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Detection of Leakage Point via Frequency Analysis of a Pipeline Flow

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Fast Fourier Transformation is employed to convert the head variation of a pipeline in the time domain to the amplitude of the frequency domain. Applying method of characteristics to a pipeline provides a significant frequency range for a surge introduced from the valve modulation. Inverse Fast Fourier Transformation and a Finite Impulse Response Filter can be used to remove any possible noise existing from the significant frequency range of an unsteady condition. A filtered signal shows higher potential for the inverse calculation of leakage detection than the noise-added signal does. The respective performances of Inverse Fast Fourier Transformation and a Finite Impulse Response Filter are compared in terms of leakage detection capability. Characteristics of the frequency range for multiple leakages were investigated to validate the effectiveness of the noise control method in the frequency domain.

Key Words : Fast Fourier Transformation, Finite Impulse Response Filter, Inverse Calculation for Leakage Detection

Nomenclature

A : Cross-sectional area of pipe
a : Wave speed
D : Pipe diameter
d_k : Hamming window function
F : Amplitude in frequency domain
f : Darcy-Weisbach friction factor
g : Gravitational acceleration
H : Piezometric head
l_k : Digital filter coefficient
M : Truncation factor for hamming window
Q : Discharge of flow
s : Frequency domain
T : Sampling interval
t : Time
w_c : Cutoff frequency
w_s : Digital sampling rate
x_{n-k} : Input signal
y_n : Filtered output signal
Z : Z transform
Dx : Space step

1. Introduction

The supervisory control and data acquisition system (SCADA) is used in many pipelines to remotely monitor the flow data on a real time basis. The reliability of data frequently suffers from data noise (Liou, 1995; 1998). The existence of noise signals might be a significant obstacle for the proper management of a pipeline system (Liou, 1998). In particular, the head variation for inverse calculations will be extremely sensitive to noise (Liggett and Chen, 1994). An extraction of a systems impulse response by a cross correlation

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method suggests a methodology to remove impact of the detrimental (Dallabetta, 1996; Liou, 1998).

The flow characteristics of a pipeline system can be defined in terms of a frequency domain. The characteristics method can be a useful tool to acquire a systems original response, signal without noise, in some unsteady conditions (Roberson *et al*., 1998; Wylie and Streeter, 1993). The range of original signals in a frequency domain is an important criterion in filtering the noise data in an actual pipeline system. However, it was assumed in this study that a filtered signal can relax the noise impact of the inverse calculation of leakage detection in a pipeline system. The objectives of this study are as follows:

1. To develop a noise filtering method for the data acquisition system in a pipeline.
2. To validate through inverse transient analysis the effectiveness of this approach for the leakage detection problem.

2. Governing Equations in a Pipeline

The momentum and continuity equations for the transient flow in the pipeline are given as follows:

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{f|Q|Q}{2DA}=0 \tag{1}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{2}$$

where x=distance along a pipe, t=time, a=the wave speed, g=the gravitational acceleration, A=the cross sectional area of pipe, Q=discharge of flow, H=the head, f=the Darcy-Weisbach friction factor, and D=the pipe diameter (Roberson et al., 1988; Wylie and Streeter, 1993).

The governing differential equations are transformed by the method of characteristics into an integrable form. The characteristic equations are as follows:

$$\frac{dH}{dt}+\frac{a}{gA}\frac{dQ}{dt}+\frac{fa}{2gDA^2}|Q|Q=0 \tag{3}$$

$$\frac{dH}{dt}-\frac{a}{gA}\frac{dQ}{dt}+\frac{fa}{2gDA^2}|Q|Q=0 \tag{4}$$

Equations (3) and (4) can be integrated on the x-t plane by employing appropriate initial conditions and boundary conditions. Known head and flows at nodes in a pipeline can be the efficient boundary conditions for a transient analysis.

3. Frequency Analysis

3.1 FFT and inverse FFT

The variation of a head in the time domain can be translated to a frequency domain specified by giving its amplitude F as a function of frequency f. The number of operations for a frequency transformation can be significantly reduced by using the Fast Fourier Transformation(FFT). A discrete Fourier transformation of length N can be rewritten as the sum of two discrete Fourier transformations as:

$$F_n=\sum_{k=0}^{N-1}H_k e^{2\pi kin/N}=F_n^e+W^k F_n^o \tag{5}$$

where, $W=e^{2\pi i/N}$, F_n^e=the nth component of length N/2 formed from the even components of the original h_k^s, and F_n^o=the nth component of length N/2 formed from the odd components.

One restriction of the FFT is that the original N is an integer power of 2, so that the data can be subdivided to the length of 1. A brief explanation of the theory and methods of the FFT appears in Press *et al.* (1994).

The signal in the frequency domain should be properly treated before it is reconverted to the time domain. The frequency amplitude relationship of computational head variation provides a significant range of associated signals in the frequency domain. When a signal is contaminated by noises, the disturbance outside of the significant range of the original signal can be recognized as redundant signal from noise.

The formula for the discrete inverse Fourier transform is as follows:

$$H_k=\frac{1}{N}\sum_{n=0}^{N-1}F_n e^{-2\pi ikn/N} \tag{6}$$

The Inverse Fast Fourier Transformation (IFFT) can be readily made using subprogram made by Press *et al*., (1994).

3.2 Finite impulse response (FIR) digital filter

The ideal filter characteristics can be applied to a design synthesis formula (Stearns, 1975) to determine the filter coefficient, l_k, as follows:

$$l_k=\frac{T}{2\pi}\int_{-w_s/2}^{w_s/2}H(w)e^{iwkT}dw \qquad (7)$$

where, w_s is the digital sampling rate in radians per second, H(w) is the frequency transfer function, and T is the sampling interval. This formula is actually a discrete version of the Inverse Fourier Transformation, and it yields the l_k coefficients, which can be directly inserted into the non-recursive algorithm to yield the filtered output sequence y_n from the input signal x_{n-k} as follows:

$$y_n=\sum_{k=-M}^{M}l_k x_{n-k} \qquad (8)$$

Since the design synthesis only depends on the desired transfer function, formula of l_k's can be formulated for various cases based on the cutoff frequency w_c. For a lowpass filter design, the synthesis formula reduces to following:

$$\begin{aligned}l_k&=\frac{T}{2\pi}\int_{-w_c}^{w_c}1\cdot e^{iwkt}dw\\&=\frac{T}{2\pi}\frac{e^{iw_ckT}-e^{-iw_ckT}}{jkT}\end{aligned} \qquad (9)$$

Applying Euler's formula, Eq. (9) can be simplified to a SINC function expression:

$$l_k=\frac{Tw_c}{\pi}\frac{\sin(w_ckT)}{w_ckT} \qquad (10)$$

Since the SINC function has singularity at k=0, SINC(0) is defined to unity employing L' Hopital's rule.

Although this filter will approach the ideal design model as the number of terms in the summation becomes infinite, for practical implementation, the FIR filter must be truncated. Selection of a truncation factor (M) passes a problem in that ripples appear in the magnitude of H(w) as shown in Fig. 1. These ripples are called the Gibbs phenomenon and can be eliminated by the using an appropriate window function (Hamming, 1983) stated as the following:

$$d_k=0.54+0.46\cos\left(\frac{k\pi}{M}\right) \qquad (11)$$

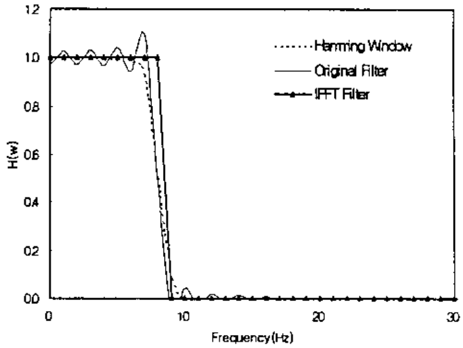


Fig. 1 The frequency transfer function of FIR filter with or without hamming window and IFFT filter

The effect of the Hamming windows is shown Fig. 1 for a truncation factor of M=50.

4. Inverse Solution for Leakage Detection

Calibration of leakage determination or the unauthorized use of water is readily performed by a pipe network transient model, INVCHAR (Chen, 1995). INVCHAR was designed to find the adjoint solution for a system that can provide a method for efficient calculation. A linear system of characteristic equations can be written as follows:

$$[M]\{v\}=\{R\} \qquad (12)$$

where, [M]=the coefficient matrix for forward transient analysis, {v}=the vector of the dependent variable, and {R}=right hand side vector. The chi-square merit function of the unity variance is to minimize the discrepancy between the measured and the calculated data

$$E=\sum(h_i^m-h_i)^2 \qquad (13)$$

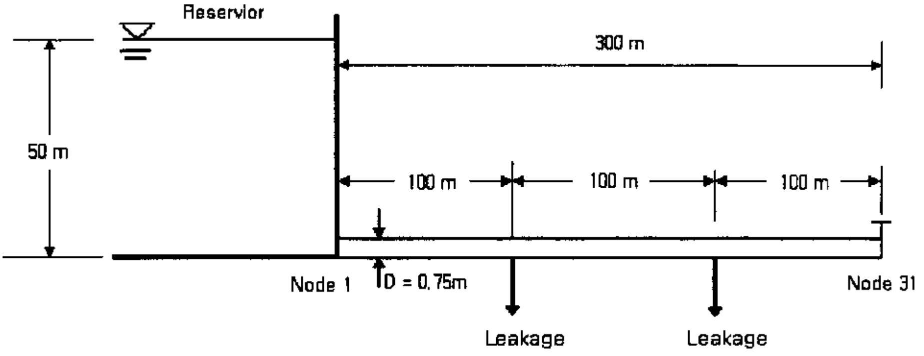
where, h_i^m=the measured head, h_i=the calculated head.

The derivative of the merit function to the leakage area a_i is as follows:

$$\frac{\partial E}{\partial a_i}=\left\{\frac{\partial E}{\partial h}\right\}^T\left\{\frac{\partial h}{\partial a_i}\right\}=-2\{h^m-h\}^T\left\{\frac{\partial h}{\partial a_i}\right\} \qquad (14)$$

A set of linear equations for the minimization of the Jacobian matrix is follows:

$$[H]\{\delta a\}=\left\{\frac{\partial E}{\partial a}\right\} \qquad (15)$$



Leakage area = 0.5E-4 m2

Bulk modulus of elasticity = 2.2E10 Pascal

Wave speed a = 1000 m/sec

Darcy-Weisbach friction factor = 0.04

Fig. 2 Reservoir-pipeline-valve definition sketch

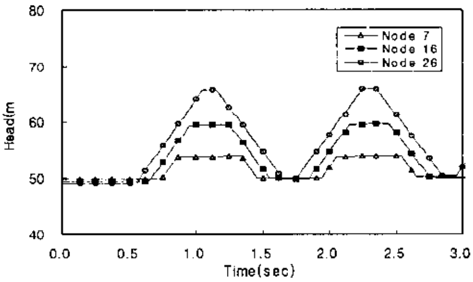


Fig. 3 Head variations at node 7, 16 and 26 about a surge introduced by the control valve closure from 0.53m^3/sec to 0.1m^3/sec in 5 sec.

where, [H] = a coefficient matrix based on the Hessian matrix. The Levenberg Marquardt method (Press *et al.*, 1994) was employed to improve the merit function. By introducing an arbitrary multiplier λ into the diagonal component of [H], as show below,

$$H_{ij} \rightarrow (i+\lambda)H_{ij},\ i=j \tag{16}$$

the variation of the λ value makes the parameter fitting process efficient in terms of the computation. Because the first and second derivatives of the merit function are involved, the evaluation of the Jacobian matrix using the Levenberg Marquardt method is computationally intensive. However, iteration improves the merit function until there is no change in the leakage area.

5. Application Example

Consider a simple horizontal pipeline in which water passes from a constant head supply reservoir to a control valve at a flow rate of 0.531m^3/sec. As Fig. 2 illustrates, the pipeline is 300m in length and 0.1m diameter. A control valve is located at the end of the pipeline and it executes a linear closure from a full gate opening. Detailed hydraulic and physical properties of the pipeline are given in Fig. 2. In particular, two leakage points exist in one-third and two-thirds points along the pipeline with a 5 * E-5 m^2 leakage area.

The pipeline is divided into 30 sections for the application of the characteristics method. A computational time step of 0.01 seconds fulfill the Courant number 1 using the length of each section, 10m, and 1,000m/sec wave propagation speed. A water hammer is introduced from a valve closing in 5 seconds.

Figure 3 shows head variations at nodes 7, 16, and 26 from a transient flow analysis. The head variation characteristics can be analyzed in the frequency domain. Application of the FFT to the time history of the head at nodes 7, 16, and 26 provides frequency-amplitude relationships, as shown in Fig. 4. The distribution of the ampli-

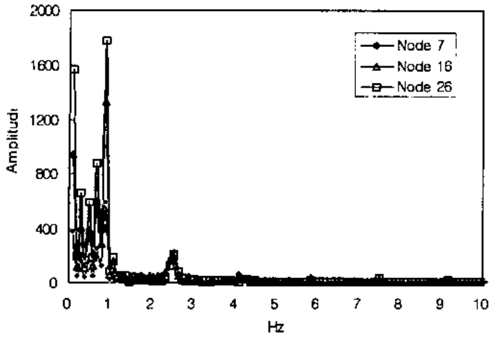


Fig. 4 The frequency amplitude relationships at node 7, 16, and 26

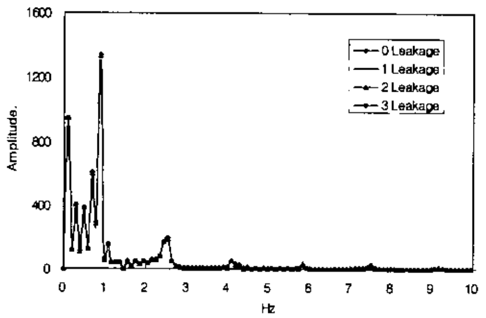


Fig. 5 The frequency asmplitude relationships of various leakage status

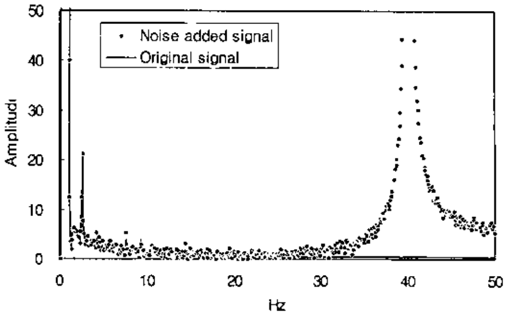


Fig. 6 The frequency amplitude relationships of original signal and noise added signal

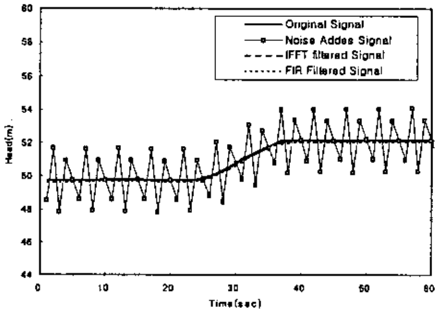


Fig. 7 Head variations of original dignal, noise-added signal, IFFT filtered signal and FIR filtered signal

tude primarily exists from 0 to 10 Hz frequency range. Higher amplitude can be observed on the node located near the valve than the node close to reservoir.

Numerical experiments of various leakage conditions, such as no leakage, one leakage, and so on, are performed in a pipeline. The frequency characteristics of each leakage condition are illustrated in Fig. 5. No significant differences can be found in the frequency-amplitude relationship between the head variation of no leakage and that of cases of multiple leakage. The existence of leakage does not have any impact on the head variation characteristics in terms of frequency response.

Considering the existence of noise in SCADA, 60Hz noise and white noise were generated and the superimposed onto the original head variation signal of 5 second valve closing transient flow.

Figure 6 compares the frequency characteristics of the original signal, noise free signal, and a noise-contaminated signal. While white noise exists within the whole range of the frequency domain, the effect of 60 Hz noise is only shown in the region near 40 Hz due to the 100 Hz sampling frequency.

Careful observation of Fig. 4 indicates that the original signal does not show much significant amplitude in the range beyond 8 Hz. The noise-added signal can be divided at 8 Hz for the purpose of noise filtering. After removing the frequency characteristic beyond 8 Hz, either the Inverse Fast Fourier Transformation (IFFT) or the Finite Impulse Response (FIR) filter can be used to retransform the treated signal from the frequency domain to the time domain. Figure 7 shows that head variations of an original signal, a noise-added signal, a noise treated signal by the IFFT and the noise treated signal by a FIR filter. As shown in Fig. 7, even though a noise treated signal by a FIR filter shows good approximation of an original signal, filter capacity of a FIR filter is limited, due to the hamming window introduced to relax the Gibbs phenomenon at the

Table 1 Inverse calculation results of leakage areas and locations in case of surge introduced by the control valve closure from 0.53m^3/sec to 0.1m^3/sec in 5 sec.

	Mon	Suspected leakage node (Value *E-5)					Chi-square value
	#	5	11	15	21	24	
Ori dat	6	0.00	1.02	0.00	1.00	0.00	.57E-4
	17	0.00	1.02	0.00	1.01	0.00	.57E-4
	27	0.00	1.02	0.00	1.01	0.00	.57E-4
Noi Add	6	0.00	0.67	0.96	0.00	0.85	198.2
	17	0.10	2.76	0.00	3.61	0.00	199.9
	27	0.21	0.95	0.00	5.41	0.00	204.1
Ifft Filter	6	0.16	0.60	0.23	1.23	0.00	.112
	17	0.00	1.16	0.00	1.26	0.00	.120
	27	0.00	1.20	0.00	1.87	0.00	.452
Fir Filter	6	0.95	0.87	0.96	1.50	0.00	.793
	17	0.12	1.41	0.66	1.52	0.00	.988
	27	0.00	0.81	0.00	1.80	0.20	1.06

edge of the filter. However, a FIR filter has an advantage over the IFFT. Because real time operation is not possible in the IFFT, due to its data number constraint of 2n, but it is available in the FIR filter.

The performance of a noise treatment filter can be effectively diagnosed by employing the inverse calculation model for leakage detection. INV-CHAR (Chen, 1995) is an excellent inverse transient flow analysis program for the detection of leakage locations and areas. A water hammer is introduced from node 31 as the control valve is closed. Table 1 shows inverse calculation results for the surge from the valve closing from 0.531 m^3/sec to 0.1 m^3/sec in 5 seconds. Depending on the data logging position, a noise-added signal shows misleading in the prediction of leakage locations and areas. Noise filtered signals show better capability in predicting leakage locations and leakage areas than the noise-contaminated signals can. However, the IFFT filter shows higher accuracy in predicting leakage conditions than the FIR filer can. The chi-square values of the IFFT are also lower than those of the FIR filter cases.

Table 2 Inverse calculation results of leakage areas and locations in case of surge introduced by the control valve closure from 0.53m^3/sec to 0.4m^3/sec in 10 sec.

	Mon	Suspected leakage node (Value *E-5)					Chi-square value
	#	5	11	15	21	24	
Ori dat	6	0.00	1.02	0.00	1.00	0.00	.55E-4
	17	0.00	1.02	0.00	1.00	0.00	.26E-4
	27	0.00	1.02	0.00	1.00	0.00	.55E-4
Noi Add	6	0.00	0.73	1.15	0.00	0.98	199.1
	17	0.03	2.80	0.00	3.84	0.00	200.6
	27	0.20	1.51	0.00	5.29	0.00	204.5
Ifft Filter	6	0.00	1.60	0.01	0.97	0.00	.0109
	17	0.00	1.01	0.01	1.13	0.00	.0159
	27	0.00	1.36	0.13	0.96	0.00	.0366
Fir Filter	6	0.47	1.40	0.72	1.24	0.20	.743
	17	0.28	1.23	0.75	1.37	0.00	.845
	27	0.07	1.31	0.00	0.94	0.65	.782

The transient flow of a reduced scale is generated by a 10-sec closing control valve from 0.53m^3/sec to 0.4m^3/sec. Table 2 shows inverse calculation results of a reduced-scale water hammer for an original signal, a noise added signal, a noise treated signal by IFFT, and a noise treated signal by an FIR filter. The performance of an IFFT filter is significantly improved, compared to bigger scale surge cases. A FIR filter also shows an improvement in leakage detection. The chi square value of the IFFT is 0.1 ∗ E-1, which is significantly lowered than chi square values of Table 1 and Table 2. This result shows that a noise filter can have high efficiency in smaller and slower surges than in bigger and abruptly introduced transient surges.

6. Conclusions

An efficient noise removing technique is developed for the SCADA system. The procedure for noise treatment is as follows: First, to simulate the transient flow of a pipeline by using a method of characteristics. Second, to apply the FFT to the time history of the head to acquire the frequency amplitude of a signal at any particular point of the pipeline. Third, to determine the cutoff frequency range from the frequency characteristics. Fourth, to retransform the properly treated signal from the frequency domain to the time domain.

Both the IFFT and the FIR filters are used in the signal reconstruction process. Given the cutoff frequency information at a particular unsteady condition, real-time noise filtering operation are possible merely by employing a FIR filter.

The inverse calculation of leakage detection program, INVCHAR, was used to test the performance of two different filters. The IFFT is found to be superior to the FIR filter in terms of predictability of leakage locations and areas. The refinement of filter design processes such as band stop or pass filter and the introduction of advanced filter types, can be future research topics to improve the performance of filters.

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