# Dynamics and Control of 2 DOF 5-bar Parallel Manipulator with Closed Chain

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## ABSTRACT

A method is proposed to obtain the Jacobian matrix of the 5-bar parallel manipulator by employing the orthogonality between position and velocity vectors of rotating rigid-body around a fixed point. The dynamics of the 5-bar parallel manipulator is analyzed and utilized to design the computed-torque controller by developing a transformation matrix of the passive joints with respect to the active ones. In experimental demonstration, it shows that high-speed and accuracy tasks are performed by the proposed computed-torque control.

**Keywords** : Parallel manipulator, computed-torque controller, Jacobian matrix, closed chain, inverse dynamics

## 1. Introduction

Now days, the structure of manipulator has been changed from an open chain to a closed chain There are many examples of mechanism using a closed chain. Because of the high mechanical stiffness, a manipulator with a closed chain is used in many industrial applications.

Because of being many links that form closed chain loops, the Jacobian analysis of parallel robot is a more difficult than serial robot. Various methods of the Jacobian analysis have been studied.

Mohamed[1] introduced the screw theory, and Sugmoto[2] applied motor algebra for the Jacobian analysis. Although two methods above are very useful, application to a parallel robot is often hindered by the presence of many passive joints. In this paper, we propose the method to obtain efficiently Jacobian of a parallel robot in the section 2.3. Although this method is applied to obtain the Jacobian of 5-bar parallel manipulator, that can be used to general parallel manipulator such as Delt and Stewart-Gough platform.

Although a parallel manipulator has various practical advantages, the theory of its dynamics has not been studied as extensively as that of the serial manipulator.

The systematic dynamics computational scheme of the closed chain manipulator was studied by Smith [3]. Paul [4] had a similar approach to this problem involving the Lagrange multipliers. Singh [5] had solved the dynamics of constrained system using singular value decomposition (SVD). Chung.Y.H[6] had solved the forward dynamics of the 5-bar parallel robot by using the singular value decomposition without computing the Lagrange multipliers. Nakamura [7] developed a systematic computational scheme of inverse dynamics of a manipulator with the closed chain.

The classical control methods such as joint independent PID control are satisfactory for point-to-point control, but they may have problems in dealing with tracking control. The dynamic control allow for accurate tracking[8][9].

A problem of dynamic control of parallel robot is that it is more difficult to design dynamic controller than that of serial robot and to compute and excute the controller based on dynamics in real time. Therefore, in most case, only simple joint independent control which is not based on dynamic model has been applied[12].

Uchiyama et al.[10] realized dynamic control by adopting a simplified dynamic model to reduce the computation time of dynamic model. Komine et al.[11] executed the computation in parallel on Super Real-time Controller. Even now real-time dynamic control without simplifying the dynamic model of the parallel manipulator are not yet ready for practical use.

In this paper, without simplifying the dynamic model and computing the lagrange multipliers, we realize easily the dynamic control of by 2 DOF 5-bar parallel manipulator in real time

## 2. Kinematics Analysis

The coordinate of 2 DOF 5-bar parallel manipulator is shown in Fig. 1.

The 5-bar parallel manipulator have one closed loop chain, two active revolute joints ( $\theta_1, \theta_3$ ), two passive revolute ones ( $\theta_2, \theta_4$ ) and the joint **P** is end-effector. And then the manipulator has translational two degree of freedom. The manipulator is symmmetry when $l_1 = l_3, l_2 = l_4$ .

Two motors are attached to the joint A, C, respectively. The distance between joint A and C is **d**. The fixed coordinate frame{**O**} of the mechanism is located at point O, the middle of between joint A and C(see Fig. 1). The input links are the link 1 and link 3, the output is the joint **P** as shown in Fig.1.

### 2.1 Forward Kinematics

The forward kinematics is that solve the position and orientation of the end-effector (P) for a given active joints. The position of the end-effector can be represented easily, but the value of passive joints must be solved.

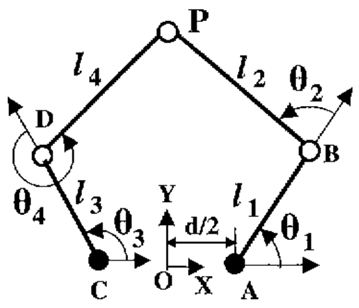


Fig. 1 Coordinate of 2 DOF 5-bar parallel manipulator.

After by virtually cutting the end-effector(P), the equation (1) is derived from the condition that the position of end-effector for a right chain is equal to that of end-effector for a left chain.

$$P_x = \frac{d}{2} + l_1C_1 + l_2C_{12} = -\frac{d}{2} + l_3C_3 + l_4C_{34}$$
$$P_y = l_1S_1 + l_2S_{12} = l_3S_3 + l_3S_{34} \qquad (1)$$

where, $C_{ij} = \cos(\theta_i + \theta_j)$ , $S_{ij} = \sin(\theta_i + \theta_j)$

If $l_1 = l_3, l_2 = l_4$ , the angle of passive joints are obtained

$$\theta_2 = -\gamma + \arcsin(\sqrt{(\alpha^2 + \beta^2)}/2) - \theta_1$$
$$\theta_4 = -\gamma - \arcsin(\sqrt{(\alpha^2 + \beta^2)}/2) - \theta_3 \qquad (2)$$

where, $\alpha = (-d - l_1C_1 + l_1C_3)/l_2$

$\beta = (-l_1S_1 + l_1S_3)/l_2$

$\gamma = \mathrm{atan2}(\alpha, \beta)$ .

From the equation (2), we know that the passive joints are represented by a nonlinear function of the active joints.

### 2.2 Inverse Kinematics

The inverse kinematics is that solve the active joints angles for a given position of the end-effector. The position of end-effector for a left chain is represented by equation (3.a) and (3.b) and then the active joints and passive joints are separated,

$$P_x + (\frac{d}{2} - l_3C_3) = l_4C_{34} \qquad (3.a)$$

$$P_y - l_3S_3 = l_4S_{34} \qquad (3.b)$$

and then summing the square of those equations.

$$[P_x + (\frac{d}{2} - l_3C_3)]^2 + [P_y - l_3S_3]^2 = l_4^2 \qquad (4)$$

From the Eq.(4), one obtains the angle of active join( $\theta_3$ ).

$$\theta_3 = \mathbf{2atan2}(\gamma_1 \pm \sqrt{\gamma_1^2 - (\alpha_1^2 - \beta_1^2)}, \alpha_1 + \beta_1) \qquad (5.a)$$

where, $\alpha_1 = P_x^{\ 2} + P_y^{\ 2} + P_x d + (\frac{d}{2})^2 + l_3^{\ 2} - l_4^{\ 2}$

$\beta_1 = 2\,P_x l_3 + l_3 d$

$\gamma_1 = 2\,P_y l_3$

For a right chain, we can obtain the $\theta_1$ in the same way above mentioned.

$$\theta_1 = \mathbf{2atan2}(\gamma_2 \pm \sqrt{\gamma_2^2 - (\alpha_2^2 - \beta_2^2)}, \alpha_2 + \beta_2) \quad (5.b)$$

where, $\alpha_2 = P_x^{\ 2} + P_y^{\ 2} + l_1^{\ 2} - l_2^{\ 2} - P_x d + (\frac{d}{2})^2$

$\beta_2 = 2P_x l_1 + l_1 d$

$\gamma_2 = 2P_y l_1$

From the equation (5.a) and (5.b), we know that there are four solutions of the inverse kinematics in this manipulator.

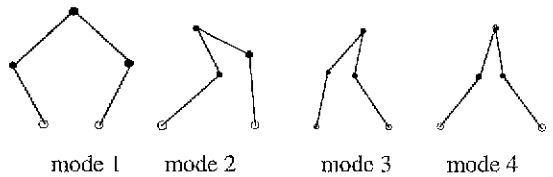


Fig. 2 Cofigurations of multiful solution of inverse kinematics

Corresponding to the sign in front of square root of Eq(5.a) and (5.b), the solutions of inverse kinematics are represented as Table.1.

Table.1 Solutions of inverse kinematics corresponding to modes of Fig.2.4

| | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
|---|---|---|---|---|
| Eq (5.a) | + | - | + | - |
| Eq.(5.b) | - | - | + | + |

### 2.3 Jacobian Analysis

In robotics, the Jacobian is relation between the velocity of end-effector and the angular velocity of active joint. In Fig.1, the relation of velocity of right chain, eqation (8), is obtained from the time derivitive of the position relation, equation.(7)

$$\boldsymbol{P} = \boldsymbol{r}_1 + \boldsymbol{r}_2 = \begin{bmatrix} (d/2) + l_1 C_1 \\ l_1 S_1 \end{bmatrix} + \begin{bmatrix} l_2 C_{12} \\ l_2 S_{12} \end{bmatrix} \qquad (7)$$

where, $\boldsymbol{r}_1 = \overline{\boldsymbol{AB}}$ , $\boldsymbol{r}_2 = \overline{\boldsymbol{BP}}$ , $\boldsymbol{P} = \overline{\boldsymbol{OP}}$ ,(see Fig.1)

$$\dot{\boldsymbol{P}} = \dot{\boldsymbol{r}}_1 + \dot{\boldsymbol{r}}_2 = \begin{bmatrix} -l_1 S_1 \\ l_1 C_1 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} -l_2 S_{12} \\ l_2 C_{12} \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2) \qquad (8)$$

In Eq.(8), it is very difficult to obtain an angular velocity of passive joint( $\dot{\theta}_2$ ) analytically

For the case of parallel manipulator, the problem of Jacobian analysis is to compute the velocity of passive joints. In this paper, we propose efficient method to obtain the Jacobian using the relation between position vector and velocity vector of rigid body to rotate about a fixed point Although this method is applied to obtain the Jacobian of 5-bar parallel manipulator, it can be used to general parallel manipulator such as Delt and Stewart-Gough platform.

Multiplying equation (8) by vector $r_2$ , because of normal property of the $r_2 \bullet \dot{r}_2$ , equation (8) can be represented by only angular velocity( $\dot{\theta}_1$ ) of active joint as follow,

$$\begin{aligned} \boldsymbol{r}_2 \bullet \dot{\boldsymbol{P}} &= \boldsymbol{r}_2 \bullet \dot{\boldsymbol{r}}_1 + \boldsymbol{r}_2 \bullet \dot{\boldsymbol{r}}_2 \\ &= l_1 l_2 S_2 \dot{\theta}_1 \end{aligned} \qquad (9)$$

Similarly, applying this method to the left chain and then representing matrics form yields

$$\dot{P} = \begin{bmatrix} r_2^{\ T} \\ r_4^{\ T} \end{bmatrix}^{-1} \begin{bmatrix} l_1 l_2 S_2 & 0 \\ 0 & l_3 l_4 S_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_3 \end{bmatrix} \qquad (10)$$

where, $J = \dfrac{1}{S_{(34-12)}} \begin{bmatrix} l_1 S_2 S_{34} & -l_3 S_4 S_{12} \\ -l_1 S_2 C_{34} & l_3 S_4 C_{12} \end{bmatrix}$

## 3. Inverse Dynamics Analysis & Controller Design

### 3.1 Inverse Dynamics Analysis

The Method to compute the inverse dynamics of the this manipulator consists of the following steps:

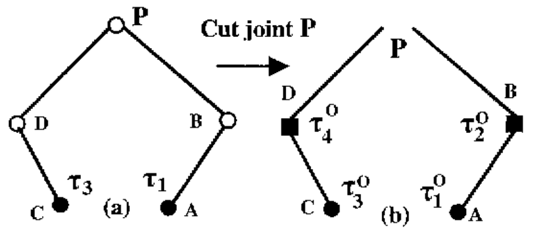


Fig. 3 The open loop structure of the closed chain system.: The ● represents active joint and ○ represents passive joint, ■ represents the virtual driven joint.

**Step 1** : By virtually cutting joint **P** as the Fig.3, we regard the parallel manipulator(Fig.3(a)) as two serial manipulators(Fig.3(b)).

**Step 2** . We assume that the passive joints(joint B and D) of the two serial manipulators have virtual actuators( $\tau_2^o, \tau_4^o$ ).

**Step 3** : We assume that there is no constraints force or torque at the joint P which is cut virtually.

When the end-effector of the two serial manipulators follows the desired trajectory which the end-effector of the original manipulator (Fig.3(a)) has to tracks, we compute the joint torques( $\boldsymbol{\tau}^o = [\tau_1^o, \tau_3^o, \tau_2^o, \tau_4^o]$ ) of the two serial manipulator.

**Step 4** : From the computed torque vector( $\boldsymbol{\tau}^o$ ) in step 3, the torque vector( $\boldsymbol{\tau}_a^c = [\tau_1, \tau_3]^T$ ) of the original manipulator(Fig.3(a)) is computed by using equation (11) and (12).

$$\boldsymbol{\tau}_a^c = \boldsymbol{W}^T \boldsymbol{\tau}^o \qquad (11)$$

where, $\boldsymbol{\tau}_a^c = [\tau_1, \tau_3]^T$

The $\boldsymbol{W}$ matrix is obtained as follows,

$$\delta\boldsymbol{\theta} = \boldsymbol{W}\delta\boldsymbol{\theta}_a \qquad (12)$$

where, $\boldsymbol{W} = \begin{bmatrix} \boldsymbol{I} \\ \dfrac{\partial \boldsymbol{\theta}_p}{\partial \boldsymbol{\theta}_a} \end{bmatrix}, \boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_a \\ \boldsymbol{\theta}_p \end{bmatrix} = \begin{bmatrix} [\theta_1, \theta_3]^T \\ [\theta_2, \theta_4]^T \end{bmatrix}$

$\boldsymbol{\theta}_a$ : active joint vector,

$\boldsymbol{\theta}_p$ : passive joint vector

In Eq.(12), we can know that $\boldsymbol{W}$ includes the Jacobian matrix of the passive joints with respect to the active ones. Hence, the $\boldsymbol{W}$ plays a role destributing 1 by 4 the torque vector( $\boldsymbol{\tau}^o = [\tau_1^o, \tau_3^o, \tau_2^o, \tau_4^o]$ ) of two serial manipulator to 1 by 2 the original torque vector( $\boldsymbol{\tau}_a^c = [\tau_1, \tau_3]^T$ ).

### 3.2 Controller Design Based on Dynamic Model

The equation of motion of 2 DOF 5-bar parallel manipulator can be represented by the second order ordinary differential equation (13) and algebra equation (14)

$$\boldsymbol{M}^c(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{V}^c(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{\Phi}_q^T \lambda = \boldsymbol{\tau}^c \qquad (13)$$

$$\boldsymbol{\Phi}_\theta \dot{\boldsymbol{\theta}} = 0 \qquad (14)$$

where, $\boldsymbol{\tau}^c = [(\boldsymbol{\tau}_a^c)^T, 0, 0]^T = [\tau_1^c, \tau_3^c, 0, 0]^T$

$\boldsymbol{\theta} = [\theta_1, \theta_3, \theta_2, \theta_4]^T$

$\boldsymbol{M}^c$ is 4 by 4 inertia mantrix, $\boldsymbol{V}^c$ is 1 by 4 centrifugal and coriolis torque vector, $\boldsymbol{\Phi}_\theta$ is constraints matrix( The c is used to denote the variables of a paralle manipulator)

For the serial manipulator, there are actuators attached to each links and each actuators can compensate for nonlinear manipulators dynamics of each links. Therefore, to applying inverse dynamic control called computed-torque controller to serial manipulators is easy[8][9].

Unfortunately, the design of computed-torque controller for parallel manipulator is not as straighforward as for serial manipulators.

For designing the computed-torque controller of 2 DOF 5-bar parallel manipulator,

**Step 1** : we regard the parallel manipulator as the two serial manipulator (Fig.3(b))such as step 1 and 2 of section 3.1. And then we can write equation of motion as following,

$$\boldsymbol{M}^o(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{V}^o(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{\tau}^o \qquad (15)$$

where, $\theta = [\theta_1, \theta_3, \theta_2, \theta_4]^T$ is 1 by 4 joint vector, $\tau^o = [\tau_1^o, \tau_3^o, \tau_2^o, \tau_4^o]^T$ is 1 by 4 torque vector, $M^o$ is 4 by 4 inertia mantrix, $V^o$ is 1 by 4 centrifugal and coriolis torque vector.( The **o** is used to denote the variables of a serial manipulator)

**Step 2 :** In order for the end-effector of the two serial manipulators(Fig.3(b)) to track the desired trajectory which the end-effector of the original manipulator(Fig.3(a)) must track, we design the computed-torque controller as follow;

$$\tau^o = M^o u + V^o(\theta, \dot{\theta}) \qquad (16)$$

$$u = \ddot{\theta}_d + K_d(\dot{\theta}_d - \dot{\theta}) + K_p(\theta_d - \theta) \qquad (17)$$

In equation (16), the torque vector($\tau^o$) is not the actual torque vector($\tau_a^c = [\tau_1, \tau_3]^T$) of the original manipulator of Fig.3(a).

**Step 3 :** For computing the actual torque $\tau^c$ of the original parallel manipulator, we use the relation of equation (11).

Therefore the actual torque($\tau^c = [\tau_1^c, \tau_3^c, 0, 0]^T$) of the original parallel manipulator is obtained as Eq.(19)

$$\tau^c = \mathbf{P}\tau_a^c \qquad (18)$$

Substituting Eq.(11) and (16) into Eq(18),

$$\tau^c = PW^{-1}\tau^o$$
$$= PW^{-T}[M^o u + V^o(\theta, \dot{\theta})] \qquad (19)$$

where, $\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T$

**P** is permutation matrix.

Therefore, the control block diagram of 5-bar parallel manipulator can is represented in Fig. 4

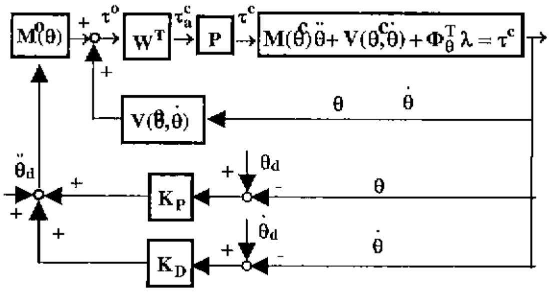


Fig. 4 Block diagram of dynamic control including the computed-torque controller.

## 4. Feedback Control Experiment

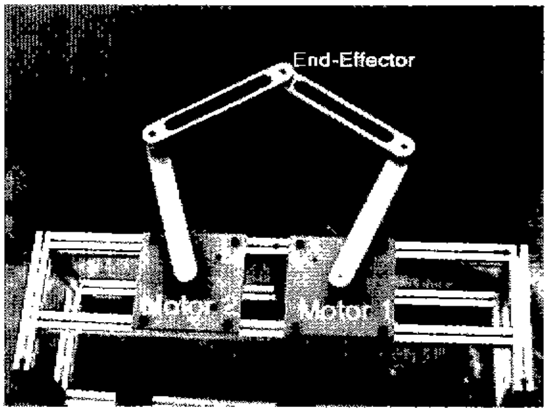


Fig. 5 Photograph of 2 DOF 5-bar parallel manipulator.

The Fig.5 is 5-bar parallel manipulator that uses the AC motor as the actuator. The AC motors which have not a reduction gear are attached to the link1 and 3 directly. The maximum torque of AC motor are 1 Nm. The resolution of encoders which are attached to the AC motor is 32,000 (pulse / rev).

The sampling time is 400Hz. With the average velocity, 0.3142 m/s, of the end-effector, it draws the circle whose diameter is 0.1m.

The performance of the PD controller and computed torque controller is represented in Fig. 6 and Fig. 7 respectively. The gain of PD controller is tuned by using the method of trial and error. As shown in Fig.7, we know that the computed-torque controller enable to track the desired trajectory accurately. The computed torque controller haves the better performance than PD controller as shown in Fig. 6 and Fig. 7.

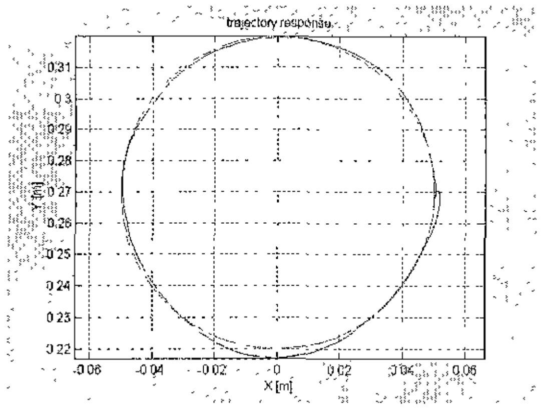


Fig. 6 Tracking performance of PD controller

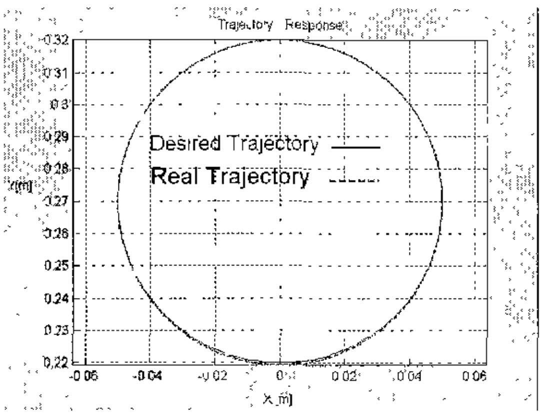


Fig. 7 Tracking performance of computed torque controller using $W$ matrix.

## 5. Conclusions

We demonstrated the possibility of the high speed and high accuracy work using the 2 DOF 5-bar parallel manipulator in this research.

In kinematic analysis, we proposed the method that easily computes the Jacobian of the general parallel manipulator by using the property that the position vector is normal to the velocity vector of rigid body rotating about a fixed point

Using the transformation matrix of the passive joints with respect to the actuated ones, we have done dynamic analysis of a parallel manipulator Without simplifying the dynamics of parallel manipulator, we easily have designed the real time computed-torque controller that shows the good tracking performance using the transformation matrix.

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