Bull. Korean Math. Soc. **38** (2001), No. 2, pp. 317–324

NORMAL EDGE-TRANSITIVE CIRCULANT GRAPHS

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ABSTRACT. A Cayley graph of a finite group G is called *normal edge-transitive* if its automorphism group has a subgroup which both normalizes G and acts transitively on edges. In this paper, we consider Cayley graphs of finite cyclic groups, namely, finite circulant graphs. We characterize the normal edge-transitive circulant graphs and determine the normal edge-transitive circulant graphs of prime power order in terms of lexicographic products.

1. Introduction

Let G be a finite group and S a subset of G not containing the identity 1_G. The *Cayley graph* $X = \mathrm{Cay}(G, S)$ of G on S is a graph defined by

$$V(X) = G, \quad E(X) = \{\, (g, sg) \mid g \in G, s \in S \,\}.$$

In particular, if $S^{-1} := \{\, s^{-1} \mid s \in S \,\}$ is equal to S, then the Cayley graph is said to be *undirected*. In this case, (x, y) is an edge if and only if (y, x) is an edge, and such a graph can be viewed as a usual undirected graph by coalescing the two edges (x, y) and (y, x) into a single *undirected* edge.

It is easy to see that the group G acts regularly on the vertex set G by right multiplication, and so G may be viewed as a regular subgroup of the automorphism group of the Cayley graph. In particular, the automorphism group $\mathrm{Aut}X$ of the Cayley graph X acts transitively on the vertex set G. The normalizer $\mathbf{N}_{\mathrm{Aut}X}(G)$ of the regular subgroup G is the semidirect product

$$\mathbf{N}_{\mathrm{Aut}X}(G) = G \cdot \mathrm{Aut}(G, S), \text{ where } \mathrm{Aut}(G, S) := \{\, \sigma \in \mathrm{Aut}(G) \mid S^\sigma = S \,\}.$$

Received May 30, 2000. Revised November 29, 2000.

2000 Mathematics Subject Classification: 05C25, 20B25.

Key words and phrases: edge-transitive Cayley graphs, circulant graphs, lexicographic products.

*The present study was supported by the Korea Research Foundation, Project No. 1998-015-D00008.

A Cayley graph $X = \mathrm{Cay}(G, S)$ is said to be *edge-transitive* if its automorphism group $\mathrm{Aut}X$ is transitive on the edges. Also, if X is undirected and $\mathrm{Aut}X$ is transitive on the undirected edges, then X is said to be *edge-transitive as an undirected graph*. It is difficult to find the full automorphism group of a graph in general, and so this makes it difficult to decide whether it is edge-transitive, even for a Cayley graph. As an accessible kind of edge-transitive graphs, Praeger[8] focuses attention on those graphs for which $\mathbf{N}_{\mathrm{Aut}X}(G)$ is transitive on edges, and those undirected graphs X for which $\mathbf{N}_{\mathrm{Aut}X}(G)$ is transitive on the undirected edges. Such a graph is said to be *normal edge-transitive*, or *normal edge-transitive as an undirected graph*, respectively. In [8], Praeger gave an approach to analyzing normal edge-transitive Cayley graphs as a subfamily of central importance. Using the strategy suggested in [8] to construct normal edge-transitive Cayley graphs from quotients, Houlis [3] was able to determine the isomorphism types of all connected normal edge-transitive undirected Cayley graphs for $\mathbf{Z}_{pq}$, where p, q are primes; for $G = \mathbf{Z}_p \times \mathbf{Z}_p$, p a prime, Houlis also made a classification which gives all normal edge-transitive undirected Cayley graphs $\mathrm{Cay}(G, S)$ such that $\mathrm{Aut}(G, S)$ acts reducibly on G. In this paper, we consider finite circulant graphs, namely Cayley graphs of finite cyclic groups.

Two Cayley graphs $\mathrm{Cay}(G, S)$ and $\mathrm{Cay}(G, T)$ are said to be *equivalent* if there exists $\alpha \in \mathrm{Aut}(G)$ such that $T = S^{\alpha}$. Equivalent Cayley graphs are of course isomorphic.

Let $\mathbf{Z}_n := \{0, 1, ..., n-1\}$ denote the additive group of integers modulo n, and let U_n denote the multiplicative group of units integers modulo n. We may identify U_n with $\mathrm{Aut}(\mathbf{Z}_n)$. The following theorem characterizes all connected normal edge-transitive circulant graphs of order n.

THEOREM 1.1. *Every connected normal edge-transitive circulant graph* $\mathrm{Cay}(\mathbf{Z}_n, T)$ *is equivalent to* $\mathrm{Cay}(\mathbf{Z}_n, S)$ *for some subgroup S of U_n. Conversely, each subgroup S of U_n defines a connected normal edge-transitive circulant graph of order n, different choices of subgroups S giving nonisomorphic graphs.*

An abelian group G has an automorphism which maps each element to its inverse, and so if an undirected Cayley graph of G is normal edge-transitive as an undirected graph then it is also normal edge-transitive. Therefore, a connected circulant undirected graph of order n is normal edge-transitive as an undirected graph if and only if it is isomorphic to $\mathrm{Cay}(\mathbf{Z}_n, S)$ for some subgroup S of U_n containing -1.

We focus our attention on the special case when n is a prime power. Let p be an odd prime. For each positive divisor r of $p-1$, there is a unique subgroup of order r in the cyclic group U_{p^i}. The Cayley graph of $\mathbf{Z}_{p^i}$ on the subgroup of order r in U_{p^i} is denoted by $X(p^i, r)$.

For $p = 2$, let $X(2^i, 1) = \mathrm{Cay}(\mathbf{Z}_{2^i}, \{1\})$, $X(2^i, 2) = \mathrm{Cay}(\mathbf{Z}_{2^i}, \{1, -1\})$, and $X(2^i, 3) = \mathrm{Cay}(\mathbf{Z}_{2^i}, \{1, -1+2^{i-1}\})$. Denote the Cayley graph of $\mathbf{Z}_n$ on the empty set by $n\mathrm{K}_1$. Then we have:

THEOREM 1.2. (i) *For an odd prime p and a positive integer m, every connected normal edge transitive circulant graph of order p^m is isomorphic to the lexicographic product $X(p^i, r)[p^{m-i}K_1]$ for some positive divisor r of $p-1$ and an integer i with $1 \le i \le m$, different choices of i or r giving nonisomorphic graphs.* (ii) *Every connected normal edge transitive circulant graph of order 2^m is isomorphic to one of the lexicographic products $X(2^i, j)[2^{m-i}K_1]$ for some integers i, j with $1 \le j \le 3 \le i \le m$ or $1 \le j \le i = 2$ for $3 \le m$, and $1 \le j \le i = m$ for $m = 1, 2$, different choices of i or j giving nonisomorphic graphs.*

We note that the analogous result can be given for $n = 2p^m$ for an odd prime p since the automorphism groups of $\mathbf{Z}_{p^m}$ and $\mathbf{Z}_{2p^m}$ are isomorphic.

2. Basic facts

In this section we give some facts on Cayley graphs, which will be useful for our purpose.

First we make some comments about the normalizer $\mathbf{N}_{\mathrm{Aut}X}(G)$ of the regular subgroup G. The normalizer of the regular subgroup G in the symmetric group $Sym(G)$ is the holomorph of G, that is the semidirect product $G \cdot \mathrm{Aut}(G)$. Thus,

$$\mathbf{N}_{\mathrm{Aut}X}(G) = (G\cdot\mathrm{Aut}(G))\cap\mathrm{Aut}X = G\cdot(\mathrm{Aut}(G)\cap\mathrm{Aut}X) = G\cdot\mathrm{Aut}(G, S).$$

The following lemma to characterize normal edge-transitivity is fundamental, and a more general description can be also found in [8].

LEMMA 2.1. *Let $X = \mathrm{Cay}(G, S)$ be a Cayley graph for a finite group G with $S \neq \emptyset$. Then X is normal edge-transitive if and only if $\mathrm{Aut}(G, S)$ is transitive on S.*

We close this section with some observations about lexicographic products of graphs.

Given two graphs X and Y the *lexicographic product*, $X[Y]$ is defined as the graph with vertex set $V(X) \times V(Y)$ and the following adjacency relation:

$$\begin{aligned} & (x, y) \text{ is adjacent to } (x', y') \text{ in } X[Y] \\ \Longleftrightarrow\ & \text{either } x \text{ is adjacent to } x' \text{ in } X, \\ & \text{or } x = x',\ y \text{ is adjacent to } y' \text{ in } Y. \end{aligned}$$

We then have the following basic result.

LEMMA 2.2. *Let $X = \mathrm{Cay}(G, S)$ be a Cayley graph for a finite group G with $S \neq \emptyset$. If S is a union of cosets of a normal subgroup M of G of order m, then $X \cong \mathrm{Cay}(G/M, S/M)[m\mathrm{K}_1]$, where S/M denotes the set of cosets Ms, $s \in S$.*

Proof. Write Y for $\mathrm{Cay}(G/M, S/M)$ and Z for $\mathrm{Cay}(M, \emptyset)$. Then $Z \cong m\mathrm{K}_1$. Let T be a set of coset representatives for M in G, so that for every $g \in G$ there exists a unique $t \in T$ such that $Mg = Mt$. Define a map $\gamma : G \longrightarrow G/M \times M$ by $\gamma(g) = (Mg, gt^{-1})$. It is routine to show that this map is an isomorphism from X onto $Y[Z]$. □

3. Proof of Theorem 1.1

For the first part of the theorem, let $X = \mathrm{Cay}(\mathbf{Z}_n, T)$ be a connected normal edge-transitive circulant graph of order n. Then by Lemma 2.1, $\mathrm{Aut}(\mathbf{Z}_n, T)$ is transitive on T. Write S for the subgroup of U_n corresponding to $\mathrm{Aut}(\mathbf{Z}_n, T)$ under the identification of U_n and $\mathrm{Aut}(\mathbf{Z}_n)$. Since $\langle T \rangle = \mathbf{Z}_n$, there exists t in $T \cap \mathrm{U}_n$. Since the action of U_n on $\mathbf{Z}_n$ is by multiplication modulo n, it follows that $T = St$. Therefore the map $i \mapsto it$, $i \in \mathbf{Z}_n$ yields an auto morphism of $\mathbf{Z}_n$ that maps S onto T, and so $\mathrm{Cay}(\mathbf{Z}_n, S)$ is equivalent to $\mathrm{Cay}(\mathbf{Z}_n, T)$.

Let S be a subgroup of U_n. Since $1 \in S$, we have $\mathbf{Z}_n = \langle S \rangle$ and so $\mathrm{Cay}(\mathbf{Z}_n, S)$ is connected. The subgroup $\{ a \in \mathrm{U}_n \mid Sa = S \}$ acts on S by multiplication as $\mathrm{Aut}(\mathbf{Z}_n, S)$ does on S. Since $\{a \in \mathrm{U}_n \mid Sa = S\} = S$, we see that $\mathrm{Aut}(\mathbf{Z}_n, S)$ is transitive on S. It follows from Lemma 2.1 that $\mathrm{Cay}(\mathbf{Z}_n, S)$ is normal edge-transitive.

We now consider the isomorphism problem of connected normal edge-transitive circulant graphs. Ádám [1] conjectured that if two circulant graphs $\mathrm{Cay}(G, S)$ and $\mathrm{Cay}(G, T)$ are isomorphic then they are equivalent; the conjecture was shown to be false in general (see for example, [2]). While the conjecture is true if the number of vertices is either square-free or twice square-free (see [6, 7]). We will show that the conjecture for our case is true, namely:

LEMMA 3.1. *If two connected normal edge-transitive circulant graphs* $\mathrm{Cay}(\mathbf{Z}_n, S)$ *and* $\mathrm{Cay}(\mathbf{Z}_n, T)$ *are isomorphic, then they are equivalent.*

We note that Klin and Pöschel in [4] first applied the method of Schur rings to solve isomorphism problems of circulant graphs and they succeeded in solving the isomorphism problem for circulant graphs of odd prime-power order in [5]. Our isomorphism problem can be solved by using some basic properties of Schur ring theory.

Let $X = \mathrm{Cay}(\mathbf{Z}_n, S)$ be a Cayley graph of $\mathbf{Z}_n$ on a subset S. Let A be the automorphism group of the graph X and let A_0 be the subgroup of all automorphisms of X that fix 0. Let $S_1, S_2, \ldots, S_k$ be all orbits of the natural action of A_0 on $\mathbf{Z}_n$. Let $\mathbf{Z}[\mathbf{Z}_n]$ be the group ring of $\mathbf{Z}_n$ over the integer ring $\mathbf{Z}$, which consists of the formal sums $c_0\underline{0}+c_1\underline{1}+\cdots+c_{n-1}\underline{n-1}$, where c_i are integers. For each subset $T = \{t_1, t_2, \ldots, t_s\}$ of $\mathbf{Z}_n$, we denote $\underline{t_1} + \underline{t_2} + \cdots + \underline{t_s}$ by $\underline{T}$ and call it a *simple quantity*. Then the transitivity module $\mathbf{Z}(\mathbf{Z}_n, A_0)$ belonging to A_0 is the module generated by $\underline{S_1}, \underline{S_2}, \ldots, \underline{S_k}$, which are called the basic quantities. It is known as Schur's fundamental theorem (see [9]) that the transitivity module is a subring of the group ring.

Let $X = \mathrm{Cay}(\mathbf{Z}_n, S)$ and $Y = \mathrm{Cay}(\mathbf{Z}_n, T)$ be isomorphic circulants with automorphism groups A and B, respectively. Let $\lambda : \mathbf{Z}_n \longrightarrow \mathbf{Z}_n$ be an isomorphism of X onto Y such that $\lambda(0) = 0$. Since A is vertex-transitive we can choose such λ. Obviously, we have $B = \lambda A \lambda^{-1}$ and $B_0 = \lambda A_0 \lambda^{-1}$ is the group of all automorphisms of Y that fix 0. The isomorphism λ extends to a linear operator of $\mathbf{Z}(\mathbf{Z}_n, A_0)$ onto $\mathbf{Z}(\mathbf{Z}_n, B_0)$. Of course λ sends each basic quantity of $\mathbf{Z}(\mathbf{Z}_n, A_0)$ to a basic quantity of $\mathbf{Z}(\mathbf{Z}_n, B_0)$. Let x be an element of $\mathbf{Z}_n$ and let $R(x)$ be the regular representation of x defined by $R(x)(i) = i+x$ for all i in $\mathbf{Z}_n$. Then $\lambda(S_i + x) = R(\lambda(x))\lambda\lambda^{-1}R(-\lambda(x))\lambda R(x)(S_i) = R(\lambda(x))\lambda\,\alpha_0(S_i) = \lambda(S_i) +$

$\lambda(x)$ where $\alpha_0 = \lambda^{-1}R(-\lambda(x))\lambda R(x)$ is an automorphism of X such that $\alpha_0(0) = 0$. Since $\lambda(\underline{S_i} \cdot \underline{x}) = \underline{\lambda(S_i + x)}$, we have $\lambda(\underline{S_i} \cdot \underline{x}) = \lambda(\underline{S_i}) \cdot \lambda(\underline{x})$. This implies that the linear operator λ preserves multiplication of the subring, and so λ is a ring-isomorphism between $\mathbf{Z}(\mathbf{Z}_n, A_0)$ and $\mathbf{Z}(\mathbf{Z}_n, B_0)$.

Let a be an automorphism of $\mathbf{Z}_n$. By Theorem 23.9(a) in [9], $\underline{S_i}^a$ is a basic quantity of $\mathbf{Z}(\mathbf{Z}_n, A_0)$. By the same idea of the proof of Theorem 23.9(a) in [9], we also have $\lambda(\underline{S_i}^a) = \lambda(\underline{S_i})^a$. Since S is again a union of some basic quantities S_i, we have $\lambda(\overline{S^a}) = \lambda(S)^a$. Since $\lambda(S) = T$, we have the following immediate consequence of this observation.

LEMMA 3.2. *If* $\mathrm{Cay}(\mathbf{Z}_n, S)$ *and* $\mathrm{Cay}(\mathbf{Z}_n, T)$ *are isomorphic, then* $\mathrm{Aut}(G, S) = \mathrm{Aut}(G, T)$.

Now let $X = \mathrm{Cay}(\mathbf{Z}_n, S)$ and $Y = \mathrm{Cay}(\mathbf{Z}_n, T)$ be isomorphic connected normal edge-transitive circulant graphs. As we have shown in the first part of this section, X and Y are equivalent to $\mathrm{Cay}(\mathbf{Z}_n, S')$ and $\mathrm{Cay}(\mathbf{Z}_n, T')$ respectively for some subgroups S' and T' of U_n. Then by Lemma 3.2, $\mathrm{Aut}(G, S') = \mathrm{Aut}(G, T')$, that is $S' = T'$. This completes the proof of Lemma 3.1, and so Theorem 1.1 is now proved.

4. Proof of Theorem 1.2

Let p be an odd prime. Then U_{p^m} is a cyclic group of order $(p-1)p^{m-1}$. Let S be a subgroup of U_{p^m} and let B be the Sylow p-subgroup of S. Then $B = \{1 + kp^i \mid k = 0, 1, 2, ..., p^{m-i}-1\}$ for some integer $i = 1, 2, ..., m$ and $|B| = p^{m-i}$. Let $M := \{kp^i \mid k = 0, 1, 2, ..., p^{m-i}-1\}$. Then M is the subgroup of $\mathbf{Z}_{p^m}$ of order p^{m-i}, and $B = 1 + M$. We see that

$$S = \cup_{a \in S}\, aB = \cup_{a \in S}\, a(1 + M) = \cup_{a \in S}\, a + M.$$

So S is a union of cosets of M in $\mathbf{Z}_{p^m}$. Let r denote $|S/B|$, which is a divisor of $p-1$. If $a + M = a' + M$ for some a, a' in S, then $a^{-1}a' \in 1 + M = B$, and so $aB = a'B$. Therefore $|S/M| = r$. By Lemma 2.2, we have

$$\mathrm{Cay}(\mathbf{Z}_{p^m}, S) = \mathrm{Cay}(\mathbf{Z}_{p^m}/M, S/M)[p^{m-i}\mathrm{K}_1].$$

We want to show that

$$\mathrm{Cay}(\mathbf{Z}_{p^m}/M, S/M)[p^{m-i}\mathrm{K}_1] \cong X(p^i, r).$$

Let $\theta : \mathbf{Z}_{p^m} \longrightarrow \mathbf{Z}_{p^i}$ be the natural homomorphism, namely $\theta(x) \equiv x \bmod p^i$. Then $\mathrm{Ker}\theta = M$ and $\theta(S)$ is a subgroup of U_{p^i} since θ preserves the multiplication as well. The homomorphism θ induces the isomorphism $\bar{\theta}$ from $\mathbf{Z}_{p^m}/M$ onto $\mathbf{Z}_{p^i}$. Note that $\bar{\theta}(S/M) = \theta(S)$. Therefore $\bar{\theta}$ is also an isomorphism between $\mathrm{Cay}(\mathbf{Z}_{p^m}/M, S/M)$ and $\mathrm{Cay}(\mathbf{Z}_{p^i}, \theta(S))$. Since $\theta(S)$ is the unique subgroup of order r in U_{p^i}, it follows that $\mathrm{Cay}(\mathbf{Z}_{p^i}, \theta(S)) = X(p^i, r)$. The proof of (i) of the theorem is now complete.

First consider the case when $n = 2^m$. We assume that $m \geq 3$. It is well known that $\mathrm{U}_{2^m} = \langle -1 \rangle \cdot \langle 5 \rangle \cong \mathbf{Z}_2 \times \mathbf{Z}_{2^{m-2}}$. For each $i = 2, 3, ..., m$, let

$$S_i = \{\, 1 + k2^i \mid k = 0, 1, ..., 2^{m-i}-1 \,\}.$$

Then $S_2, S_3, ..., S_m$ consist of all subgroups of $\langle 5 \rangle$. For each $i = 2, ..., m-1$, let

$$T_i = S_{i+1} \cup \{\, -1 + k2^i \mid 1 \leq k \leq 2^{m-i}-1,\ k:\ \text{odd} \,\}.$$

Then T_i is a subgroup of U_{2^m} such that $|T_i| = |S_i| = 2^{m-i}$. Let T be a subgroup of U_n, and let π be the natural projection from U_n onto $\langle 5 \rangle$. Then $\pi(T) = S_i$, for some $i = 2, 3, ..., m$. Then there exits a homomorphism θ from S_i to $\langle -1 \rangle / (\langle -1 \rangle \cap T)$ such that $T = \{\, st \mid \theta(s) = t + \langle -1 \rangle \cap T,\ s \in S_i \,\}$. If T contains -1, then the only such homomorphism is trivial; therefore T is the direct product of S_i and $\langle -1 \rangle$. Suppose that T does not contain -1. If $\theta(S_i) = \langle 1 \rangle$, then θ is the trivial homomorphism, and hence $T = S_i$. If $\theta(S_i) = \langle -1 \rangle$, it follows from $\mathrm{Ker}\theta = S_{i+1}$ and $\theta(S_i) = \langle -1 \rangle \cong \mathbf{Z}_2$ that $T = T_i$ for some $i = 2, ..., m-1$. Consequently, we have the following lemma.

LEMMA 4.1. *$\{\, S_i, T_j, \langle -1 \rangle \cdot S_i \mid i = 2, 3, ..., m,\ j = 2, 3, ..., m-1 \,\}$ is the set of all subgroups of U_{2^m}.*

We then consider (ii) of the theorem. We first assume that $m \geq 3$. Let S be a subgroup of U_{2^m}. Then S is one of those listed in Lemma 4.1. Write B for $S \cap \langle 5 \rangle$. Then $B = \{\, 1 + k2^i \mid k = 0, 1, ..., 2^{m-i}-1 \,\}$ for some $i = 2, 3, ..., m$. Let $M := \{\, k2^i \mid k = 0, 1, 2, ..., 2^{m-i}-1 \,\}$. Then M is a subgroup of order 2^{m-i} of $\mathbf{Z}_{2^m}$. Since $B = 1 + M$, S is a union of some cosets of M in $\mathbf{Z}_{2^m}$. In fact either i) $S = 1 + M$, or ii) $S = (1+M) \cup (-1+M)$, or iii) $S = (1+M) \cup (-1+2^{i-1}+M)$, $3 \leq i \leq m$ from Lemma 4.1. Let $\theta : \mathbf{Z}_{2^m} \longrightarrow \mathbf{Z}_{2^i}$ be the natural homomorphism such that $\theta(x) \equiv x \bmod 2^i$. Then $\mathrm{Ker}\theta = M$, and $\theta(S) = \{1\}$ for i), $\theta(S) = \{1, -1\}$ for ii) and $\theta(S) = \{1, -1 + 2^{i-1}\}$, $3 \leq i \leq m$ for iii). Since $\mathrm{Cay}(\mathbf{Z}_{2^m}/M, S/M) \cong \mathrm{Cay}(\mathbf{Z}_{2^i}, \theta(S))$, it follows from Lemma 2.2 that $\mathrm{Cay}(\mathbf{Z}_{2^m}, S) \cong \mathrm{Cay}(\mathbf{Z}_{2^i}, \theta(S))[2^{m-i}\mathrm{K}_1]$, where i varies from 2 to m for i) and ii), while i varies from 3 to m for iii). We observe that $\mathrm{Cay}(\mathbf{Z}_{2^i}, \theta(S)) = X(2^i, 1)$, $2 \leq i \leq m$ for i), $\mathrm{Cay}(\mathbf{Z}_{2^i}, \theta(S)) = X(2^i, 2)$, $2 \leq i \leq m$ for ii), and $\mathrm{Cay}(\mathbf{Z}_{2^i}, \theta(S)) = X(2^i, 3)$, $3 \leq i \leq m$ for iii). Consequently $\mathrm{Cay}(\mathbf{Z}_{2^m}, S) \cong X(2^i, j)[2^{m-i}\mathrm{K}_1]$ where $3 \leq i \leq m$, $1 \leq j \leq 3$, or $i = 2$, $j = 1, 2$. For $m = 2$ or 1, $S = \{1\}$, or $S = \{1, -1\}$, and so $\mathrm{Cay}(\mathbf{Z}_{2^m}, S) = X(2^m, j)$ where $1 \leq j \leq m$. This proves (ii) of the theorem.

ACKNOWLEDGEMENT. The authors thank the refree for several improvements of the exposition.

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