Development of a Numerical Methodology for Analysis and Design of Weldments

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There are many analysis formulas for determining the resultant shear force in welds. However, there is no general procedure which is applicable to a joint with all six possible loadings exerted simultaneously. A numerical methodology and computer program for such a problem were developed, and they are capable of analyzing a weld of any shape composed of straight or circular line segments. The computer program developed in this study can also display the design procedures and results using computer graphics. The development of such a design procedure and an interactive computer program for weldments analysis will lead to lower cost.

1. Introduction

There are many aspects of welding which are important and rightly deserve a place in machine design considerations. For example, overwelding increases the cost of welding. Specifying the correct weld size is a step in obtaining low cost welds. For the designer, an immediate problem is the determination of the size of weld necessary for a given part, and this requires a stress analysis of the designed weld joint. Once the forces and moments acting on the weld are known, the weld is designed using the procedures recommended by the American Welding Society (AWS) with modifications as recommended by the Lincoln Electric Company(Blodgett, 1963).

Groove welds must be made all the way through the plate, i.e. 100% penetration, and when properly made, must have equal or greater strength than the plate. There is no need for calculating the stress in a groove weld. However, the size of a groove weld may be needed for

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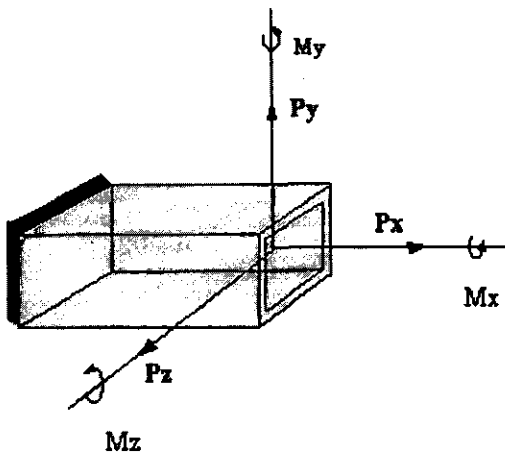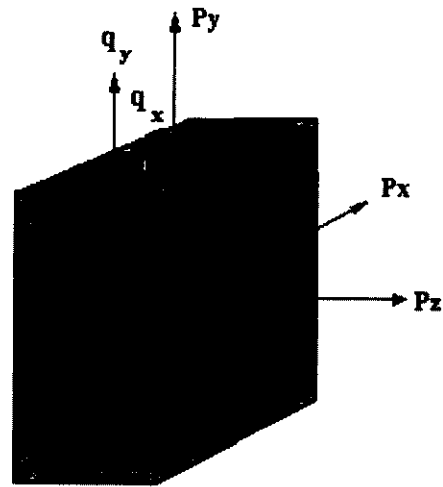
estimating the production cost. When welding alloy steels, it is necessary to match the weld metal strength and the plate strength but this is primarily a matter of proper electrode selection and welding technique.

It is possible to have too small or too large a fillet weld. Therefore, it is important that the proper size of a fillet weld be determined. The objective of this work is to develop an interactive computer design procedure for sizing fillet welds.

There are three force components and three moment components which must be considered when determining the size of a fillet weld. The three force components are Px, Py and Pz, acting in the x, y and z coordinate directions, and the three moment components Mx, My and Mz, acting about the x, y and z-axes, as shown in Fig. 1. The magnitudes of these components are obtained using the standard structural analysis techniques.

Most machine design books have equations for designing regular weld-joint shapes including equations for the centroid and moments of inertia. A computer program must be able to generate these equations plus those for any other shape. The computer program developed in this study is capable of analyzing a weld of any shape composed of straight or circular line segments.
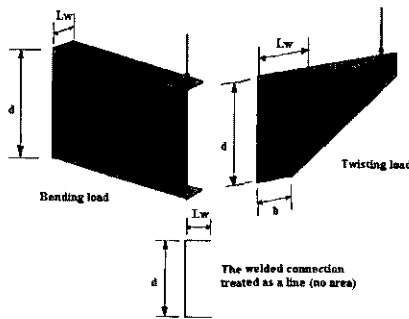
Fig. 1 Six loading components

Fig. 2 Treating welded connection as a line

2. Development of Matrix Algorithm

2.1 Determination of resultant shear flow

A weld connection is modeled as a single or series of line segments. The area of a weld connection is the thickness times the length but the thickness is associated with the stress component to obtain a shear flow type quantity. Therefore, the length of the weld is an important quantity and is denoted by Lw, Fig. 2.

There are many analysis formulas for determining the resultant shear force in the weld. However, there is no general procedure which is applicable to a joint with all six possible loadings exerted simultaneously. It is simple to compute the shear flow on the weld for forces acting along the x, y and z-axes. As shown in Fig. 3, the resultant shear flow acting in the x direction, q_x (Sander, 1978), is
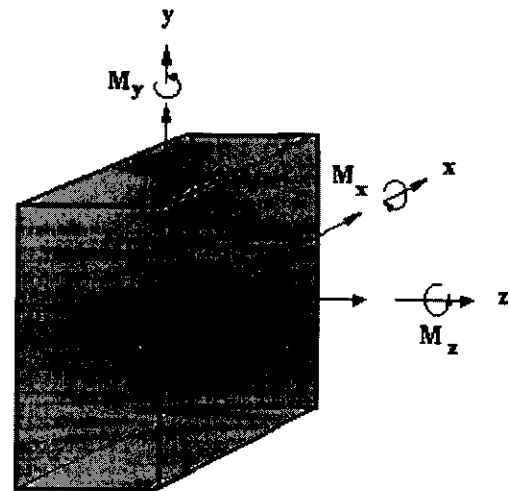

Fig. 3 Resultant shear flow for force components

Fig. 4 Three moment components

$$q_x = \frac{P_x}{L_w} \tag{1}$$

Similarly,

$$q_y = \frac{P_y}{L_w} \tag{2}$$

$$q_z = \frac{P_z}{L_w} \tag{3}$$

where P_x, P_y, and P_z are the input forces acting in the x, y, and z directions, respectively.

The shear flows resulting from the three moments shown in Fig. 4 are illustrated in Figs. 5, 6 and 7. The values are calculated using (Blodgett, 1963),
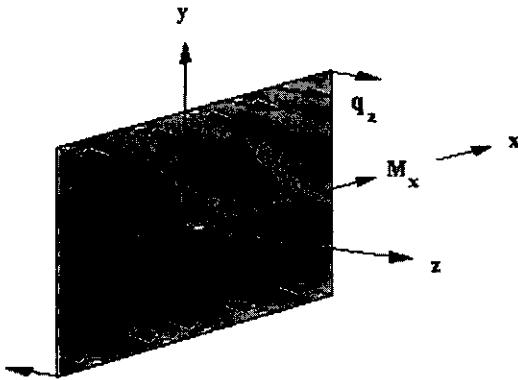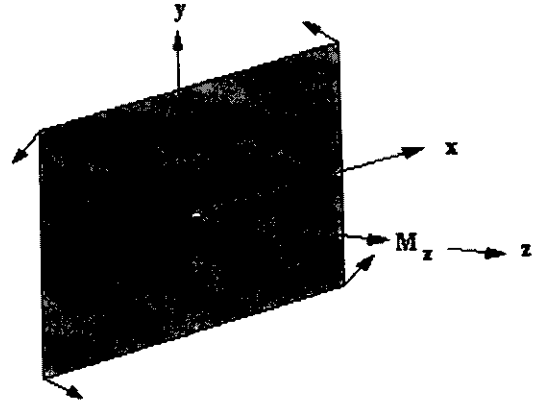
Fig. 5   Shear flow resulting from moment, M_x

Fig. 7   Shear flow resulting from moment, M_z

The resultant shear flow for any value of x and y is

$$q_r = \sqrt{q_x^2 + q_y^2 + q_z^2} \tag{9}$$

2.2   Centroid of weld

The matrix equation Eq. (8), clearly indicates that the weld shape properties I_x, I_y and J must be known before the shear flow components q_x, q_y and q_z can be calculated. The values of I_x, I_y and J are evaluated relative to the centroid of the weld. The evaluation of the centroid is discussed in this section.

The centroid of an area which is defined by the coordinates $\overline{x}$ and $\overline{y}$ are given by (McLean, 1972 & Tuma, 1975)
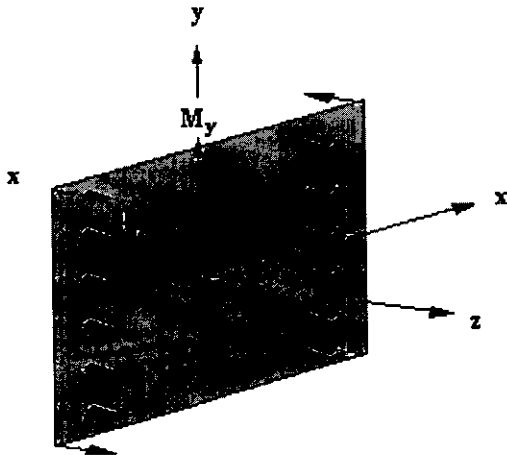

Fig. 6   Shear flow resulting from moment, M_y

$$q_z = \frac{M_x y}{I_x} \tag{4}$$

Similarly, from Figs. 5, 6 and 7

$$q_z = -\frac{M_y x}{I_y} \tag{5}$$

$$q_x = -\frac{M_z y}{J} \tag{6}$$

$$q_y = \frac{M_z x}{J} \tag{7}$$

where I_x, I_y and J are the three moment properties of the weld.

The three components of shear flow can be summarized, using a matrix multiplication, as

$$
\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} =
\begin{bmatrix}
\dfrac{1}{L_w} & 0 & 0 & 0 & 0 & \dfrac{-y}{J} \\
\dfrac{1}{L_w} & 0 & 0 & 0 & \dfrac{x}{J} \\
0 & \dfrac{1}{L_w} & \dfrac{y}{I_x} & -\dfrac{x}{I_y} & \dfrac{x}{J} \\
0 & 0 & \dfrac{1}{L_w} & \dfrac{y}{I_x} & -\dfrac{x}{I_y} & 0
\end{bmatrix} \tag{8}
$$

$$\overline{x} = \frac{\int x\, dA}{A} \tag{10}$$

$$\overline{y} = \frac{\int y\, dA}{A} \tag{11}$$

where A denotes the area, $\overline{x}$ indicates the x coordinate of the centroid and $\overline{y}$ indicates the y coordinate of the centroid. The centroid of an area is the point at which the area can be considered to be concentrated with unchanged first moment of the area about any axis.

The centroid of a few areas of basic shapes is easily calculated. In a symmetrical area such as a circle or a square, the centroid coincides with the geometric center of the area.

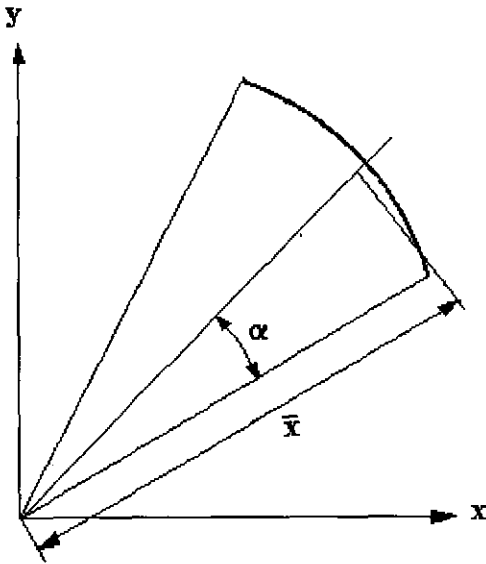A weld is analyzed by assuming the welded

Fig. 8 Centroid of a curved element, $\bar{x}$

Fig. 9 Straight line segment

region is a line rather than an area. The centroid of a line segment is defined by

$$\bar{x} = \frac{\int x\, dL}{L} \tag{12}$$

$$\bar{y} = \frac{\int y\, dL}{L} \tag{13}$$

where L denotes the length of the weld. The centroid of a straight line is at the center of the line. The centroid of a circular arc, $\bar{x}$ (McLean, 1972), is

$$\bar{x} = \frac{r \sin \alpha}{\alpha} \tag{14}$$

where r is the radius of the circle for the arc, and α is the angle of the arc, as shown in Fig. 8.

2.3 Moment of inertia

The fillet welds in this study are assumed to consist of straight or circular line segments. The moment properties of these line segments are now discussed.

2.3.1 Straight element

The moment of inertia of a irregular line about any axis in the plane of the weld is given by the summation of the moments of inertia of the individual line segments about that axis. If the
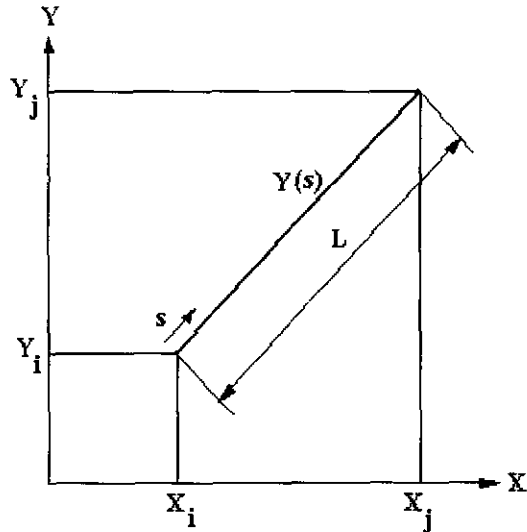
moment of inertia of the finite line about the x-axis is denoted by I_x and about y-axis by I_y, then

$$I_x = \int y^2\, ds \tag{15}$$

$$I_y = \int x^2\, ds \tag{16}$$

A straight line segment can be defined by

$$y(s) = [N_i N_j] \begin{bmatrix} Y_i \\ Y_j \end{bmatrix} \tag{17}$$

where N_i, N_j are linear shape functions(Segerlind, 1976)

$$N_i = \left(1 - \frac{s}{L}\right), \quad N_j = \frac{s}{L} \tag{18}$$

and Yi and Yj are the y coordinates of the ends of the line, Fig. 9.

Substituting Eq. (17) into Eq. (15) gives

$$I_x = \int [Y_i Y_j] \begin{bmatrix} N_i \\ N_j \end{bmatrix} [N_i N_j] \begin{bmatrix} Y_i \\ Y_j \end{bmatrix} ds \tag{19}$$

Evaluating the integrals involving the shape functions gives

$$I_x = \frac{L}{6} [Y_i Y_j] \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} Y_i \\ Y_j \end{bmatrix} \tag{20}$$

Equation (20) can be written in a more convenient form using, Fig. 10,

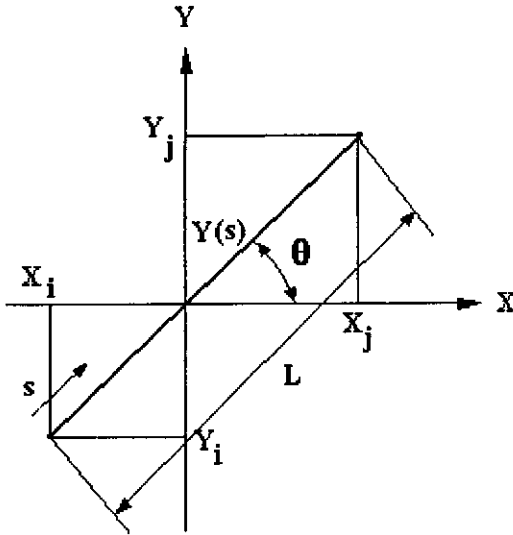$$Y_i = \frac{L}{2} \sin \theta \tag{21}$$

Fig. 10   Transformed line segment

Fig. 11   Perpendicular distance between two axes

$$Y_j = -\frac{L}{2}\sin\theta \tag{22}$$

Substituting these into Eq. (20) gives

$$I_x' = \frac{L^3\sin^2\theta}{12} \tag{23}$$

where I_x' is the moment of inertia of a straight line about its own centroidal axis.

Similarly,

$$I_y' = \frac{L^3\cos^2\theta}{12} \tag{24}$$

Using the parallel-axis theorem gives (see Fig. 11 for d_x and d_y)

$$I_x = L(d_y)^2 + L^3\frac{\sin^2\theta}{12} \tag{25}$$

$$I_y = L(d_x)^2 + L^3\frac{\cos^2\theta}{12} \tag{26}$$

The polar moment of inertia, J, is needed to determine the shear flow resulting from the torsional moment M_z. The polar moment of inertia is defined by

$$J = \int_L \rho^2 ds = \int_L (x^2 + y^2) ds \tag{27}$$

where ρ is the radial distance from the origin of the coordinate system to the length segment ds. Using Egs. (25) and (26) gives

$$J = \rho^2 L + \frac{L^3}{12} \tag{28}$$

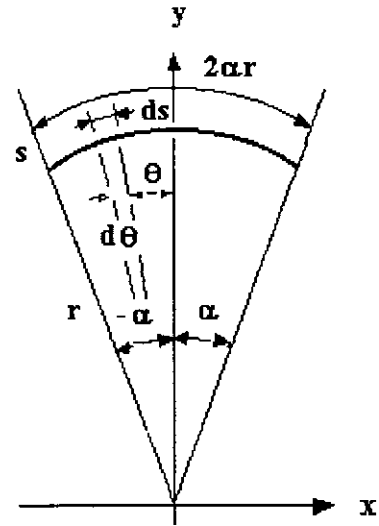**Fig. 12**   Curved element

2.3.2   Curved element

The moment of inertia for a curved element is defined by

$$I_x' = \int y^2(s) ds \tag{29}$$

where s is the distance along the curve. For the geometry shown in Fig. 12, we know that $ds = rd\theta$ and $y = r\sin\theta$. Therefore,

$$I_x' = \int_{-\alpha}^{\alpha} r^3\sin^2\theta d\theta \tag{30}$$

Evaluating the integral gives

$$I_{x'} = r^3\left(\alpha - \frac{\sin 2\alpha}{2}\right) \qquad (31)$$

Similarly,

$$I_{y'} = r^3\left(\alpha + \frac{\sin 2\alpha}{2}\right) \qquad (32)$$

In general, we have to compute the moment of inertia about any axis. Using the transformation equations and the parallel-axis theorem, we have

$$I_{xt} = r^3(\alpha - \sin 2\alpha \cos 2\theta) \qquad (33)$$
$$I_{yt} = r^3(\alpha + \sin 2\alpha \cos 2\theta) \qquad (34)$$

where I_{xt}, I_{yt} are the moments of inertia with respect to x_t and y_t-axes shown in Fig. 13. The quantities $L(d_y)^2$ and $L(d_x)^2$ must be added to Eqs. (33) and (34). Thus, we have
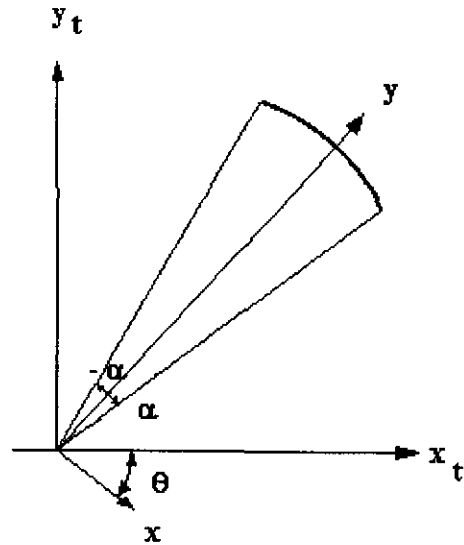

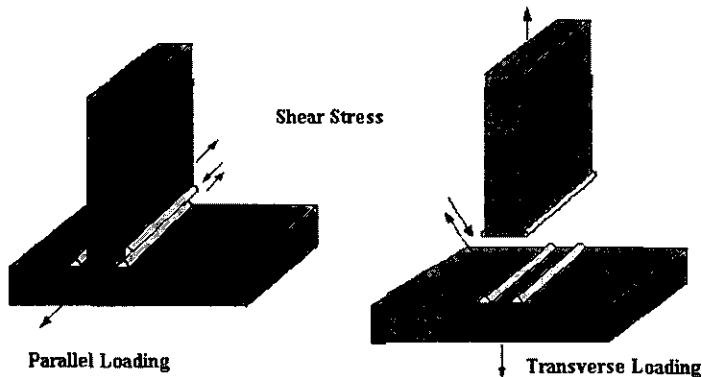Fig. 13 Curved element for the transformation equations
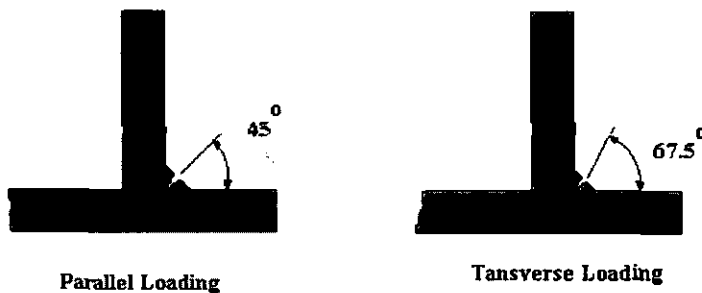

Fig. 14 Direction of the loads

Fig. 15 Plane of maximum shear stress

$$I_x = L(d_y)^2 + r^3(a - \sin 2a \cos 2\theta) \qquad (35)$$

$$I_y = L(d_x)^2 - r^3(a + \sin 2a \cos 2\theta) \qquad (36)$$

In addition to I_x and I_y, we also need J (Shigley, 1977 ; Hall, 1961), the polar moment of inertia, which is

$$J = I_x + I_y \qquad (37)$$

2.4 Analysis of weldments

Weld size is obtained by dividing the resultant shear force on the weld, by the allowable strength of the particular type of weld being used. Basically, there are three allowable stresses for fillet welds, 13.6 ksi, 18 ksi and 21 ksi (Blodgett, 1963).

The latter two correspond to E60xx and E70xx electrodes. The allowable shear stress of 13. 6 ksi is specified in the AWS code.

Fillet welds are classified according to the direction of the load, parallel loads and transverse loads, Fig. 14. The plane of maximum shear stress in the conventional 45° throat when subjected to a parallel load, and that in the 67.5° throat when subjected to a transverse load are shown in Fig. 15. A 45° fillet weld result in greater weld strength for a transverse load. The size of a fillet weld is specified by the leg length of the largest inscribed isosceles right triangle or the leg lengths of the largest inscribed right triangle. The leg length of a fillet weld with equal legs is given by w and the leg lengths of a fillet weld with unequal leg are given by a and b, as shown in Fig. 16. The throat dimension t for a fillet weld having equal legs is obtained by multiplying the size of the fillet weld by 0.707, i.e. t=0.707w, as shown in Fig. 17. Stress in a fillet weld is considered as shear on the throat in the direction of applied load.

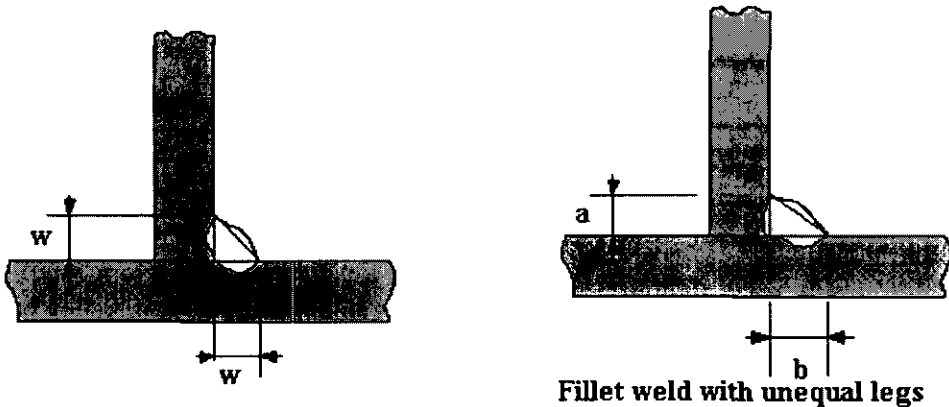The allowable parallel load per inch of weld in a statically loaded fillet weld is (Blodgett, 1963)

Fillet weld with unequal legs

Fig. 16   Fillet weld with equal legs and unequal legs

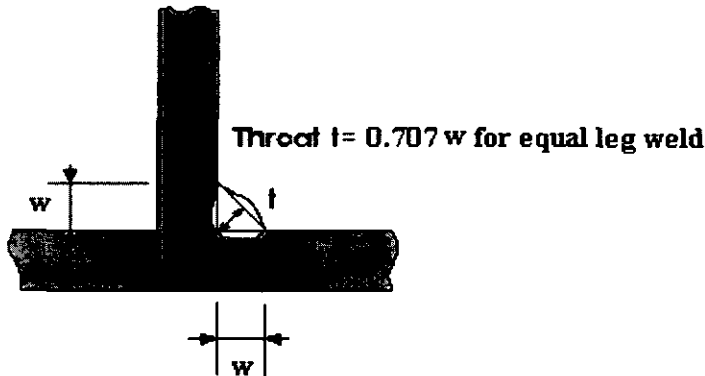**Throat t= 0.707 w for equal leg weld**

Fig. 17   Throat dimension

$$q_r = S_{all}(0.707w) \qquad (38)$$

where S_{all} is the allowable shear stress, and w is the weld size, inches.

Next, the allowable transverse load per inch of weld in a statically loaded fillet weld is

$$q_r = S_{all}(0.707w)/\cos 22.5° \qquad (39)$$

According to AWS, if a part of the load is applied in the parallel direction and another part transverse, the allowable parallel load must be used. Therefore, the weld size in this study has been computed using Eq. (38), and the weld size is given by

$$w = \frac{q_r}{0.707 S_{all}} \qquad (40)$$

3. Computer Program

(Prueitt, 1975 ; College of Engineering,
1966 ; Case Center, 1981)

A computer program was written to solve the equations discussed in the previous section. The weld properties Ix, Iy and J are calculated from the geometry of welds and the resultant shear forces and moments are input and the resultant shear flow is calculated for each node point. Coordinate values of additional points can be input. The weld size for each node and those for other specified node points are displayed on the graphics screen.

The program presented here for the design of welded joint consists of a main program and eight subroutines. The program uses interactive graphics to display the weld joint and weld sizes.

A brief outline of the program sequence is as follows:

a) The program asks for inputs, such as the allowable stresses, number of straight elements, number of curved elements, end coordinates of the straight elements and the coordinates of the radius center of the curved elements.
b) The program draws the weld joint shape.
c) The program computes the centroid and the moments of inertia.
d) The program asks if the shape is correct.
e) The three forces acting along the x, y and z axes and three moments acting about x, y and

z axes are input.
f) The computer calculates the weld size for the node points.
g) The computer draws the weld joint shape and gives the magnitude of weld size at the specified locations.
h) The computer summarizes the weld size.

Some of the subroutines and their functions are:

a) Subroutine MARKER draws circles for the weld shape.
b) Subroutine NUMELE writes the integer element numbers in parentheses, for example, (5).
c) Subroutine NUMNOD writes the integer node numbers.
d) Subroutine NUMNOD2 writes the characters for the weld size.
e) Subroutine AXIS2 draws the three-dimensional axes system.
f) Subroutine COORD writes x and y-axis scales.
g) Subroutine REDRAW re-draws the axes, weld shape and weld sizes.
h) Subroutine FIND checks if the specified location is on the line.

The computer program calculates the weld size for any coordinate pair (x, y) specified by the user. If the user makes a mistake during input, this coordinate pair may not lie on the weld. A check has been programmed so that coordinate pairs which do not lie on the weld are deleted.
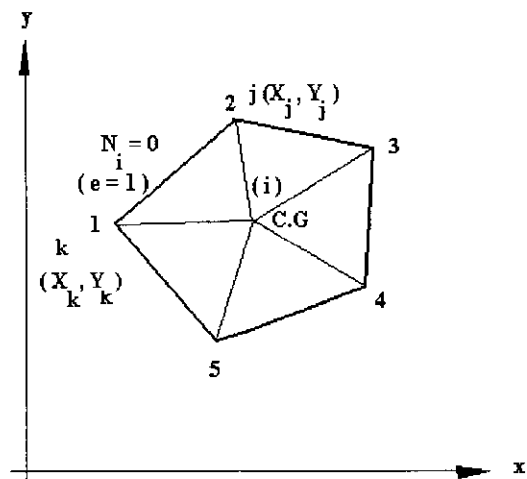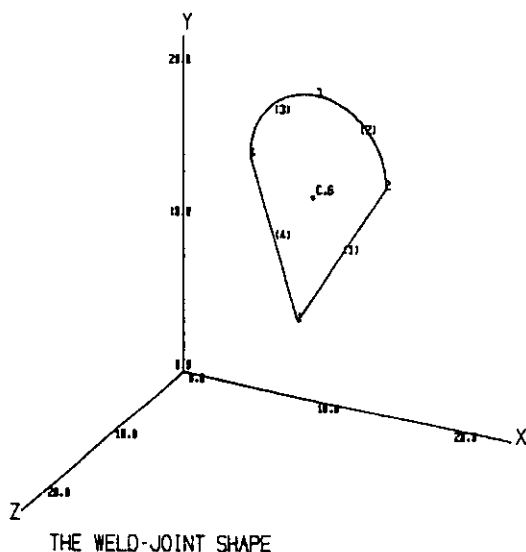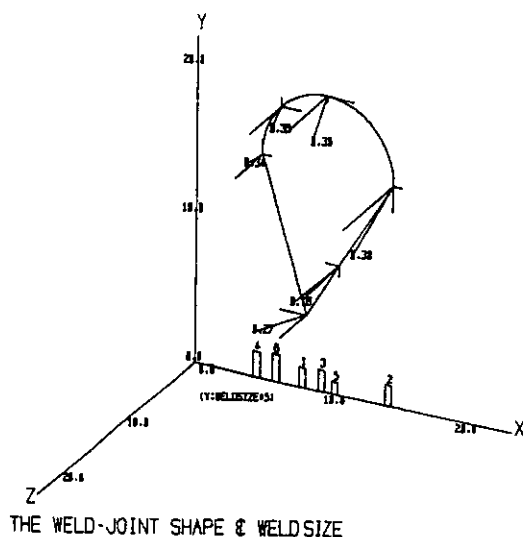

Fig. 18   Triangular region

Table 1 The computed weld sizes for the second example

Node #	X-coordinate	Y-coordinate	Weld size (inch.)
1	3.000	0.000	0.069
2	3.000	1.000	0.052
3	3.000	2.000	0.040
4	3.000	3.000	0.036
5	3.000	4.000	0.043
6	2.000	4.000	0.026
7	1.000	4.000	0.062
8	0.000	4.000	0.106

THE WELD-JOINT SHAPE

Fig. 19 The first example of weld joint shape

THE WELD-JOINT SHAPE & WELD SIZE

Fig. 20 Result plot for the first example

The basis for this check is discussed here.

For circular elements the quantity $r^2-(x^2+y^2)$ is checked where r is the radius of the curve and x and y are the coordinates of the input point relative to the center of curvature. For straight line segments, a triangular region is constructed from ends of the line segment to the center of gravity of the weld, Fig. 18. If the center of gravity is defined as node i, then the triangular shape function (Segerlind, 1976),

$$N_i = -\frac{1}{2A}(X_jY_k-X_kY_j)+(Y_j-Y_k)x+(X_k-X_j)y \quad (41)$$

is zero on the weld segment. A tolerance of 0.05 was allowed when checking for the circular arc or straight welds.

4. Examples

Here we employ this program to find weld sizes for two examples.

(a) The computer input and output for the first example problem shown in Fig. 19 are given in this section. The weld is subjected to shear forces, $P_x=10000$ lbs, $P_y=-15000$ lbs and $P_z=20000$ lbs, and bending moments, $M_x=100000$ in-lbs, $M_y=200000$ in-lbs and $M_z=-380000$ in-lbs acting at its centroid. The output of the example program is shown in Figs. 20 and 21.

(b) The second example is shown in Figs. 22. A $3''\times4''$ angle for support of a pipe extends out from the transverse intermediate stiffeners on a

```
SELECT 1_PX,2_PY,3_PZ,4_MX,6_MY,6_MZ WANT TO CHANGE
2
THE VALUE OF  2 WANT TO CHANGE
-15808
ENTER 1 TO CHANGE MORE VALUE
       0 IF NOT
1
SELECT 1_PX,2_PY,3_PZ,4_MX,5_MY,6_MZ WANT TO CHANGE
3
THE VALUE OF  3 WANT TO CHANGE
20808
ENTER 1 TO CHANGE MORE VALUE
       0 IF NOT
0
   ** THE INPUT VALUES **

      FORCE(PX)    FORCE(PY)    FORCE(PZ)
         10808.0     -15808.8     20808.8

      MOMENT(MZ)   MOMENT(MY)   MOMENT(MZ)
         100808.8    200808.8    -380808.8

ENTER 1 FOR SATISFIED
      0 FOR UNSATISFIED
1
      NODE#  X-COORD.   Y-COORD.   WELDSIZE
        1      0.500      5.008      0.270
        2     15.808     15.808      0.304
        3     18.808     20.808      0.302
        4      5.808     15.808      0.538
ENTER 1 IF SATISFIED WITH THE WELDSIZE
      0 IF NOT
       (CONFORM(( IF WELDSIZE GREATER THAN 1.)
1
ENTER 1 TO COMPUTE OTHER LOCATION
      0 IF NOT
1
ENTER COOR. AT WHICH YOU WANT TO KNOW WELDSIZE
11*2 0.806
YOUR COORD. WAS NOT ON THE LINE!!
ENTER COOR. AT WHICH YOU WANT TO KNOW WELDSIZE
11 8.806
      NODE#  X-COORD.   Y-COORD.   WELDSIZE
        5     11.088      8.806      0.188
ENTER 1 TO CHANGE THE LOCATION
      0 IF NOT
1
ENTER COOR. AT WHICH YOU WANT TO KNOW WELDSIZE
6.464 18.535
      NODE#  X-COORD.   Y-COORD.   WELDSIZE
        6      6.464     18.536      0.363
ENTER 1 TO CHANGE THE LOCATION
      0 IF NOT
0
   ** THE RESULTANT WELDSIZE **

      NODE#  X-COORD.   Y-COORD.   WELDSIZE
        1      0.508      5.008      0.270
        2     15.808     15.808      0.304
        3     18.808     20.808      0.302
        4      5.808     15.808      0.538
        5     11.808      8.806      0.188
        6      6.464     18.536      0.363
ENTER 1 FOR SATISFIED
      0 FOR UNSATISFIED
1
```

Fig. 21 Result on the screen for the first example

plate girder. This must be field welded. It will be difficult to weld in the overhead position along the bottom edge of the angle as well as to make the vertical weld along the end of the angle next to the girder web because of poor accessibility. The pipe's weight on the hanger is 300 lbs and a possible horizontal force of approximately 200 lbs is applied to the hanger during the erection of the pipe.

The output of the second example is shown in Table 1.

The output for the two examples showed that the computed weld sizes were exactly the same as the values in Reference (Blodgett, 1963).

5. Conclusions

There are many analysis formulas for determining the resultant shear force in the weld. However, there is no general procedure which is applicable to a joint with all six possible loadings exerted simultaneously. A numerical methodology and computer program for such problems were developed, and they are capable of analyzing a weld of any shape composed of straight or circular line segments.

The program uses interactive graphics to display the weld joint and weld sizes. The weld size for each node and those for other specified node points are displayed on the graphics screen. A check has been programmed which deletes the erroneously input coordinate pairs of nodes that do not lie on the weld. The weld properties are

Isometric view of weldment structure   Detail of 'A'
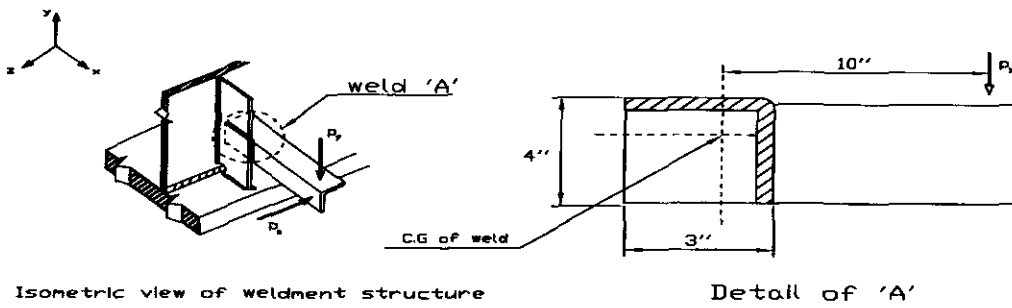
Fig. 22 The second example of weldment analysis

calculated from the coordinates of the welds, and the resultant shear flow is calculated. The computer calculates the weld size and draws the weld joint shape, and then gives the magnitude of weld size at the specified locations.

The results show that the weld sizes computed using the program were exactly the same as the values from manual calculations in the literature. Furthermore, the development of the design procedure and an interactive computer program for sizing fillet welds will lead to low cost welds.

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