \hline 0 & I\end{array}\right]^T\left[\begin{array}{cc|c}AS+SA^T & SB_1^T & C_1^T\\ B_1R & -\gamma I & D_{11}^T\\ \hline C_1 & D_{11} & -\gamma I\end{array}\right]\left[\begin{array}{c|c}N_S & 0\\ \hline 0 & I\end{array}\right]<0$$

$$\begin{bmatrix}R & 0\\ 0 & S\end{bmatrix}\geq 0\tag{8}$$

where, N_R and N_S denote bases of the null spaces of (B_2^T, D_{12}^T) and (C_2, D_{21}), respectively. In addition, there exist such controllers of order $k<n$ (reduced order), if and only if the above three LMIs hold for some R and S that further satisfy

$$\text{rank}(I-RS)\leq k.\tag{9}$$

Suppose that some solution (R, S) of the LMI systems (8) and (9) has been computed. We propose a method to construct an H_∞ controller from the obtained data. Here, we collect the controller parameters into a single variable Ξ which is to be used in the next formulation.

$$\Xi=\begin{bmatrix}A_c & B_c\\ C_c & D_c\end{bmatrix}.\tag{10}$$

The matrices of closed-loop system are obtained by

$$\begin{aligned}A_{cl}&=A_0+\tilde{B}\Xi\tilde{C},\ B_{cl}=B_0+\tilde{B}\Xi\tilde{D}_{21}\\ C_{cl}&=C_0+\tilde{D}_{12}\Xi\tilde{C},\ D_{cl}=D_0+\tilde{D}_{12}\Xi\tilde{D}_{21}\end{aligned}\tag{11}$$

where,

$$\begin{aligned}&A_0=\begin{bmatrix}A & 0\\ 0 & 0\end{bmatrix},\ B_0=\begin{bmatrix}B_1\\ 0\end{bmatrix},\ C_0=[C_1\ 0]\\ &\tilde{B}=\begin{bmatrix}0 & B_2\\ I & 0\end{bmatrix},\ \tilde{C}=\begin{bmatrix}0 & I\\ C_2 & 0\end{bmatrix},\ \tilde{D}_{12}=[0\ D_{12}],\\ &\tilde{D}_{21}=\begin{bmatrix}0\\ D_{21}\end{bmatrix}.\end{aligned}\tag{12}$$

From the state-space realization of the plant and controller, let

$$\begin{aligned}\dot{x}_{cl}&=A_{cl}x_{cl}+B_{cl}w\\ z_1&=C_{cl1}x_{cl}+D_{cl1}w\\ z_2&=C_{cl2}x_{cl}+D_{cl2}w\end{aligned}\tag{13}$$

$$z=\begin{bmatrix}z_1\\ z_2\end{bmatrix},\ C_{cl}=\begin{bmatrix}C_{cl1}\\ C_{cl2}\end{bmatrix},\ D_{cl}=\begin{bmatrix}D_{cl1}\\ D_{cl2}\end{bmatrix}$$

be the corresponding closed-loop state-space equations.

We are now in the position to state the procedure.

[H_∞ Controller Construction Procedure]

1. Compute two full-column-rank matrices

N, $N \in \mathbb{R}^{n \times k}$ satisfying

$$MN^T = I - RS. \tag{14}$$

2. Solve the following linear equation about X_{cl} :

$$\begin{bmatrix} S & I \\ N^T & 0 \end{bmatrix} = X_{cl} \begin{bmatrix} I & R \\ 0 & M^T \end{bmatrix}. \tag{15}$$

3. Solve the matrix inequality

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix}$$
$$= \begin{bmatrix} A_0^T X_{cl} + X_{cl} A_0 & X_{cl} B_0 & C_0^T \\ B_0^T X_{cl} & -\gamma I & D_0^T \\ C_0 & D_0 & -\gamma I \end{bmatrix}$$
$$+ \begin{bmatrix} \tilde{C}^T \\ \tilde{D}_{21}^T \\ 0 \end{bmatrix} \Xi^T [\tilde{B}^T X_{cl} \ 0 \ \tilde{D}_{12}^T]$$
$$+ \left\{ \begin{bmatrix} \tilde{C}^T \\ \tilde{D}_{21}^T \\ 0 \end{bmatrix} \Xi^T [\tilde{B}^T X_{cl} \ 0 \ \tilde{D}_{12}^T] \right\}^T < 0 \tag{16}$$

for the matrix variable Ξ. Then, the matrices A_c, B_c, C_c and D_c are obtained from the solution Ξ.

This procedure will be applied to compute H_∞ output feedback controller for simultaneous optimization design problem.

5. The Algorithm to Integrate Structure and Control System Design Based on Optimization Design Method

Here, our problem of finding a solution to inequality (16) can be embedded in the parametrized family of problems :

$$\Omega(X_{cl}, p, y_c(\delta, \Xi), \gamma) > 0 \tag{17}$$

where, $y_c(\delta, \Xi)$ denotes the function of actuator parameter and dynamic controller.

At first, we consider the following subproblems.

5.1 (X_{cl}, p)-Optimization

Here, we consider the optimal selection of the structure parameter p.

Suppose the solution of inequality (16), $(X_{cln}, p_n, y_{cn}(\delta, \Xi), \gamma_n)$ is given for the nominal system (4), and let the matrix $y_{cn}(\delta, \Xi)$ be fixed. Then we consider the following Matrix Inequality.

$$\begin{bmatrix} A_{cl}^T(p) X_{cln} + X_{cln} A_{cl}(p) & X_{cln} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cln} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{18}$$

Theorem 1 : Suppose the actuator parameter and dynamic controller $y_{cn}(\delta, \Xi)$ is given for the nominal system. Then the (X_{cl}, p)-Optimization problem is represented as follows:

$$\min_{(X_{cl}, p, \gamma)} \gamma, \quad \text{subject to Inequality (18)}, \quad \underline{p} < p < \bar{p} \tag{19}$$

The optimal solution is denoted as $(X_{cln+1} p_{n+1}, \gamma_{n+1})$. Such that the sub-optimal dynamic controller and actuator parameters are determined from $y_{cn}(\delta, \Xi)$, yield $\gamma_{n+1} \le \gamma_n$.

5.2 $(X_{cl}, y_c(\delta, \Xi))$-Optimization

Here, we consider the selection for the actuator location parameter δ and dynamic controller parameter Ξ.

Suppose the solution of the (X_{cl}, p)-Design Loop, op, $(X_{cln+1} p_{n+1}, \gamma_{n+1})$ is given for the system with a fixed $y_{cn}^*(\delta, \Xi)$. Then the problem of $(X_{cl}, y_c(\delta, \Xi))$-Optimization is to find a new controller subject to H_∞ constraints. This problem can be formulated as follows.

Theorem 2 : Let the solution $(X_{cln+1}, p_{n+1}, \gamma_{n+1})$ to the (X_{cl}, p)-Design Loop be given. Then the $(X_{cl}, y_c(\delta, \Xi))$-Optimization is given as follows:

$$\min_{(X_{cl}, y_c(\delta, \Xi), \gamma)} \gamma,$$
subject to
$$\begin{bmatrix} A_{cl}^T(y_c) X_{cl} + X_{cl} A_{cl}(y_c) & X_{cl} B_{cl}(y_c) & C_{cl}^T(y_c) \\ B_{cl}^T(y_c) X_{cl} & -\gamma I & D_{cl}^T(y_c) \\ C_{cl}(y_c) & D_{cl}(y_c) & -\gamma I \end{bmatrix} < 0$$
$$\underline{\delta} < \delta < \bar{\delta}. \tag{20}$$

The optimal solution is denoted as $(X_{cln+2}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$. Then $\gamma_{n+2} \le \gamma_{n+1}$.

This problem gives the optimal (in general suboptimal) value for the parameter δ and new controller Ξ, satisfying the H_∞ constraint. Now, an algorithm is given to integrate structure and control system design. The basic idea is to iterate the (X_{cl}, p)-Optimization and the $(X_{cl}, y_c(\delta, \Xi))$-Optimization Loop.

Algorithm : *Consider the system (4), and set* $p_0=0$, $\delta_0=0$, $n=0$. *Suppose that* Ξ *is given for the nominal system with the numerical tolerance* $\varepsilon>0$.

Step 1 : Solve the (X_{cl}, p)*-Design Loop with a fixed* $y_{cn}(\delta, \Xi)$ *to get* $(X_{cln+1}p_{n+1}, \gamma_{n+1})$.

Step 2 : Solve the $(X_{cl}, y_c(\delta, \Xi))$*-Optimization problem to get* $(X_{cln+2}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$.

Step 3 : If $|\gamma_{n+1}-\gamma_{n+2}|>\varepsilon$, *go to Step 1, otherwise, output* $(X_{cln+2}, p_{n+1}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$.

Step 4 : The actuator and controller parameters are obtained as follows : $y_{cn+1}(\delta, \Xi)$.

6. Simulation

The model of the design object shown in Fig. 1 is considered. The equation of motion for this system is given by

$$\begin{aligned} m_1(\ddot{x}_1+\ddot{d}) &= -c_1\dot{x}_1-k_1x_1-c_2(\dot{x}_1-\dot{x}_2) \\ &\quad -k_2(x_1-x_2) \\ m_2(\ddot{x}_2+\ddot{d}) &= -f-c_2(\dot{x}_2-\dot{x}_1)-k_2(x_2-x_1) \\ &\quad -c_3(\dot{x}_2-\dot{x}_3) \\ m_3(\ddot{x}_3+\ddot{d}) &= f-c_3(\dot{x}_3-\dot{x}_2) \end{aligned} \tag{21}$$

where, m_i, $c_i(i=1, 2, 3)$ and $k_i(j=1, 2)$ are mass, damping and stiffness, respectively. Especially, m_3 is active mass to suppress the vibration. And x_i denotes the state displacement. If we let

$$x_{1d}=x_1+d,\ x_{2d}=x_2+d,\ x_{3d}=x_3+d,\ f=k_Tu$$

Table 1 Estimated parameters

Parameters		Values	Unit
Mass	m_1	150.30	kg
	m_2	116.50	kg
	m_3	to be designed	kg
Damping coef.	c_1	29.12	N/(m/s)
	c_2	14.22	N/(m/s)
	c_3	11.33	N/(m/s)
Stiffness coef.	k_1	28,812.00	N/m
	k_2	25,855.00	N/m
Montor torque coef.	k_T	to be designed	N/A

then the Eq. (21) is represented by

$$\begin{aligned} m_1\ddot{x}_{1d} &= -(c_1+c_2)\dot{x}_{1d}+c_2\dot{x}_{2d}-(k_1+k_2)x_{1d} \\ &\quad +k_2x_{2d}+c_1\dot{d}+k_1d \\ m_2\ddot{x}_{2d} &= c_1\dot{x}_{1d}-(c_2+c_3)\dot{x}_{2d}+c_3\dot{x}_{3d} \\ &\quad +k_2\dot{x}_{1d}+k_2x_{2d}-k_Tu \\ m_3\ddot{x}_{3d} &= c_3\dot{x}_{2d}-c_3\dot{x}_{3d}+k_Tu \end{aligned} \tag{22}$$

where, k^T and u denote the torque constant and control input, respectively. And consider

$$x_{4d}=\dot{x}_{1d},\ x_{5d}=\dot{x}_{2d},\ x_{6d}=\dot{x}_{3d} \tag{23}$$

then, the system description is given by

$$\begin{aligned} \dot{x} &= A(p)x+B_2(\delta)u+E_1\dot{d}+E_2d \\ z_1 &= u \\ z_2 &= C\ x \end{aligned} \tag{24}$$

where,

$$x=[x_{1d}\ x_{2d}\ x_{3d}\ x_{4d}\ x_{5d}\ x_{6d}]^T \tag{25}$$

In this paper, it is assumed that the passive parameter $p(m_1, m_2, k_1$ and $k_2)$ is fixed. Then, optimization parameters are $\delta(m_3, k_T)$ and Ξ (controller parameter). And the system matrices of (24) are represented by

$$A(\delta)=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -(k_1+k_2)/m_1 & k_2/m_1 & 0 & -(c_1+c_2)/m_1 & c_2/m_1 & 0 \\ k_2/m_2 & -k_2/m_2 & 0 & c_2/m_2 & -(c_2+c_3)/m_2 & c_3/m_2 \\ 0 & 0 & 0 & 0 & c_3/m_3 & -c_3/m_3 \end{bmatrix}$$

$$\begin{aligned} B_2(\delta) &= [0\ 0\ 0\ 0\ -k_T/m_2\ k_T/m_3]^T \\ E_1 &= [0\ 0\ 0\ c_1/m_1\ 0\ 0]^T \\ E_2 &= [0\ 0\ 0\ k_1/m_1\ 0\ 0]^T \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{26}$$

Suppose

$$B_1=[E_2\ E_1],\ w=\begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{27}$$

then, (24) is represented as follows :

$$\begin{aligned} \dot{x} &= A(\delta)x+B_1w+B_2(\delta)u \\ z_1 &= u \\ z_2 &= Cx. \end{aligned} \tag{28}$$

Suppose we have freedom to redesign the values for the active mass m_3 and torque constant k_T. That is, the actuator redesign parameters are

$$\delta_1 = k_T,\ \delta_2 = m_3.$$

The constraint on the redesign parameter is given by

$$\underline{\delta}_i = 0.1,\ \overline{\delta}_i = \infty,\ i = 1,\ 2.$$

The algorithm of section 5 gives the optimal variation of $\delta(k_T,\ m_3)$ with the parameter $\gamma(>0)$, which is illustrated in (a) of Fig. 2. And the control performance index γ in each iteration is plotted in (b) of Fig. 2. From this result, the parameter $k_T=1.8$, $m_3=2.2$ for optimal values are chosen. Where $\gamma=0.0086$.

The controller parameter is given by

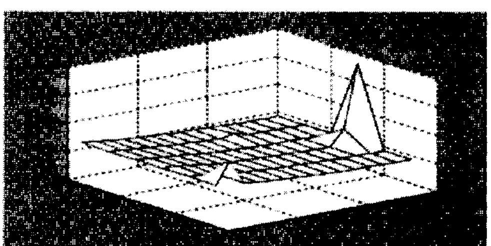
$$\Xi = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}$$

where

$$A_c = \begin{bmatrix} 1.008\times10^2 & 1.562\times10^2 & 5.556\times10 & 2.645\times10 & 2.223\times10 & 8.508 \\ 4.457 & -6.621\times10 & -2.720 & -6.903 & -1.005\times10 & -6.907 \\ 6.239\times10 & 6.615\times10 & 2.848 & 1.766 & 1.075\times10 & -4.881 \\ -2.572\times10^2 & 3.405\times10^2 & 1.290 & 4.598\times10 & 4.866\times10 & 2.278 \\ 3.767 & -1.475 & -3.248 & 3.465\times10^{-2} & -1.644\times10^{-1} & -4.339 \\ -1.016\times10^2 & 1.665\times10^2 & 5.998 & 2.760\times10 & 2.381\times10 & 8.465 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -5.653\times10 & -6.911\times10^{-2} \\ 4.167\times10 & 5.095\times10^{-2} \\ -7.252\times10 & -8.868\times10^{-2} \\ -1.049\times10^2 & -1.283\times10^{-1} \\ -1.080 & -1.313\times10^{-3} \\ -6.336\times10 & -7.746\times10^{-2} \end{bmatrix},$$

$$C_c = \begin{bmatrix} 6.504\times10^{-6} \\ -1.016\times10^{-7} \\ -4.737\times10^{-8} \\ 5.418\times10^{-7} \\ 7.963\times10^{-7} \\ -7.465\times10^{-6} \end{bmatrix},$$



(a)

(b)

Fig. 2 Performances with redesign process (k_T, m_3, γ and iteration index)

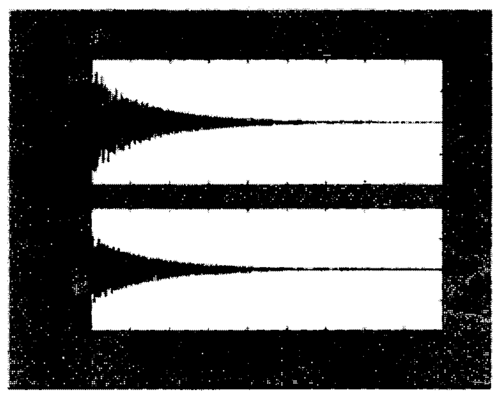


Fig. 3 Impulse responses (uncontrolled cases)

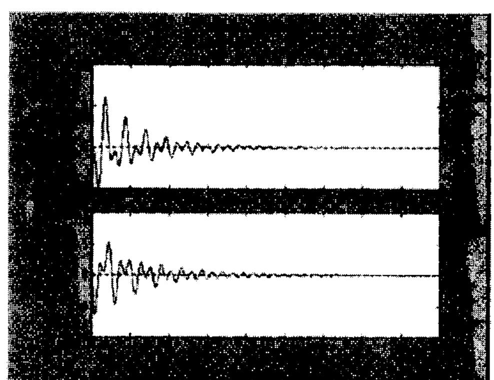


Fig. 4 Impulse responses (controlled cases) (Simultaneous optimization control)

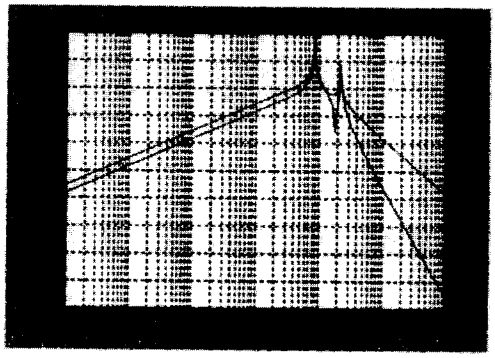


Fig. 5 Frequency responses (uncontrolled cases)

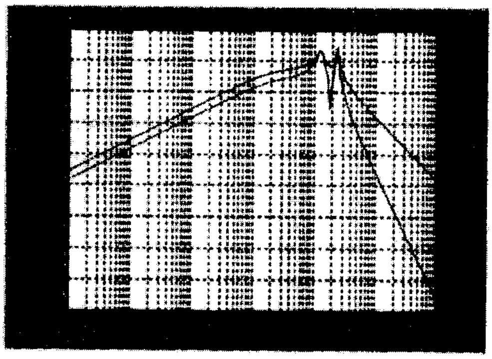


Fig. 6 Frequency responses (controlled cases)

$D_c = [-2.200 \quad -1.100 \times 10^3]$.

The impulse responses of the open-loop structure is shown in Fig. 3. Figure 4 denotes the impulse responses of the closed-loop system associated with the redesigned closed-loop structure and the controller. The frequency responses of the systems with control and without control are shown in Figs. 5 and 6.

7. Concluding Remarks

In this paper, an algorithm for integrating structure and control system design of output feedback case is proposed. The optimization problem is divided into two subproblems with iteration between the two. The first subproblem gives the optimal values for the structure passive parameters. The second subproblem gives the optimal values for the actuator location parameters and the controller with the constraints. This approach has been applied to the vibration control of a structure. In practice, it is clear that this approach is very useful to the control system design because the high demanding performance requirements can be achieved.

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