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A NOTE ON INVARIANT PSEUDOHOLOMORPHIC CURVES

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Abstract. Let (X,ω) be a closed symplectic 4-manifold. Let a finite cyclic group G act semifreely, holomorphically on X as isometries with fixed point set Σ (may be empty) which is a 2-dimension submanifold. Then there is a smooth structure on the quotient $X' = X/G$ such that *the projection* $\pi : X \to X'$ *is a Lipschitz map. Let* $L \to X$ *be the* $Spin^c$-structure on X pulled back from a $Spin^c$-structure $L' \to X'$ and $b_2^+(X') > 1$. If the Seiberg-Witten invariant $SW(L') \neq 0$ of L' is non-zero and $L = E \otimes K^{-1} \otimes E$, then there is a G-invariant pseudo-holomorphic curve $u : C \to X$ such that the image $u(C)$ represents the fundamental class of the Poincaré dual $c_1(E)$. This is an equivariant version of the Taubes' Theorem.

1. Cyclic group actions on 4-manifolds

Let X be an oriented, closed, smooth 4-manifold. Let G be a finite cyclic group. For simplicity, we assume that G acts smoothly, semifreely on X as orientation preserving isometries such that the fixed point set $\Sigma = X^G$ of G is a 2-dimensional submanifold of X.

As in [18], we may choose a smooth structure on the quotient space $X' = X/G$ such that the projection $\pi : X \to X'$, on a tubular neighbourhood $N(\Sigma)$ of Σ given by $\pi(a,r,\theta) = (\pi(a), r, n\theta)$, is Lipschitz and smooth away from Σ, where $n = |G|$ is the order of G, $a \in \Sigma$, and (r,θ) is the polar coordinate in the normal direction of Σ. The push down metrics or forms to X' of any smooth G-invariant ones on X are smooth away from Σ and have bounded coefficients near Σ with respect to the local coordinates of X'. Moreover, the projection π induces a one-to-one correspondence between G-invariant L^p metrics or forms on X and L^p metrics or forms on X'. To

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study the relations between G-invariant properties on X and ones on X', we will fix this smooth structure on X' in this note.

Let $T(N)$ be the Thom space of the normal bundle $N \to \Sigma$, and let $\phi : H^0(\Sigma) \to H^2(T(N))$ be the Thom isomorphism. Let $c : X \to T(N)$ be the collapsing map in the Thom space.

THEOREM 1.1 [1]. *The second Stiefel-Whitney classes satisfy the following formula:*

$$w_2(X) - \pi^* w_2(X') = c^* \phi(1) \mod 2$$

where 1 *is the standard generator in* $H^0(\Sigma; \mathbb{Z})$.

REMARK. Consider the Thom isomorphism and the collapsing map

$$\begin{array}{ccccc} H^0(\Sigma) & \xrightarrow[1 \mapsto \phi(1) = u]{\phi} & H^2(T(N)) & \xrightarrow{c^*} & H^2(X) \\ & & {\scriptstyle u \mapsto e}\downarrow & & \uparrow{\scriptstyle \pi^*} \\ & & H^2(\Sigma) & \longleftarrow & H^2(X') \end{array}$$

where e is the Euler class of $N \to \Sigma$.

By considering the restriction $H^2(X) \to H^2(\Sigma)$ we have $c^*\phi(1) = (n-1)$ $PD(\Sigma)$.

As in [4] and [18] we consider the following commmutative diagram :

$$\begin{array}{ccc} H^2(X'; \mathbb{Z}) & \xrightarrow{\pi^*} & H^2(X; \mathbb{Z})^G \\ {\scriptstyle |G| \cdot PD}\downarrow & & \downarrow{\scriptstyle PD} \\ H_2(X'; \mathbb{Z}) & \xleftarrow{\pi_*} & H_2(X; \mathbb{Z})^G. \end{array}$$

Then we have the following consequences

PROPOSITION 1.2 [4], [18].
(1) *The maps* π^* *and* π_* *are one-to-one.*
(2) *The Euler characteristics :* $\chi(X) = |G|\chi(X') - (|G| - 1)\chi(\Sigma)$.
(3) *The signatures :* $\tau(X) = |G|\tau(X') - (|G| - 1)\Sigma \cdot \Sigma$.
(4) *The second Betti numbers :*

$$b_2(X') = b_2^G(X),$$
$$b_2^+(X') = B_2^{G^+}(X) = \dim H_+^2(X; \mathbb{R})^G.$$

Proof.
(1) By the above diagram and the Universal coefficients theorem.
(2) and (3) By the Atiyah-Singer G-index Theorem.
(4) By the above diagram and (1). □

To see the $Spin^c$-structures on X and X', we consider the $\mathbb{Z}_2$ coefficient reduction on cohomologies:

$$\begin{array}{ccc} H^2(X';\mathbb{Z}) & \xrightarrow{\pi^*} & H^2(X;\mathbb{Z})^G \\ {\scriptstyle P'}\downarrow & & {\scriptstyle P}\downarrow \\ H^2(X';\mathbb{Z}_2) & \xrightarrow{\pi^*} & H^2(X;\mathbb{Z}_2)^G. \end{array}$$

Since G acts on X as orientation preserving isometries, for all $h \in G$, $h^*w_2(TX) = w_2(h^*TX) = w_2(TX)$, and so $w_2(TX) \in H^2(X;\mathbb{Z}_2)^G$. The set of $Spin^c$-structures on X' is $(P')^{-1}(w_2(TX'))$, and the set of the G-invariant ones on X is $P^{-1}(w_2(TX)) \subset H^2(X;\mathbb{Z})^G$.

Let $L' \to X'$ be a $Spin^c$-structure on X', then $c_1(L') = w_2(TX') \mod 2$. By Thoerem1.1, $\pi^*w_2(TX') = w_2(TX) + (|G|-1)\, PD([\Sigma])$. And $P\pi^*c_1(L') = \pi^*P'c_1(L') = \pi^*w_2(TX')$.

There is an element α' in $H^2(X';\mathbb{Z})$ such that $\pi^*(\alpha') \in P^{-1}(w_2(TX)) \subset H^2(X;\mathbb{Z})^G$ is a G-invariant $Spin^c$-structure on X, and then

$$\begin{aligned} \pi^*P'(\alpha') = P\pi^*(\alpha') &= w_2(TX) \\ &= \pi^*w_2(TX') + (|G|-1)PD([\Sigma]). \end{aligned}$$

PROPOSITION 1.3. *If* $(|G|-1)PD([\Sigma]) \subset 2H^2(X;\mathbb{Z})$, *then* π^* *pull-backs the* $Spin^c$*-structures on* X' *to* G*-invariant ones on* X.

2. Invariant Seiberg-Witten invariant

Let a cyclic group G act smoothly, semifreely on X as orientation preserving isometries with the fixed point set Σ, which is a 2-dimensional submanifold of X. Let $L' \to X'$ be a $Spin^c$-structure on X' and $(|G|-1)[\Sigma] \in 2H_2(X;\mathbb{Z})$. Then the pull-back $L = \pi^*L' \to X$ is a $Spin^c$-structure on X. Let $\mathcal{A}$ be the set of $U(1)$-connections on L. Let $W^\pm$ be the associated spinor bundles of L. Let A_0 be a G-invariant connection of L, then $\mathcal{A} = A_0 + \Omega^1(i\mathbb{R})$ is affine space with the origin A_0. Let $\omega^1_s(i\mathbb{R}) = \{ia \in \omega^1(i\mathbb{R}) | d^*a = 0\}$ be the infinitesimal slice of the gauge group $\mathcal{T} = \{g : X \to U(1)\}$ action on $\mathcal{A}$. For a pair $(A, \phi) \in \mathcal{A} \times \Gamma(W^+)$, the Seiberg-Witten equations are a pair of equations:

$$\begin{cases} D_A\phi = 0 \\ F_A^+ = q(\phi). \end{cases}$$

We review the Seiberg-Witten theory to use our applications, for details see [14], [19]. The space of the gauge equivalence classes of the solutions of the Seiberg-Witten equations is called the moduli space $\mathcal{M}(L)$.

For a fixed G-invariant connection A_0 of L, the Seiberg-Witten equations define a map

$$F : \Omega_s^1(i\mathbb{R}) \times \Gamma(W^+) \to \Omega_+^2(i\mathbb{R}) \times \Gamma(W^-),$$

$$F(ia, \phi) = (F_{A_0}^+ + d^+(ia) - q(\phi), D_{A_0}\phi + ia \cdot \phi).$$

The moduli space $\mathcal{M}(L)$ can be identified with $F^{-1}(0)/S^1$. We can write $F = F_0 + R$, where $F_0 = (d^+, D_{A_0})$, which has index :

$$\text{index}F = \text{index}F_0 = \frac{1}{4}(c_1(L)^2 - (2\chi(X) + 3\sigma(X))).$$

If $b_2^+ > 1$, then for generic $v \in \Omega_+^2(i\mathbb{R})$ the moduli space $F^{-1}(v)/S^1 = \mathcal{M}(L)$ is a smooth compact manifold with dimension $2d = \text{index}F$. Let e be the Euler class of the free S^1-action on $F^{-1}(v)$, then the Seiberg-Witten invariant:

$$SW(L) = \int_{\mathcal{M}(L)} e^d$$

is independent on the choice of metric on X.

If $b_2^+(X)^G = b_2^+(X/G) > 1$, then we can choose a G-invariant 2-form $v \in (\Omega_+^2(i\mathbb{R}))^G \setminus \text{im}{d^+}^G$ such that the perturbed moduli space $F^{-1}(v)/S^1$ is a G-space with no reducible singularities, but may not be smooth.

However, if we consider the following G-invariant map

$$F^G : [\Omega_s^1(i\mathbb{R}) \times \Gamma(W^+)]^G \to [\Omega_+^2(i\mathbb{R}) \times \Gamma(W^-)]^G,$$

then for a generic G-invariant v, the G-invariant moduli space $(F^G)^{-1}(v) / S^1 \equiv \mathcal{M}^G(L)$ is a smooth compact manifold. We can define the G-invariant Seiberg-Witten invariant: $SW(L) = \int_{\mathcal{M}^G(L)} (e^G)^{d'}$ where $2d' = \dim \mathcal{M}^G(L)$ and e^G is the Euler class of the free S^1-action on $(F^G)^{-1}(v)$ and we use the orientation $\mathcal{M}^G(L)$ given by (2) of the following theorem.

THEOREM 2.1. *If $b_2^+(X)^G > 1$, then the G-invariant moduli space $\mathcal{M}^G(L)$ has the following properties.*

(1) *$\mathcal{M}^G(L)$ has a natural orientation by choosing an orientation of $det(H^0(X;\mathbb{R})^G) \otimes det(H^1(X;\mathbb{R})^G) \otimes det(H^2_+(X;\mathbb{R})^G)$.*

(2) *For generic G-invariant $v \in \Omega^2_+\ (i\mathbb{R})^G - \mathrm{im}\ d^{+,G}$, the moduli space $\mathcal{M}^G(L,v)$ is a compact smooth manifold.*

(3) *$\mathcal{M}^G(L)$ has a virtual dimension*

$$\begin{aligned}&\frac{1}{4|G|}[c_1(L)^2 - \{2\chi(X) + 3\sigma(X)\} - (n-1)\{2\chi(\Sigma) + 3\Sigma\cdot\Sigma\}\\ &= \frac{1}{4}[c_1(L')^2 - \{2\chi(X') + 3\sigma(X')\}]\\ &= \dim\mathcal{M}(L'),\end{aligned}$$

where $\mathcal{M}(L')$ is the Seiberg-Witten moduli space of the quotient $L' \to X'$.

(4) *The G-invariant Seiberg-Witten invariant $SW^G(L)$ is independent on the choice of the generic G-invariant metrics on X.*

Proof. (2) Since $b_2^+(X)^G = \dim\ [H_+^{2,G}\ (X;\mathbb{R})] > 1$, the image $d^+\ (\Omega^{1,G}(i\mathbb{R}))^G \subset \Omega^2_+\ (i\mathbb{R})^G$ has a codimension greater than one. The generic G-invariant $i\mathbb{R}$-valued self-dual 2-forms are path connected.

(3) To compute the virtual dimension of the G-invariant moduli space, we may use the Atiyah-Singer G-index Theorem. □

Let $L' \to X'$ be the quotient of the $Spin^c$-structure $L \to X$ on X. Then L' is also a $Spin^c$-structure on X' under our assumption.

Since $\pi^* : H^n(X';\mathbb{R}) \to H^n(X;\mathbb{R})^G$ is an isomerphism for all n. We may choose an orientation of the moduli space $\mathcal{M}(L')$ as the one of the $\mathcal{M}(L)^G$. By using the finite approximation method of Furuta[9], we have the following theorem.

THEOREM 2.2. *Under the above situation, we have that the G-invariant Seiberg-Witten invariant $SW^G(L)$ of L on X is the same as the Sieberg-Witten invariant $SW(L')$ of the quotient L' on X'.*

Sketch of Proof. Essentially the map $F : \Omega^1_s(i\mathbb{R}) \times \Gamma(W^+) \to \Omega^2_+(i\mathbb{R}) \times \Gamma(W^-)$ has the restriction $F^G : [\Omega^1_s(i\mathbb{R}) \times \Gamma(W^+)]^G \to [\Omega^2_+(i\mathbb{R}) \times \Gamma(W^-)]^G$ on the G-invariant part which is the same as $F' : [\Omega^1_s(i\mathbb{R}) \times \Gamma(W^+)]' \to [\Omega^2_+(i\mathbb{R}) \times \Gamma(W^-)]'$ which is induced from the Seiberg-Witten equations on the quotient $L' \to X'$. As we know, the projection map $\pi : X \to X'$ induces a one-to-one correspondence between G-invariant L^p-settings on X and L^p-ones on X'.

Both moduli spaces $\mathcal{M}^G(L)$ and $\mathcal{M}(L')$ have the same dimensions and the same orientations, and are compact smooth manifolds. The operators $D_A^* D_A$, $D_A D_A^*$ and $d_+^* d_+$, $d_+ d_+^*$ induced by F^G and F' have the same eigenvalues. The Seiberg-Witten invariants $SW^G(L)$ and $SW(L')$ are the degrees of the maps which are induced by finite dimensional approximation, (by compactness of moduli space) and chipping out by the moduli space. Thus the G-invariant Seiberg-Witten invariant $SW^G(L)$ of L on X is the same as the Seiberg-Witten invariant $SW^G(L')$ of L' on X'. □

By Theorem 1.1, if $(|G|-1)[\Sigma] \in 2H_2(X;\mathbb{Z})$, then the pull-backs of the $Spin^c$-structures on X' are G-invariant $Spin^c$-structures on X. In particular, if either G has odd order or the homology class $[\Sigma]$ is divisible by 2, then this is the case.

Corollary 2.3. *Let $(|G|-1)[\Sigma] \in 2H_2(X;\mathbb{Z})$. If the G-invariant $SW^G \equiv 0$ are identically zero, then the Seiberg-Witten invariants $SW \equiv 0$ are identically zero on the quotient manifold $X/G = X'$.*

Remark. If $L' \to X'$ is a $Spin^c$-structure on X' such that $SW(L') \neq 0$, then the pull-back $L \to X$ is a $Spin^c$- structure on X with G-action.

By Theorem 2.2 $SW^G(L) \neq 0$, there is an irreducible G-invariant solution of Seiberg-Witten equations which may not be generic. However there is no guarantee that the Seiberg-Witten invariant of L is non-trivial.

3. G-invariant pseudo-holomorphic curves

Let (X, ω) be a closed symplectic 4-manifold with a symplectic form ω on X. Let J be a tamed almost complex structure on X. As before we assume that a cyclic group G acts semifreely, holomorphically on X as isometries with fixed point set Σ which is a 2-dimensional submanifold. We assume that $L \to X$ is the $Spin^c$-structure induced by the pull-back of a $Spin^c$-stucture $L' \to X' = X/G$ via the projection $\pi : X \to X'$.

Since X be a symplectic 4-manifold, there is an element $e \in H^2(X;\mathbb{Z})$ such that $e = c_1(E)$ and $L = E^2 \otimes K^{-1}$, $W^+ = E \oplus K^{-1} \otimes E$, where $K = \Lambda^{2,0}(T^*X \otimes \mathbb{C})$ is the canonical complex line bundle on X. Since the actions of G on X are holomorphic, each element of G acts on K as a holomorphic bundle isomorphism. Thus $h^*c_1(K) = c_1(h^*K) = c_1(K)$ for any $h \in G$, and $c_1(L) = 2c_1(E) - c_1(K)$. Hence $e = c_1(E) \in H^2(X;\mathbb{Z})^G$ is G-invariant.

Suppose that the Seiberg-Witten invariant $SW(L') \neq 0$ of L' is non-zero and $b_2^+(X') \geq 2$. There is an irreducible solution $(A', \phi') \in \mathcal{A} \times \Gamma(W'^+)$ of the Seiberg-Witten equations:

$$\begin{cases} D_{A'} = 0 \\ F_{A'}^+ = q(\phi') + \nu', \end{cases}$$

where $\nu' \in \Omega_+^2(X'; i\mathbb{R}) \setminus \mathrm{im} d'^+$ and $d'^+ : \Omega_s^1(X'; i\mathbb{R}) \to \Omega_+^2(X'; i\mathbb{R})$. Since $H_+^2(X';\mathbb{R}) \xrightarrow[\pi^*]{\sim} H_+^2(X;\mathbb{R})^G$ and $\pi^* : \Omega_+^2(X'; i\mathbb{R}) = H_+^2(X; i\mathbb{R}) \oplus \mathrm{im} d'_+ \to \Omega_+^2(X; i\mathbb{R})^G = H_+^2(X; i\mathbb{R})^G \oplus \mathrm{im} d_+^G$ are isomorphisms, there is a G-invariant self-dual 2-form $\nu \in \Omega_+^2(X; i\mathbb{R})^G \setminus \mathrm{im} d_+^G$ such that $\pi^*(\nu') = \nu$, where $d_+^G : \Omega_s^1(X; i\mathbb{R})^G \to \Omega_+^2(X; i\mathbb{R})$ is the restriction of d_+. Let $\pi^* A' = A \in \mathcal{A}^G$, $\pi^*\phi' = \phi \in \Gamma(W^+)^G$. Then the Seiberg-Witten equations

$$\begin{cases} D_A = 0 \\ F_A^+ = q(\phi) + \nu \end{cases}$$

hold. Since $b_2^+(X') = b_2^+(X)^G = \dim(H_+^2(X;\mathbb{R})^G) > 1$, for a generic G-invariant self-dual 2-form $\nu \in \Omega_+^2(X; i\mathbb{R})^G$ the Seiberg-Witten equations have G-invariant solutions $(A, \phi) \in \mathcal{A}^G \times \Gamma(W^+)^G$.

LEMMA 3.1. *If (A', ϕ') is a solution of the Seiberg-Witten equations for a Spin^c-structure $L' \to X'$ on X', then the pullback (A, ϕ) via the projection $\pi : X \to X'$ is a solution of thoses for the Spin^c-structure $L \to X$ on X pulled back by the π.*

To examine the G-invariant solution $(A, \phi) \in \mathcal{A}^G \times \Gamma(W^+)^G$, let us split the solution: $A = A_0 + 2a \in \mathcal{A}^G(K^{-1} \otimes E^2)$ and $\phi = \sqrt{r}(\alpha, \beta) \in \Gamma(W^+ \equiv E \oplus K^{-1} \otimes E)^G$. As in [17], we consider the perturbed Seiberg-Witten equations:

$$(*) \qquad \begin{cases} D_A \sqrt{r}(\alpha, \beta) = 0 \\ F_a^+ + \dfrac{ir}{8}(1 - |\alpha|^2 + |\beta|^2)\omega - \dfrac{r}{4}(\alpha\beta^* - \alpha^*\beta) = 0, \end{cases}$$

where $(\alpha\beta^* - \alpha^*\beta)$ belongs to the orthogonal complement of $\mathbb{C}\omega$ in $\Lambda^+ \otimes \mathbb{C}$, and r is a fixed real number.

THEOREM 3.2 (Taubes). *Let X be a compact, minimal symplectic 4-manifold. If $W^+ = E \oplus K^{-1} \otimes E$ is the positive spin^c-bundle with non-trivial complex line bundle E, then $SW(\det W^+) = Gr(PDc_1(E))$ where $Gr(PDc_1(E))$ is a weighted count of compact symplectic submanifolds whose fundamental class is the Poincaré dual $PD(c_1(E))$.*

Let the $Spin^c$-structure $L = E^2 \oplus K^{-1} \to X$ is the pull back of a $Spin^c$-structure $L' \to X'$. The first Chern class $c_1(E)$ of E is G-invariant. The action of G on X lifts to the bundle E. Choose a section s of the bundle $E \to X$ such that s transverses the zero section. By averaging we have a new section $s_1 = \frac{1}{|G|}\sum_{h\in G} h^*s$ of E, which is G-invariant. The zero set ${s_1}^{-1}(0) = Z \subset X$ of s_1 is a G-invariant subset of X. Indeed, if $x \in {s_1}^{-1}(0)$, then for any element $h \in G$, $s_1(h(x)) = h(s_1(x)) = h(0) = 0$. For suitable choice of s and the lifting of G on E such that the section s_1 vanishes transversaly the zero set Z. The G-invariant 2-dimensional submanifold Z in X is the Poincaré dual to the $c_1(E)$.

LEMMA 3.3. *In the above argument, the Poincaré dual of the first Chern class $c_1(E)$ can be represented by a G-invariant submanifold in X.*

Let $(A = A_0 + 2a,\ \alpha,\ \beta)$ be a solution of the Seiberg-Witten equations $(*)$ for a suffliciently larger r. By Taubes [Theorem 3.2] there is a compact complex curve C with a pseudo-holomorphic map such that

(1) the image $u_*([C])$ is the Poincaré dual of $c_1(E)$,
(2) as an element in the homology $H_2(X;\mathbb{Z})^G$, $u_*([C]) = [\alpha^{-1}(0)] = PD(c_1(E))$.

As in Lemma 3.3, we define a new section $\alpha_1 = \frac{1}{|G|}\sum_{h\in G} h^*\alpha$ of $E \to X$. Then the section α_1 is G-invariant and its zero set ${\alpha_1}^{-1}(0)$ is a G-invariant subset in X. If $(A = A_0 + 2a,\ \alpha,\ \beta)$ is a G-invariant solution of $(*)$, then $\alpha = \alpha_1$ and ${\alpha_1}^{-1}(0) = \alpha^{-1}(0)$.

THEOREM 3.4. *Let (X, ω) be a closed, symplectic 4-manifold. Let a finite cyclic group G act semifreely, holomorphically on X as isometries with fixed point set Σ (may be empty) which is a 2-dimension submanifold. Then there is a smooth structure on the quotient $X' = X/G$ such that the projection $\pi : X \to X'$ is Lipschitz.*
Let $L \to X$ be the $Spin^c$-structure on X by pull-backed a $Spin^c$-structure $L' \to X'$ and $b_2^+(X') > 1$. If the Seiberg-Witten invariant $SW(L') \neq 0$ of L' is non-zero and $L = E \oplus K^{-1} \otimes E$, then there is a G-invariant pseudo-

holomorphic curve $u : C \to X$ such that the image $u(C)$ represents the fundamental class of the Poincaré dual $c_1(E)$.

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