A MW-Mvar Investment Technique Focused on System Loss Minimization

Jae-Sun Eom, Sang-Joong Lee and Kern-Joong Kim

Abstract - In this paper, a MW-Mvar investment technique focused on minimizing the system loss is presented. An optimization technique, in which the system loss is defined as the objective function and the power flow equations as the constraints, is introduced to obtain the Lagrangian multipliers λP and λQ. The Lagrangian multipliers imply the variation of the system loss with respect to incremental bus power and are used as MW-Mvar investment indices for minimizing the system loss. ΔP MW and ΔQ Mvar are invested, step by step, by the priority of λP and λQ index given for each bus. Derivation of the index uses the information from normal power flow calculation.

Key words - MW-Mvar investment, optimization technique, Lagrangian multiplier, system loss sensitivity

1. Introduction

In order to minimize the transmission loss through MW-Mvar investment, it is desirable to use the system loss sensitivity given to each bus as the investment index. Derivation of the system loss sensitivity with respect to the bus power, however, has been a problem of incompletion for the last decades. Many system planners have been using the bus voltage information as the investment index for reducing the system loss and improving the system voltage stability. They usually invested first on the lowest-voltaged bus. Sometimes, so called B-coefficient loss formula in which the system loss is expressed strictly as a function of generator power output, has been applied to the economic load dispatch to obtain approximate solution of the system loss sensitivity. The major assumptions upon which the loss formula is predicated are[1]:

- that each load current remains a constant complex ratio of the total load current irrespective of load level.
- the var to watt ratio of all generators and ties remain constant.
- the deviation of generator voltage and angles from those incorporated in the loss formula for base case are small.

The first two assumptions are theoretically impossible in actual power system equipped with various non-linear elements and, therefore, the computation result by B-coefficient loss formula is, of course, approximation. H.Happ presented that the system loss sensitivity can be represented in terms of system Jacobian in 1974. Happ derived the loss sensitivity by using the chain rule[2] and Wood and Wollenberg applied the formulation to economic load dispatch[3]. Another Jacobian-based method for derivation of the system loss sensitivity is developed by using the optimization technique in 1990[4], which has been applied to assessment of the system voltage stability[4,5,6] and investment of the var apparatus[5,6,7], etc. In this paper, the author introduces the derivation of the system loss sensitivity using optimization technique and proposes a MW-Mvar investment technique focused on minimizing the system loss.

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2. Derivation of System Loss Sensitivity using Optimization Technique

Let us find a method by which the variation of the system loss with respect to the bus power can be traced from the result of power flow calculation. Equation (1) can be a mathematical representation for such a problem.[8]

$$\begin{aligned} &\textit{Minimize} \quad P_{LOSS} \\ &\text{s.t.} \quad P(V_D, \theta) = P^{SPEC} \\ &\qquad\quad Q(V_D, \theta) = Q^{SPEC} \end{aligned} \tag{1}$$

where PSPEC, QSPEC are P[MW] and Q[Mvar] specified for each bus and VD, θ are load bus voltage and bus angle . Defining the Lagrangian function,

$$\begin{aligned} L = P_{LOSS}(V_D, \theta) &+ \lambda_P^T[P(V_D, \theta) - P^{SPEC}] \\ &+ \lambda_Q^T[Q(V_D, \theta) - Q^{SPEC}] \end{aligned} \tag{2}$$

optimality conditions are obtained at optimal point as following.

$$\frac{\partial L}{\partial \theta} = \frac{\partial P_{LOSS}}{\partial \theta} + (\frac{\partial P}{\partial \theta})^T \lambda_P + (\frac{\partial Q}{\partial \theta})^T \lambda_Q = 0 \quad (3)$$

$$\frac{\partial L}{\partial V_D} = \frac{\partial P_{LOSS}}{\partial V_D} + (\frac{\partial P}{\partial V_D})^T \lambda_P + (\frac{\partial Q}{\partial V_D})^T \lambda_Q = 0 \quad (4)$$

$$\frac{\partial L}{\partial \lambda_P} = P(V_D, \theta) - P^{SPEC} = 0 \quad (5)$$

$$\frac{\partial L}{\partial \lambda_Q} = Q(V_D, \theta) - Q^{SPEC} = 0 \quad (6)$$

Matrix representation for equation (3) and (4) yields,

$$\begin{bmatrix} \frac{\partial L}{\partial \theta} \\ \frac{\partial L}{\partial V_D} \end{bmatrix} = J^T \begin{bmatrix} \lambda_P \\ \lambda_Q \end{bmatrix} + \begin{bmatrix} \frac{\partial P_{LOSS}}{\partial \theta} \\ \frac{\partial P_{LOSS}}{\partial V_D} \end{bmatrix} = 0 \quad (7)$$

where J is the Jacobian matrix. [λ P, λ Q] can be derived from (7) as follows,

$$\begin{bmatrix} \lambda_P \\ \lambda_Q \end{bmatrix} = -J^{-T} \begin{bmatrix} \frac{\partial P_{LOSS}}{\partial \theta} \\ \frac{\partial P_{LOSS}}{\partial V_D} \end{bmatrix} \quad (8)$$

Differentiating the Lagrangian function (2) with respect to PSPEC and QSPEC yields,

$$[\partial L / \partial P^{SPEC}] = -\lambda_P \quad (9)$$

$$[\partial L / \partial Q^{SPEC}] = -\lambda_Q \quad (10)$$

The Lagrangian multipliers λ P and λ Q have an important physical meaning. At current operating point the result of power flow calculation satisfies the optimality conditions of equations (5) and (6), which means that the optimality conditions for variables [λ P, λ Q] are already fulfilled. Consequently, the Lagrangian function (2) includes only system loss component and, therefore, the variables [λ P, λ Q] become equal to the change of system loss with respect to the incremental bus power PSPEC and QSPEC. Computation time for deriving [λ P, λ Q] is almost the same as that for power flow calculation since only a small amount of additional calculation for product operation of the Jacobian matrix and the sensitivity vector (∂ PLOSS/ ∂ θ, ∂ PLOSS/ ∂ VD)T is required.

3. MW-Mvar Investment Algorithm using λ Index

As the Lagrangian multipliers λ P and λ Q(or λ index) mean the variation of the system loss with respect to the incremental bus power, they can be used as MW-Mvar investment index for minimizing the system loss. The higher magnitude of λ index, the higher contribution to the system loss for the same amount of investment. Investment on the highest λ -indexed bus results in maximum effect on minimizing the system loss, while minimizing the system loss most effectively improves the system voltage stability[4,5,6]. Fig.1 is the flow chart for the MW-Mvar investment algorithm. ΔP [MW] is invested on the highest λ P-indexed bus, step by step, by the priority of λ P index given for each bus, and similarly ΔQ [Mvar] is invested on the highest λ Q-indexed bus according to the following procedure.

1) Compute power flow.
2) Calculate [λ P, λ Q] in equation (8).
3) Invest ΔP [MW] on the highest λ P-indexed bus, and invest ΔQ [Mvar] on the highest λ Q-indexed bus.
4) Continue 1), 2) and 3) until ΣΔP+jΣΔQ reaches pre-scheduled amount.

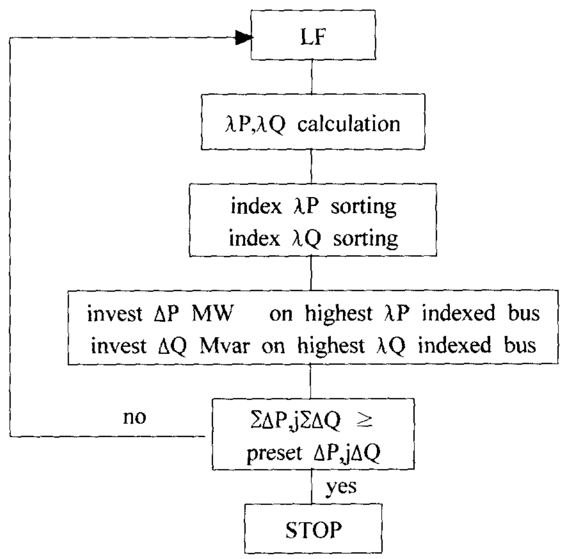

Fig. 1 MW-Mvar investment algorithm using λ index

4. Case Study

A five-bus system as shown in Fig. 2 has been studied.[9] Table 1 is the line parameters. The computation of power flow and λ indices for base case are included in table 2. V1 and V3, the voltages of bus 1 and 3 are assumed to be constant. (V1=1.0 and V3=0.98, where bus 1 is the slack bus.) Assume that we plan a new 2.2+j2.2 pu of MW-Mvar investment on load buses to improve the system loss and voltage stability. Let us find the optimal bus locations and the amount of investment by λ index. Table 2 is the comparison of the investment result by λ index with the following two cases.

case 1: All 2.2+j2.2 p.u. is invested to only one arbitrary bus.(in this case on bus 5, as shown 'investment only on bus 5' in table 2)

case 2: $\Delta P+j\Delta Q$ is invested by the priority of the voltage magnitudes given for each bus, i.e., $\Delta P+j\Delta Q$ is invested step by step on the lowest-voltaged bus according to the following procedure.

1) Compute power flow.
2) Invest a small amount of $\Delta P+j\Delta Q$ on the lowest-voltaged bus.(where $\Delta P+j\Delta Q=[2.2+j2.2]/N$, where N denotes a large number)
3) Continue 1) and 2) until $\sum\Delta P+j\sum\Delta Q$ reaches 2.2+j2.2 pu.

The results of investment by voltage magnitudes are shown as 'Investment by voltage information' in table 2.

The author defines an evaluator AVP for evaluation of the average voltage profile of total system as following.

$$AVP=\sqrt{\frac{(\Sigma V_i*P_i)^2+(\Sigma V_i*Q_i)^2}{(\Sigma P_i)^2+(\Sigma Q_i)^2}} \quad (11)$$

Loads are assumed as constant power and Si in table 2 represents the amount of MW+jMvar invested on the i-th bus.

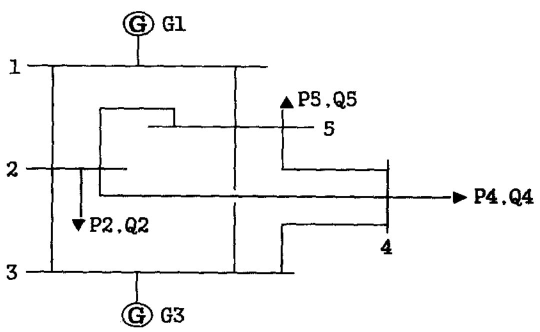

Fig. 2 Five bus system

Table 1 Line parameters

from	to	G	B	Line Charging
1	2	1.40056	-5.60224	Bus 1 0.03600
1	5	2.81647	-7.48352	Bus 2 0.01800
2	3	2.45000	-5.87000	Bus 3 0.08200
2	4	3.05000	-4.45170	Bus 4 0.03575
2	5	3.13000	-2.09335	Bus 5 0.03575
3	4	3.70000	-3.73000	
3	5	1.12985	-4.47675	
4	5	2.05362	-4.16883	

As shown in table 2, MW-Mvar investment by λ index shows the most noticeable improvement in system loss and voltages.

Table 2 Comparison of system conditions for three types of MW-Mvar investment on 5 bus system

	Base-case (before investment)	Investment only on bus 5	Investment by voltage information	Investment by λ_P, λ_Q index
S_2 S_4 S_5	0 0 0	0 0 2.2+j2.2	.809+j.809 .841+j.841 .549+j.549	0.725+j.880 1.258+j.541 0.215+j.777
P_{Loss}	1.4689	.5444	.2117	.1973
AVP	.7420	.8287	.9293	.9320
V_2 V_4 V_5	.7474 .7056 .7630	.8425 .8283 1.0038	.9293 .9293 .9293	.9317 .9309 .9324
λ_{P2}, λ_{Q2} λ_{P4}, λ_{Q4} λ_{P5}, λ_{Q5}	-1.094,-.622 -1.611,-.745 - .867,-.542	-.300, -.154 -.380, -.156 -.083, +.012	-.134, -.015 -.171, .008 -.119, -.022	-.124, -.015 -.124, -.015 -.124, -.015
P_1+jQ_1 P_3+jQ_3	5.868+j2.813 1.5 +j3.232	2.744+j.299 1.5 -j.755	2.411+j.409 1.5 +j.055	2.397+j.366 1.5 +j.088
P_2+jQ_2 P_4+jQ_4 P_5+jQ_5	2.4 + j0.9 1.5 + j0.6 2.0 + j0.8			
Remarks: All units are in p.u.				

5. An Application to Real Power System

System load and the network configuration vary anytime. λ index calculated by equation (8), of course, changes according to these variations. It is expected that the proposed investment algorithm can be efficiently applied to a relatively small power system especially when the MW-Mvar apparatus to be invested can be easily transported, e.g., diesel generators or static condensers on trailers, ship and etc. Moving the transportable MW-Mvar apparatus from a bus to higher λ -indexed bus will contribute to system loss reduction and voltage stability improvement.

6. Conclusion

In this paper, a MW-Mvar investment technique focused on minimizing the system loss is presented. An optimization technique, in which the system loss is defined as the objective function and the power flow equations as the constraints, is introduced to obtain the Lagrangian multipliers λP and λQ. The multipliers can be used as MW-Mvar investment index for minimizing the system loss since they imply the variation of the system loss with respect to the incremental bus power. ΔP MW is invested by the priority of λP index given for each bus and ΔQ Mvar is invested by λQ index. Derivation of the indices uses the information from normal power flow. Computation time for deriving λP and λQ is almost the same as that for power flow calculation. A case study on a simple test system has proved the effectiveness of the algorithm proposed.

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