# Strain Path Dependence of Forming Limits Predicted by Barlat and Lian's Non-Quadratic Anisotropic Yield Criterion for Sheet Materials

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This paper presents an analytical study that can predict the path-dependent forming limit of anisotropic sheet materials that experience various combinations of strain paths. To predict the forming limit diagrams(FLD), the proposed analytical procedure is performed within the framework of the Marciniak and Kuczynski(M-K) approach by using the Barlat and Lian's non-quadratic anisotropic yield criterion and introducing the effect of the existence of a strain gradient over a stretching punch. The predicted path-dependent forming limit of an anisotropic sheet has been compared with the published experimental results. It has been found that the predicted path-dependent forming limits are in good agreement with the experimental data.

**Key Words** : Forming Limit Diagrams(FLD), Marciniak and Kuczynski(M-K) Model, Strain Path

## 1. Introduction

Most practical sheet metal forming processes is generally subjected to different forming modes such as drawing, plane-strain deformation, and stretching. Sheet metal formability can be defined as the ability of a material to be formed into a specific desired shape without experiencing fracture or generating an unacceptable failure defect. Accordingly, in order to understand and estimate formability it is necessary to analyze all failures that occur as a result of sheet metal forming. There is no predominant type of failure associated with the sheet metal forming process; however, the most common types of observable failures include sheet wrinkling resulting from compressive dominant stresses and a localized neck that occurs when the stress state leads to an increase in the surface area of sheet with a reduction in the thickness. In sheet metal forming operations, the level of useful deformation is limited by the occurrence of a localized neck.

The formability of sheet metals is often evaluated using a strain analysis based on the concept of forming limit diagrams(FLD) originally introduced by Keeler(1965) and Goodwin(1968). Until now, FLDs have been widely accepted as a useful measure of sheet metal formability(Kim and Park, 1993).

Marciniak and Kuczynski(1967) analyzed the causes of necking and subsequent failure based on the presence of local inhomogeneities in the original sheet metal. Stören and Rice(1975) incorporated the $J_2$ deformation theory of plasticity, which is a simplified model of the corner theory, into a classical bifurcation analysis for predicting localized necking within the range of an FLD. Zhao, Sowerby, and Sklad(1996) investigated limit strains using the forming limit stress concept.

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These studies were subsequently developed further by many researchers and extended using the $J_2$ deformation theory and various other methods. It has turned out that an FLD depends upon a great number of interactive parameters such as the material behavior, strain path, strain and strain rate hardening, lubrication, and geometrical factors related to the method of deformation(Son and Kim, 1999).

In the industry, out-of-plane (punch stretching) or in-plane stretching tests are frequently used to approximate the actual automotive stamping conditions, including factors such as the testing geometries, bending and unbending, and friction conditions. The most commonly used test is the out-of-plane method that utilizes the punch stretching geometry as suggested by Hecker (1975). It has been shown by experiments(Ghosh and Hecker, 1974) that an FLD, for strain gradients resulting from the bending of a sheet, indicates an increased sheet formability. Therefore, an analysis must include the effect of strain gradients resulting from the bending of a sheet on the FLD. Shi and Gerdeen(1991) were the first to quantitatively explain the difference in the FLD as a result of in-plane and out-of-plane deformations.

Theoretical and experimental FLDs are usually determined for a linear strain path. However, this hypothesis is no longer valid as in most production processes the industrial stamping of complex shapes often involves multi-stage forming operations, therefore, the formability of the sheet metal will depend on the strains the metal has been subjected to(Barata et al., 1984). Accordingly, the ability to predict an accurate path-dependent FLD is crucial in the design and manufacturing of new components.

In this study, for the flexibility in predicting path-dependent forming limits and ability to be compared with the experimental forming limits, a theoretical FLD is calculated based on the Marciniak-Kuczynski(M-K) approach by using the Barlat and Lian's non-quadratic anisotropic yield criterion(Barlat and Lian, 1989) and a constitutive equation that considers the effect of a strain gradient in conjunction with the flow theory of plasticity. To confirm the precision of the proposed model, the theoretical FLD for punch stretching is compared with the experimental data on CHSP35E and DDQ steel. Furthermore, the effect of strain path changes on the theoretical FLD is also investigated and compared with the experimental results(Nam et al., 1997).

## 2. Theoretical Analysis

To consider the effect of the strain gradient along the thickness direction resulting from bending a flat sheet into a curved sheet of thickness, $t$, over a punch with radius, $a$, the constitutive equation proposed by Shi and Gerdeen(1991) is used, without deformation heating. That is,

$$\bar{\sigma}=K'\bar{\varepsilon}^{n}\dot{\bar{\varepsilon}}^{m}-gQ_AK[t/(a+t/2)]^{2n} \quad (1)$$

with $g=\bar{c}^{n}/Q_b[n/(8+4n)]^{n}$,

$$\bar{c}=d\bar{\varepsilon}/d\bar{\varepsilon}_1,\ Q_b=\bar{\sigma}_b/\sigma_b,\ Q_A=\bar{\sigma}_A/\sigma_A$$

where $\bar{\sigma}$ is the effective stress; $\bar{\varepsilon}$ is the effective strain; $K'$ is the modified coefficient of the material strength coefficient $K$; $\sigma_A$ is the average stress at the section; $\sigma_b$ is the stress resulting from the bending only; $t$ is the sheet thickness; $a$ is the punch radius; $n$ is the strain-hardening exponent; and, $m$ is the strain-rate sensitivity. Here, it is assumed that the strain ratio or stress ratio in the two non-thickness directions resulting from the bending has a constant value and it does not vary with the deformation state.

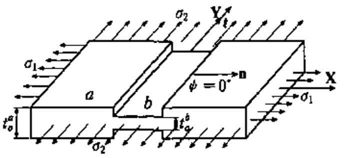
The yield surface for planar anisotropy is described using the Barlat and Lian's non-quadratic anisotropic yield criterion under plane stress, which states that:

$$\varphi=\frac{2}{2-c}\bar{\sigma}^{M}$$
$$=|k_1+k_2|^{M}+|k_1-k_2|^{M}+\frac{c}{2-c}|2k_2|^{M} \quad (2)$$

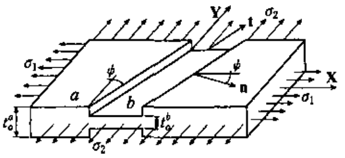
with $k_1=\dfrac{\sigma_x+h_1\sigma_y}{2}$, $k_2=\sqrt{\left(\dfrac{\sigma_x-h_1\sigma_y}{2}\right)^2+h_2^2\sigma_{xy}^2}$

where $c$, $h_1$, and $h_2$ are the anisotropic material constants. This criterion for planar anisotropy can be used to consider the shear stress term $\sigma_{xy}$ where the principal stress axes do not coincide with the symmetry axes.

The orientation of the imperfection is taken to lie perpendicular to the X-axis for a positive



(a) Geometry for positive strain ratio



(b) Geometry for negative strain ratio

**Fig. 1** Geometric configuration of plastic instability model

strain ratio [Fig. 1(a)] and is modified to lie in the direction of zero-extension for a negative strain ratio [Fig. 1(b)]. The equilibrium equations can be written in the coordinate system associated with the groove orientation as

$$\sigma_n{}^a t^a = \sigma_n{}^b t^b,\ \sigma_{nt}{}^a t^a = \sigma_{nt}{}^b t^b \tag{3}$$

The compatibility condition requires that the strains at regions "a" and "b" across the band are equal.

$$d\varepsilon_t{}^a = d\varepsilon_t{}^b \tag{4}$$

The stress components in the $n$ and $t$ directions can be expressed in the stress components in the $x$ and $y$ directions using the simple tensor transformation.

$$\begin{aligned}\sigma_n &= \sigma_x\cos^2\psi + \sigma_y\sin^2\psi + 2\sigma_{xy}\sin\psi\cos\psi \\ \sigma_t &= \sigma_x\sin^2\psi + \sigma_y\cos^2\psi - 2\sigma_{xy}\sin\psi\cos\psi \\ \sigma_{nt} &= \sigma_{xy}(\cos^2\psi - \sin^2\psi) + (\sigma_y - \sigma_x)\sin\psi\cos\psi\end{aligned} \tag{5}$$

From Eqs. (1)~(5), the following relationships are obtained: for the right-hand side (positive strain ratio),

$$\begin{aligned}&B_1(\bar{\varepsilon}^a)^n(\dot{\bar{\varepsilon}}^a)^m - fA_1(\bar{\varepsilon}^b)^n(\dot{\bar{\varepsilon}}^b)^m \\ &\quad - gQ_1^a B_1([t^a/(a+t^a/2)]^{2n} - f[t^b/(a \\ &\quad + t^b/2)]^{2n}) = 0\end{aligned} \tag{6}$$

and, for the left-hand side (negative strain ratio),

$$\begin{aligned}&(\bar{\varepsilon}^a)^n(\dot{\bar{\varepsilon}}^a)^m - f(\bar{\varepsilon}^b)^n(\dot{\bar{\varepsilon}}^b)^m - gQ_1^a([t^a/(a \\ &\quad + t^a/2)]^{2n} - f[t^b/(a+t^b/2)]^{2n} = 0\end{aligned} \tag{7}$$

with $A_1 = \left[1 + |h_1\alpha|^M + \frac{c}{2-c}|1 - h_1\alpha|^M\right]^{1/M}$,

$$\begin{aligned}&B_1 = \left[1 + |h_1\alpha^b|^M + \frac{c}{2-c}|1 - h_1\alpha^b|^M\right]^{1/M}, \\ &f = f_o\exp(\varepsilon_3^b - \varepsilon_3^a) = t_o^b/t_o^a\exp(\varepsilon_3^b - \varepsilon_3^a), \\ &Q_1^a = \bar{\sigma}^a/\sigma_x^a,\ \alpha = \sigma_2^a/\sigma_1^a = \sigma_y^a/\sigma_x^a,\ \alpha^b = \sigma_2^b/\sigma_1^b\end{aligned}$$

Although the FLD corresponding to a one-stage linear strain path remains as a basic concept in the formability research, the industrial stamping of complex shapes often involves a multi-stage forming operation, therefore, a one-stage linear strain path is no longer applicable. However, to simplify the theoretical analysis, the strain path can be considered to consist of several nearly linear sections each with a particular strain ratio. Accordingly, in this study, a complex strain path frequently experienced in sheet metal is assumed to be composed of two stages with a linear strain path in each stage, as defined by the following equation:

$$\varepsilon_2 = \begin{cases}\rho_1\varepsilon_1 & ,\ (\varepsilon_1 \leq \varepsilon_{1s}) \\ \rho_1\varepsilon_{1s} + \rho_2(\varepsilon_1 - \varepsilon_{1s}) & ,\ (\varepsilon_1 > \varepsilon_{1s})\end{cases} \tag{8}$$

where $\rho_1$ and $\rho_2$ are the ratios of the minor to major strains in the first and second stages, respectively, and $\varepsilon_{1s}$ is the major strain where the strain path changes.

In order to obtain the limit strains for a sheet metal with known mechanical properties, it is necessary to determine $\varepsilon_2{}^a$ and $\varepsilon_1{}^a$ at instability (i. e. $d\bar{\varepsilon}^a/d\bar{\varepsilon}^b \rightarrow 0$). Eqs. (6) and (7) can easily be solved using the Newton-Raphson iteration technique provided that the mechanical properties of the sheet metal and the initial values (i. e. $f_o$, $\rho$, $\Delta\varepsilon_1{}^a$, $\Delta\varepsilon_1{}^b$) are prescribed. Here, the limit strains are achieved when the ratio of these two strain rates approaches a critical value corresponding to local instability (selected as $d\bar{\varepsilon}^a/d\bar{\varepsilon}^b \leq 0.1$). Furthermore, in order to obtain the equivalent prestrains, the equivalent strains are calculated by

$$\begin{aligned}\bar{\varepsilon} &= \sum d\bar{\varepsilon} \\ &= \frac{2}{h_1(2-c)} \frac{\left[\frac{2-c}{c}\left\{|kp_1 + kp_2|^M + |kp_1 - kp_2|^M + \frac{c}{2-c}|2kp_2|^M\right\}\right]^{\frac{M-1}{M}}}{|kp_1 + kp_2|^{M-1} + |kp_1 - kp_2|^{M-1}} (h_1 + \rho)\varepsilon_1\end{aligned} \tag{9}$$

with

$$\rho=\frac{h_1\Big[|kp_1+kp_2|^{M-1}\Big(\frac{1}{2}-\frac{1-h_1\alpha}{4kp_2}\Big)+|kp_1+kp_2|^{M-1}\Big(\frac{1}{2}+\frac{1-h_1\alpha}{4kp_2}\Big)-\frac{c}{2-c}|2kp_2|^{M-1}\Big(\frac{1-h_1\alpha}{2kp_2}\Big)\Big]}{\Big[|kp_1+kp_2|^{M-1}\Big(\frac{1}{2}+\frac{1-h_1\alpha}{4kp_2}\Big)+|kp_1-kp_2|^{M-1}\Big(\frac{1}{2}-\frac{1-h_1\alpha}{4kp_2}\Big)+\frac{c}{2-c}|2kp_2|^{M-1}\Big(\frac{1-h_1\alpha}{2kp_2}\Big)\Big]}$$

$$kp_1=\frac{1+h_1\alpha}{2},\quad kp_2=\sqrt{\Big(\frac{1-h_1\alpha}{2}\Big)^2+h_2^2\Big(\frac{\sigma_{12}}{\sigma_1}\Big)^2}$$

Eq. (9) is used to assign the initial equivalent strain, $\bar{\varepsilon}_o$, in Eq. (1) as

$$\bar{\varepsilon}=\sum_{i=1}^{N}\Delta\bar{\varepsilon}_i+\bar{\varepsilon}_o \quad (N\text{: critical number corresponding to local instability}) \qquad (10)$$

In order to compare the theoretical FLD with the experimental results(Nam et al., 1997), the results for three typical cases of a two-stage strain path are presented in the following section. However, it should be noted that the present analysis is also capable of dealing with an arbitrary strain path and extending to multi-stage forming occurring in a conventional sheet metal forming process.

## 3. Results and Discussions

Forming limits for various stress states ranging from equibiaxial to uniaxial tension have been calculated using the proposed analytical model. To confirm the accuracy of the proposed analysis, the predicted FLDs have been compared with the experimental data resulting from punch stretching tests proposed by Hecker(1975), as shown in Figs. 2 and 3. According to the M-K formulation, the material contained an initial imperfection, $f_o$, which has been characterized by the thickness ratio, $t_o^b/t_o^a$, and although the variation of $f_o$ has affected the FLD level, it has shown little impact on the shapes. Therefore, first of all it has been found important to select a reasonable $f_o$ value for comparisons with the experimental data. Consequently, it has been concluded that the value of $f_o$ is about 0.996 for CHSP35E 0.75t and

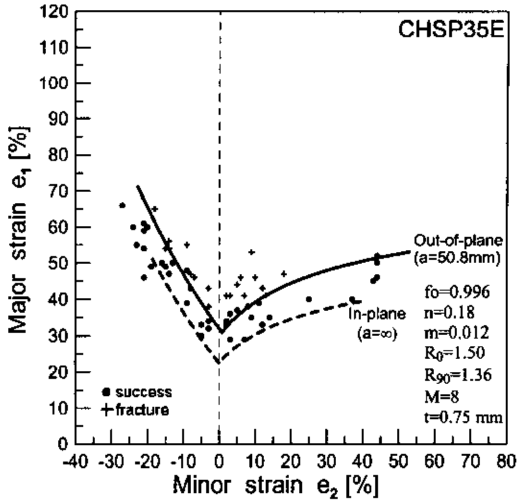



**Fig. 2** Forming limit diagrams for CHSP35E 0.75t

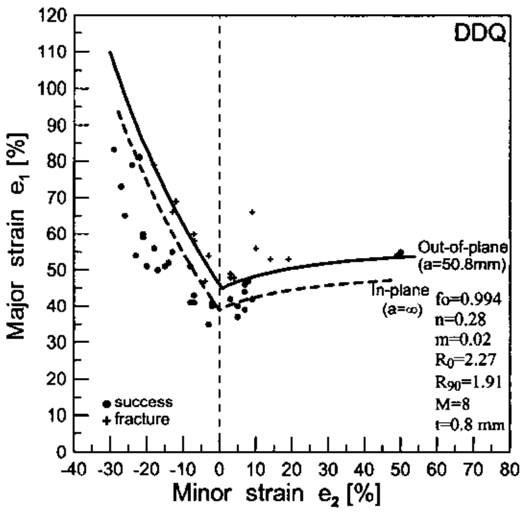



**Fig. 3** Forming limit diagrams for DDQ 0.8t

0.994 for DDQ 0.8t. Also, a reasonable $f_o$ value of 0.994 has been used to predict the theoretical path-dependent FLD for DDQ steel. With a reasonable $f_o$ value, it is found that the agreement between the theoretical FLD and the experimental data is quite good for CHSP35E and DDQ steel, as shown in Figs. 2 and 3.

The results in Fig. 4 show the effect of the punch radius, $a$, on the FLD. It is clear that the forming limit of the sheet increases as the punch radius decreases, and the $FLD_o$ defined at the lowest point on the FLD shifts slightly to the right as the punch radius decreases. This shift to

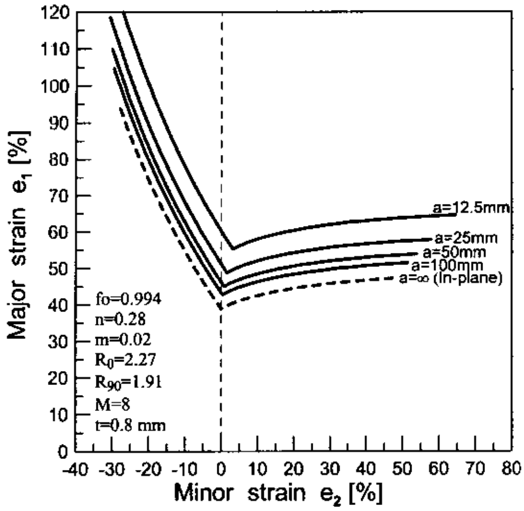



**Fig. 4** Effect of punch radius on forming limit diagrams

the right may have been caused by the effect of the bending strains on the strain gradient as determined by the specimen conformation to the punch. As previously reported by Charpentier(1975), Figs. 2~4 show that the FLDs for in-plane stretching lie below those obtained with punch stretching; that is, the strain gradient resulting from the bending of a sheet material would appear to increase the formability of the sheet material. Here, the symbol ∞ denotes the limit where the punch radius approaches infinity; that is, the case of in-plane stretching.

In order to set up a more realistic model for predicting a path-dependent FLD, the proposed analytical method has been adopted to follow the experimental procedure of Nam's work(1997), where sheets of DDQ 0.8t were prestrained in uniaxial tension and equibiaxial tension, and vertically directed along the rolling direction. In uniaxial prestrains specimens with a 200-mm width have been used, and prestrains in equibiaxial tension have been achieved with a 250-mm diameter using ring-shaped polyurethane spacer and beef tallow, as indicated in Fig. 5. That is, the 1st punch radius $a^{1st}=\infty$ has changed to the 2nd radius $a^{2nd}=50.8\ mm$.

Figures 6 and 7 show the effects of the strain path on the forming limits, where the predicted FLDs are compared with the experimental results.

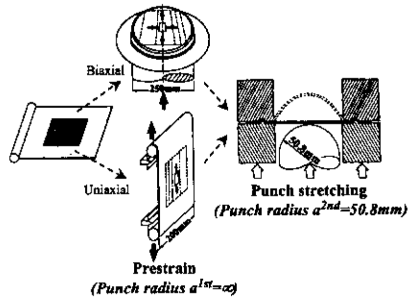



**Fig. 5** Schematic of experimental FLD for equibiaxial prestrains and uniaxial prestrains

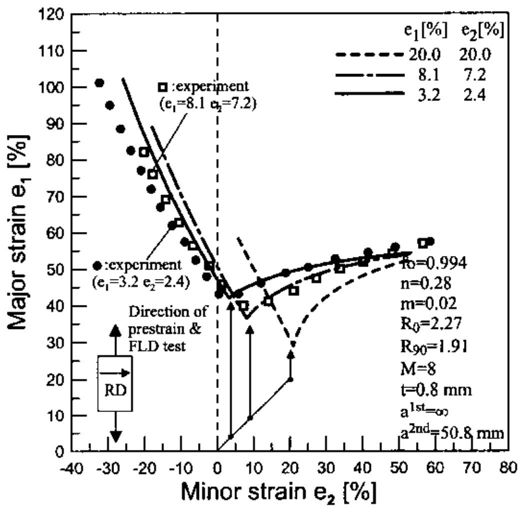



**Fig. 6** Effect of prestrains in biaxial stretching on FLD

In Fig. 6, FLDs with prestrains in equibiaxial tension are compared with FLDs constructed under conventional proportional loading. It is found that prestrains in equibiaxial tension generally lowers the entire FLD level, and shifts the $FLD_o$ to the right. The effect on the FLD of prestrains in uniaxial tension is shown in Fig. 7. Prestrains in uniaxial tension considerably increases the FLD on the right-hand side, with little effect on the left-hand side, and the $FLD_o$ shifts to the left. As can be seen from the two figures, the strain path shows a pronounced influence on the shape of the FLD. Furthermore, in order to assess the accuracy of the path-dependent FLD prediction, the theoretical FLDs have been compared with the published experimental results of Nam(1997) for DDQ steel 0.8t. It has

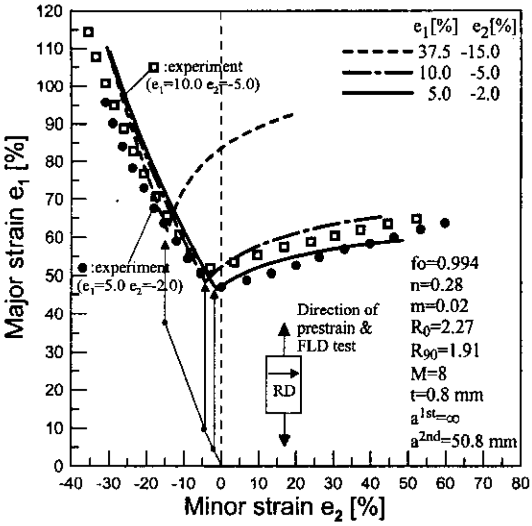



**Fig. 7** Effect of prestrains in uniaxial tension on FLD

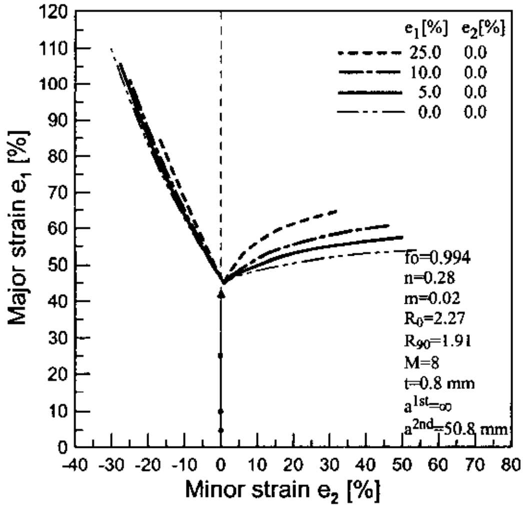



**Fig. 8** Effect of prestrains in plane strain on FLD

clearly been shown that the path-dependent FLD is a good match with the experimental data, with a proper choice of the initial imperfection parameter $f_o$.

Figure 8 shows theoretically how prestrains in a plane strain affect the forming limits. In this graph, no comparable experimental data is available due to difficulties in conducting such experiments. The prestrains in this mode has shown no effect on the $FLD_o$. However, the slopes of both sides of the FLD increase as the prestrain increases, so that the shape of the FLD on the left-hand side is nearly the same as that of an FLD for conventional proportional loading; yet, the shape of the FLD on the right-hand side produces different results.

## 4. Conclusions

An analytical model for predicting a path-dependent FLD based on the M-K approach has been presented. The effect on the FLD of the strain gradient in the constitutive equation has been included in the present study.

The following conclusions may be obtained. A change in the strain path in sheet metal forming significantly influences the shape of the FLD. It has also been observed that prestrains in equibiaxial tension generally lower the entire FLD level, whereas prestrains in uniaxial tension significantly raise the forming limits on the right-hand side of the FLD, with little effect on the left-hand side.

Therefore, this study reemphasizes the significance of the strain path, which influences the formability of sheet metals, in determining the level and shape of the FLD. Moreover, it appears that a theoretically predicted FLD could be used effectively to assess the formability of sheet metals and document forming processes in the press shop.

## Acknowledgments

This work was supported by the Brain Korea 21 Project, Kyungpook National University(2000). The authors would like to thank Dr. Nam in POSCO Technical Research Institute for his many helpful suggestions and experimental data.

## References

Barata Da Rocha, A., Barlat, F. and Jalinier, J. M., 1984, "Prediction of Forming Limit Diagrams of Anisotropic Sheets in Linear and Non-Linear Loading," *Mater. Sci. Engng*, Vol. 68, pp. 151 ~164.

Barlat, F. and Lian, J., 1989, "Plastic Behavior and Stretchability of Sheet Metals, Part I : A Yield Function for Orthotropic Sheets Under Plane Stress Conditions," *Int. J. Plasticity*, Vol. 5, pp. 51～66.

Charpentier, P. L., 1975, "Influence of Punch Stretching Curvature on the Stretching Limits of Sheet Steel," *Met. Trans*., Vol. 6A, pp. 1665～1669.

Ghosh, A. K. and Hecker, S. S., 1974, "Stretching Limit in Sheet Metals : In-Plane Versus Out-of-Plane Deformations," *Met. Trans*., Vol. 5A, pp. 2161～2164.

Goodwin, G. M., 1968, "Application of Strain Analysis to Sheet Metal Forming Problems in the Press Shop," *SAE Paper*, No. 680093

Hecker, S. S., 1975, "A Simple Technique for Determining Forming Limit Curves," *Sheet Metal Ind*. Vol. 52, pp. 671～676.

Keeler, S. P., 1965, "Determination of Forming Limits in Automotive Stampings," *SAE paper*, No. 650535

Kim, Y. S. and Park, K. C., 1993, "A Consideration on the Simulation Tests for Evaluating Stamping Formability," *J. Korean Soc. Mech. Engng*., Vol. 33, pp. 47～65.

Marciniak, Z. and Kuczynski, K., 1967, "Limit Strains in the Processes of Stretch-Forming Sheet Metals," *Int. J. Mech. Sci*., Vol. 9, pp. 609～620.

Nam, J. B., Park, K. C. and Jeong, K. J., 1997, "Determination of Forming Limit Curves of Steel Sheets with Considering the Effect of Strain-Path Changes," *J. Korean Soc. Tech. Plasticity*, Vol. 6, pp. 36～45.

Shi, M. F. and Gerdeen, J. C., 1991, "Effect of Strain Gradient and Curvature on Forming Limit Diagrams for Anisotropic Sheets," *J. Mater. Shaping Technol*., Vol. 9, pp. 253～268.

Son, H. S., Kim, Y. S. and Lim, S. E., 1999, "Prediction of Forming Limit Diagrams for Anisotropic Sheet Metals," *Proc. 12th Comp. Mech. conf. JSME*, pp. 281～282.

Stören, S. and Rice, J. R., 1975, "Localized Necking in Thin Sheets," *J. Mech. Phys. Solids*, Vol. 23, pp. 421～441.

Zhao, L., Sowerby, R. and Sklad, M. P., 1996, "A Theoretical and Experimental Investigation of Limit Strains in Sheet Metal Forming," *Int. J. Mech. Sci*., Vol. 38, pp. 1307～1317.