A Study on the Robust Control Algorithm for an Axially Moving Film

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This paper presents the non-linear modeling of the web(film) transfer system and design of (robust) sliding mode controller bared on the developed model. The system model shows change in the roll-radius during winding and unwinding of the web. The change of inertia makes the web transfer system time-varying and nonlinear. In designing a robust controller, two major objectives are considered. The first one is that the web transferring velocity tracks a given reference velocity; the second one is that the web tension is regulated at a certain value. To verify the control algorithm, the proposed algorithm is compared with a conventional PID controller through a campater simulation based on simulink model. Computer simulation study shows that the proposed robust sliding mode controller is an improvement over the PID controller for various control inputs.

Key Words: Web, Thin Film, Sliding Mode Controller, Tension Control, Speed Control

Nomenclature

B_u, B_w	: Damping coefficient of roller bearing
B_{mu}, B_{mw}	: Damping coefficient of (un) winder motor
b	: Damping coefficient of film
h_{web}	: Thickness of film
I_u, I_w	: Motor current input
J_{mu}, J_{mw}	: Moment of inertia of (un)winder motor
J_u, J_{wr}	: Total moment of inertia of (un) winder
J_{ur}, J_w	: Moment of inertia of (un)winder roller
$J_{web,Ru}, J_{wb,Rw}$	: Moment of inertia of (un)winder side web which is in hollow shape
$J_{web,Rur}, J_{web,Rwr}$	: Moment of inertia of (un) winder side web which is in roller shape
K_{mw}, K_{mu}	: Motor constant of (un)winder motor
κ	: Stiffness of film
R_{ui}, R_{wi}	: initial radius of (un)winder roll
R_u, R_w	: Radius of (un)winder roll
W	: Width of film
θ_u, θ_w	: Rotating angle of (un)winder
ρ_{rol}, R_{web}	: Density of roll and web

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1. Foreword

Web or thin film is a general material that is long and easy to bend. The web transfer system is the device which moves the web in certain conditions. It is used in film, paper, and semi-conductor industries for various transferring systems. Generally, this system tends to bend, tear, and change form easily. Moreover, it shows non-linear characteristics due to changes in the rolling radius of the web winder and un-winder. Mainteaining constant velocity and tension of the web is difficult but important.

Various researches about tension control of the web have been conducted. The mechanisms of tension control include the following: a method which moves the friction plate to control frictional torque; a method which changes the position of the dancer roll; a method which changes the angle of the looper arm; a method which uses the ERF(electro-magnetic rheological fluid) clutch; and a method which drives motors synchronously, using the velocity difference between winder and unwinder as in the present study. Stangroom(1990) investigated the basic tension control of the spool through the use of the ERF clutch. Yokohama(1993) studied the tension control of magnetic tape through the use of the frictional torque of a plate. Stoten(1993) used the minimal control synthesis algorithm to control the tension of the magnetic tape, whereas Yeung (1995) used the fuzzy control algorithm for the tension control of wire material. Shin(1996) modeled the longitudinal dynamics of a continuous transfer system and used the variable gain PID control algorithm for controlling the tension of the web or wire. Many of these studies, however, lacked robustness as they were based on the linear model and theory.

This paper presents non-linear modeling and the design of a (robust) sliding mode controller for the web transfer system. This system model shows change in the roll-radius during winding and unwinding of the web. The change of inertia makes the web transfer system time-varying and nonlinear. Two major objectires are considered in the design of the robust controller. The first one is that the web transferring velocity should track a given reference velocity. The second one is that the web tension be regulated at a certain value. For enhancing robustness, the sliding mode controller was designed to handle both external disturbance and parameter uncertainty in the system.

2. System Description

The film or web transferring system is the mechanical system which moves film from unwinder to winder while maintaining the constant tension. Precision processing of the film is closely related with the tension, and it varies with the velocity difference between winder and unwinder. Hence, in order to control the tension of a film, each roll needs mechanisms which are able to control velocities independently to each other. To achieve this, each roll must be connected with separated actuators. In this system model, three assumptians were made: first, film consists of spring and damper elements; second, the radius of each roll varies as film is transferred; third, frictional force at each support is so small as to be insignificant. The lumped mass model of the film transferring system is shown in Fig. 1.

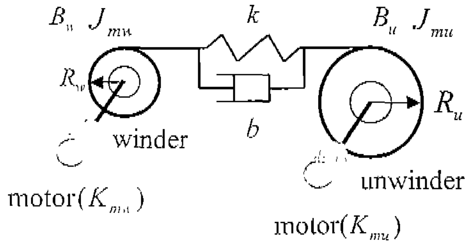

Fig. 1 System model

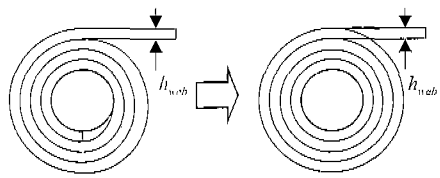

Fig. 2 Idealized winding states of film

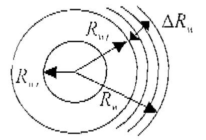

Fig. 3 Schematic of winder

2.1 Modeling of winder and unwinder

As shown in Fig. 2, the film winds the winder and unwinder rolls in a spiral shape. If we assume that the film material is much thinner than the radius of the roll, we can assume that the film winds the roll in a regular circle shape.

In the case of the winder, the radius and the moment of inertia are increased over time and these can be expressed as a function of the rotational angle. Radius of the winding roll shown in Fig. 3 is expressed as follows:

$$R_w(\tau)=R_{wi}+\frac{\theta_w}{2\pi}h_{web} \tag{1}$$

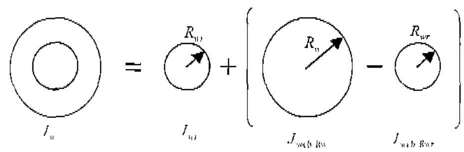

Fig. 4 Schematic of moment of winder

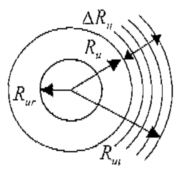

Fig. 5 Schematic of unwinder

The moment of inertia of a winder is the sum of a roller's moment of inertia and the film's moment of inertia. Figure 4 represents the calculation of the moment of inertia of a roller and the film.

The result is as follows:

$$J_w = J_{wr} + (J_{web,Rw} - J_{web,Rwr})$$
$$= \frac{1}{2}\pi w\left[\rho_{roll}R_{wr}^4 + \rho_{web}\left(R_{wi} + \frac{\theta_w}{2\pi}h_{web}\right)^4 - \rho_{web}R_{wr}^4\right] \quad (2)$$

On the right-hand side of Eq. (2), the first term represents the moment of inertia of the roller. The second and third terms represent the moment of inertia of the film around the winding roller.

As shown in Fig. 5, the radius of the unwinder is decreased over time and this can be expressed as follows:

$$R_u(\tau) = R_{ui} - \frac{\theta_u}{2\pi}h_{web} \quad (3)$$

The moment of inertia of an unwinder can also be expressed by using the moment of inertia of the roller and the film as follows:

$$J_u = J_{ur} + (J_{web,Ru} - J_{web,Rur})$$
$$= \frac{1}{2}\pi w\left[\rho_{roll}R_{ur}^4 + \rho_{web}\left(R_{ui} - \frac{\theta_u}{2\pi}h_{web}\right)^4 - \rho_{web}R_{ur}^4\right] \quad (4)$$

2.2 Modeling of the whole system

The film between the winder and the unwinder can be modeled as a spring and damping element as shown in Fig. 1. Many research results show the

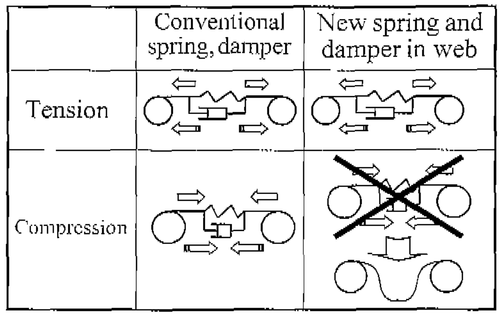

Fig. 6 Spring and damping elements in web model

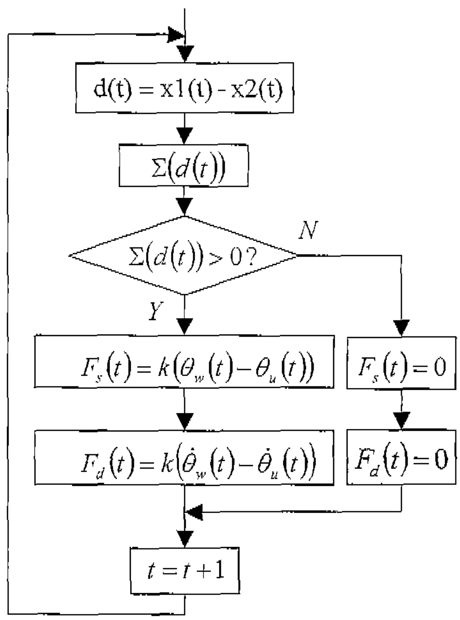

Fig. 7 Non-linear spring and damping forces of film

tension of the film fluctuates around its reference value. This means that if the reference tension is zero, the film element will generate compression force in some interval. These elements, however, differ from the general spring and damping elements in a compression case. The differences between the two models can be seen in Fig. 6.

In the case of compression, the film does not generate compression forces from the spring and damping elements. Hence, the non-linear spring and damping model, which calculates forces from a summation of the difference between the linear displacement of the winder and the unwinder, must be used. The block diagram in Fig. 7 re-

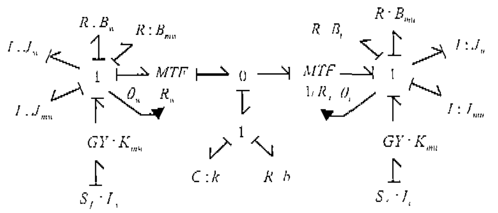

Fig. 8 The bond graph model for a film transferring system

presents this algorithm.

Assuming that motor torques are proportional to the current input and there is no slip between film and rollers, the bond graph model can be attained by using Eqs. (1)~(4). The bond graph model for the film transferring system is shown in Fig. 8.

Two-degrees-of-freedom motions are obtained by using the following Eqs. (5)~(6).

$$\frac{d}{dt}\{(J_w+J_{mw})\dot{\theta}_w\}=-R_wF_s(t)-R_wF_d(t)-(B_w+B_{mw})\dot{\theta}_w+K_{mw}I_w \quad (5)$$

$$\frac{d}{dt}\{(J_u+J_{mu})\dot{\theta}_u\}=R_wF_s(t)+R_wF_d(t)-(B_u+B_{mu})\dot{\theta}_u+K_{mu}I_u \quad (6)$$

where, F_s: non-linear spring force of the film
F_d: non-linear damping force of the film.

3. Control Algorithm

The concept of sliding mode control is employed to choose a suitable surface in state space, called switching surface. The control input is then chosen to guarantee that the trajectories near the sliding surface are directed towards the surface. As long as the resulting trajectories are pointing towards the surface, any control input will suffice. After the system falls into the sliding surface, the closed-loop dynamics are completely governed by the equations that define the surface. Since the parameters defining the surface are chosen by the designer, the closed-loop dynamics of the system will be independent of perturbations in the parameters of the system and robustness is achieved. This means that it is possible to design a robust controller for a film transferring system with disturbance and noise rejection characteristics. Moreover, using the sliding mode controller's robustness characteristics, it is possible to compensate for unmodeled non-linear system dynamics caused by non-linear radius changes and spring-damping forces.

The sliding mode controller differs from simple relay controllers in that it relies on extremely high-speed switching of the control inputs. Recent advances in microelectronics, however, have made it possible to implement high-speed switching for many classes of systems.

3.1 Transferring speed control

The film transferring system in general needs constant transferring speed for many reasons. In this study, the DC motor of the winder is controlled in order to maintain constant transferring speed. From the Eq. (5), the transferring speed dynamics of the winder system can be expressed as:

$$\ddot{\theta}_w=f_w+b_wI_w+\Delta f_w+D_w \quad (7)$$

where, Δf_w : modeling error and parameter uncertainties
D_w: disturbance in the winder dynamics

The speed error is as follows:

$$e_s=y_s-y_{sd} \quad (8)$$

where, $y_s=R_w\dot{\theta}_w$: transferring speed of film
y_{sd}: desired speed.

In order for the film speed to track the desired speed, i.e. $y_s=y_{sd}$, the sliding surface is defined as:

$$s=e_s=y_s-y_{sd} \quad (9)$$

$$\dot{s}=\dot{y}_s-\dot{y}_{sd}=\left(R_{wi}+\frac{h_{web}}{2\pi}\theta_w\right)\ddot{\theta}_w+\frac{h_{web}}{2\pi}\dot{\theta}_w^2 \quad (10)$$

The sliding surface can be interpreted as the surface of velocity tracking error between the winder linear speed and the reference speed. As S goes to zero, the film tracks the reference speed perfectly. The best approximation of $\hat{I}_w$ for a continuous control law that would achieve $\dot{s}=0$ is thus:

$$\hat{I}_w=\frac{1}{b_w}(\dot{y}_{sd}-f_w)=\frac{J_w+J_{mw}}{K_{mw}}\left[\frac{-h_{web}\dot{\theta}_w^2/2\pi}{R_w}\right. \quad (11)$$

$$+\frac{R_wF_s(t)+R_wF_d(t)+(B_w+B_{mw})\dot{\theta}_w}{J_w+J_{mw}}\Bigg].$$

To satisfy sliding conditions regardless of uncertainty in the model, a discontinuous term is added to $\hat{I}_w$(Ro, 1996):

$$I_w=\hat{I}_w-\frac{J_w+J_{mw}}{K_{mw}}\kappa_w\mathrm{sgn}(s) \tag{12}$$

$$\text{where } \begin{cases}\mathrm{sgn}(s)=+1 \text{ if } s>0\\ \mathrm{sgn}(s)=-1 \text{ if } s<0.\end{cases} \tag{13}$$

By replacing I_w in Eq. (7) by Eq. (12), the Lyapunov candidate can be tested as(Slotine, 1991):

$$\frac{1}{2}\frac{d}{dt}s^2\leq-\eta_w\,|\,s\,| \tag{14}$$

$$\text{or} \quad s\dot{s}\leq-\eta_w\,|\,s\,| \tag{15}$$

Hence, η_w should be positive and the derivative of the sliding parameter is described as:

$$\dot{s}\leq-\eta_w\mathrm{sgn}(s), \tag{16}$$

the derivative of S is

$$\begin{aligned}\dot{s}&=f_w+b_wI_w+\Delta f_w+D_w-\dot{y}_{sd}\\ &=f_w+b_w\left(\hat{I}-\frac{\kappa_w}{b_w}\mathrm{sgn}(s)\right)+\Delta f_w+D_w\\ &\quad-\dot{y}_{sd}\leq-\eta_w\mathrm{sgn}(s)\end{aligned} \tag{17}$$

Substituting $\hat{I}_w$ in Eq. (11), the stable switching condition is reached.

$$\begin{aligned}\dot{s}=&f_w+b_w\left(\frac{1}{b_w}(\dot{y}_{sd}-f_w)-\frac{\kappa_w}{b_w}\mathrm{sgn}(s)\right)\\ &+\Delta f_w+D_w-\dot{y}_{sd}\leq-\eta_w\mathrm{sgn}(s)\end{aligned} \tag{18}$$

Finally, gain κ_w in Eq (12) should satisfy the following condition as (Ro, 1996):

$$\kappa_w\geq|\,\Delta f_w+D_w\,|+\eta_w \tag{19}$$

In practical systems, it is impossible to achieve the high switching control that is necessary to realize most of the sliding mode controller design. There are two reasons for this. One reason is the presence of finite delay in computation for control and the other is the limitation of physical actuators. Since it is impossible to switch the control at an infinite rate, a chattering always occurs on the sliding surface of a sliding mode control system. One approach for overcoming the undesirable effects of control chattering is to introduce what is known as a boundary layer around the sliding surface and to approximate the switching control by a continuous control inside this boundary layer(Slotine, 1991). Thus, the control law in Eq. (12) can be replaced by the continuous approximation as:

$$I_w=\hat{I}_w-\frac{J_w+J_{mw}}{K_{mw}}\kappa_w sat\left(\frac{S}{\Phi}\right) \tag{20}$$

where Φ is the boundary layer thickness and the function $sat(\cdot)$ is defined as:

$$\begin{cases}sat\left(\frac{s}{\Phi}\right)=\frac{s}{\Phi} & if\left|\frac{s}{\Phi}\right|\leq1\\ sat\left(\frac{s}{\Phi}\right)=\mathrm{sgn}\left(\frac{s}{\Phi}\right) & if\left|\frac{s}{\Phi}\right|>1\end{cases} \tag{21}$$

3.2 Tension control

There are several methods to control the tension (as mentioned previously); however, in this system, minimization of the tension method was used. To achieve this, the unwinder displacement should be controlled to track winder displacement for tension minimization.

The displacement error is as follows:

$$e_t=y_u-y_w \tag{22}$$

where, $y_u=R_u\theta_u$: displacement of unwinder
$y_w=R_w\theta_w$: displacement of winder

The above error dynamics of the tension has relative degrees of second order. The best approximation of $\hat{I}_u$ for a continuous control law is:

$$\hat{I}_u=\left(\frac{J_u+J_{mu}}{K_{mu}}\right)\left[\frac{\left(\begin{aligned}&\frac{h_{web}}{\pi}\dot{\theta}_u^2+R_w\dot{\theta}_w+\frac{h_{web}}{\pi}\theta_w^2\\ &-\lambda R_u\dot{\theta}_u+\lambda R_w\dot{\theta}_w\end{aligned}\right)}{R_u}-\frac{R_uF_s(t)+R_uF_d(t)-(B_u+B_{mu})\dot{\theta}_u}{J_u+J_{mu}}\right] \tag{23}$$

In order to satisfy sliding conditions regardless of uncertainty in the model, a discontinuous term is added to $\hat{I}_u$.

$$I_u=\hat{I}_u-\frac{J_u+J_{mu}}{K_{mu}}\kappa_u sat\left(\frac{s}{\Phi}\right) \tag{24}$$

Gain κ_u in Eq. (24) should satisfy the following condition:

$$\kappa_u\geq|\,\Delta f_u+D_u\,|+\eta_u \tag{25}$$

where, Δf_u: modeling error and parameter uncertainties

Table 1 Model parameters

Parameter	Value	Units
Web stiffness	50	N/m
Web damping ratio	0.001	Ns^2/m
Web thickness	6e−7	m
Motor damping ratio	0.02	Nm/(rad/s)
Motor constant	0.206	Nm/A
Moment of inertia of motor	0.003	kgm^2

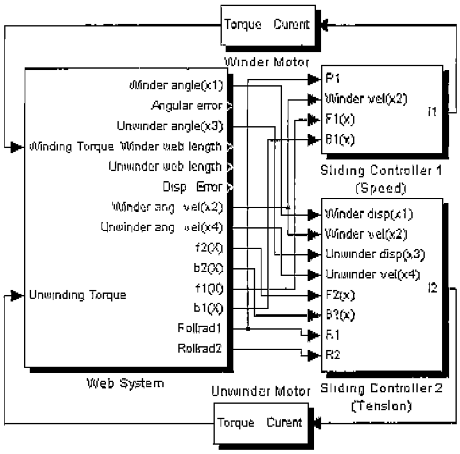

Fig. 9 Simulink model of the entire system

D_u: disturbance at the unwinder dynamics

4. Simulation

For the simulation, the film transferring model and the proposed control algorithms are represented by the Simulink as in Fig. 9.

For the verification of the control algorithm, a computer simulation was carried out and its result was compared with the result of the PID controller. Model paramcters used in the simulation are shown in Table 1.

4.1 Reference speed tracking performance

To investigate the reference speed tracking performance, the trapezoidal speed input is applied. The reference speed profile is shown in Fig. 10. First, film is accelerated in 1 second, and then a constant speed of 2m/s for 10 seconds is maintained. Finally the film is decelerated in 1 second.

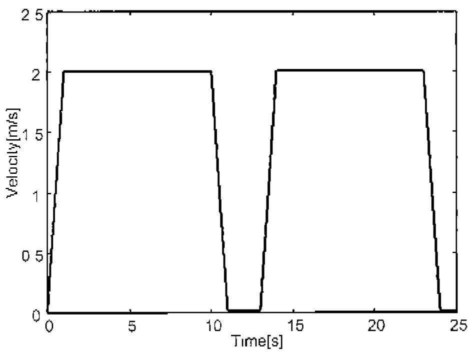

Fig. 10 Reference speed profile

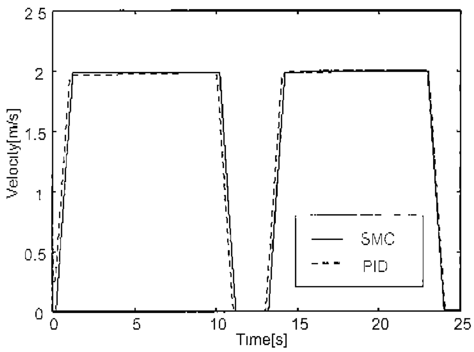

Fig. 11 Transferring speed about trapezoidal input

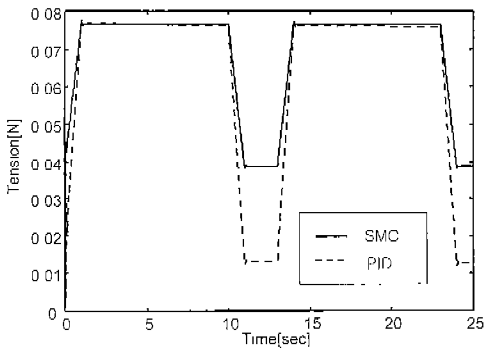

Fig. 12 Tension variation about trapezoidal input

Figures 11 and 12 show the transferring speed and the tension of the film. The dotted and the continuous lines indicate the results of the PID and the sliding mode controllers, respectively. In these Figures, the sliding controller performs better than the PID controller in transient region. The performances of these controllers are, however, satisfactory.

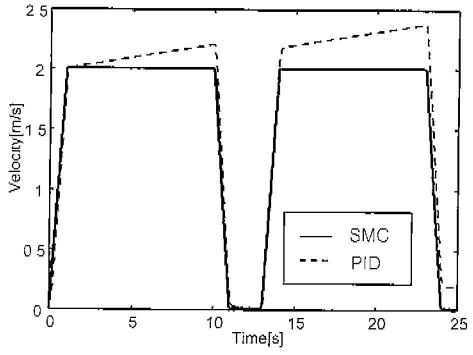

Fig. 13 Transferring speed for the parameter change

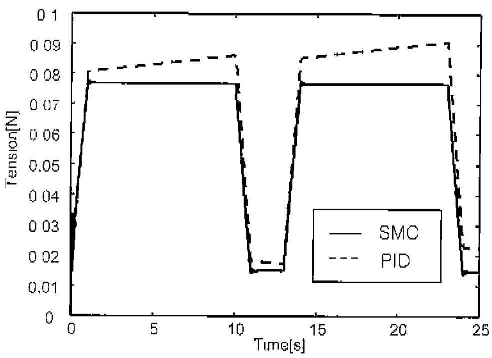

Fig. 14 Tension variation for the parameter change

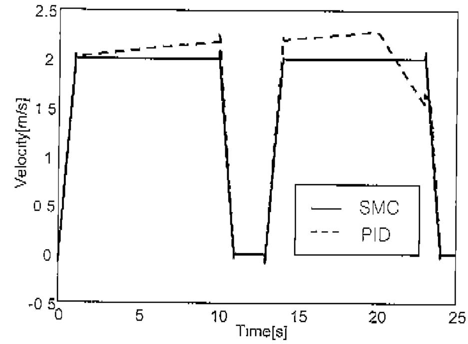

Fig. 15 Transferring speed for the thickness change

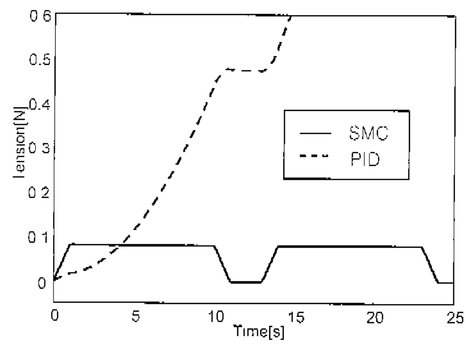

Fig. 16 Tension variation for the thickness change

4.2 Robust performance for the system parameter change

For model-based control, the control scheme is designed upon a nominal model of a dynamic system. For robust sliding mode control, however, the controller is designed with considering both the nominal model and some characterization of the model uncertainties.

For 10 percents variations of film stiffness, the speed and tension change of the sliding mode controller are much smaller than those of PID, as shown in Figs. 13 and 14. In this simulation, all gains were the same as in the previous simulation. Therefore, the sliding mode control, which is insensitive to system parameter variations, overcomes undesirable behaviors.

The non-linearity of the system is mainly due to the thickness of the film. If the thickness of the film is changed or much larger than the previous value, the initial control gain will not work properly due to the system's nonlinearity. For variation of film thickness of up to 3 times, the speed and tension change of the sliding mode controller are much smaller than those of PID, as shown in Figs. 15 and 16. The sliding mode control is robust to film thickness variations.

4.3 The disturbance rejection for unexpected motor torque

When a film is transferred, it may experience unexpected disturbance during processing. In this situation, disturbance will cause speed and tension variation of the film. This result is shown in Fig. 17 and Fig. 18. Since sliding mode controller changes model structure autonomously, the transient response of the sliding mode controller shows much better disturbance rejection characteristics than does the PID controller.

5. System Construction and Experiment

In this study, the controller is verified by constructing a web transferring system. Fig. 19

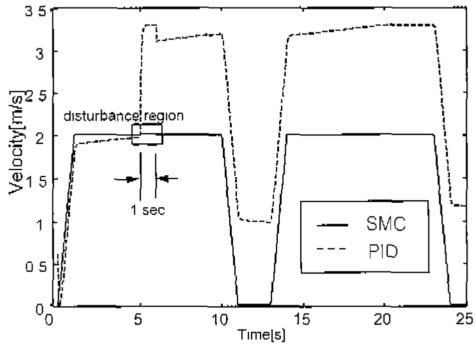

Fig. 17 Transferring speed for the external disturbance

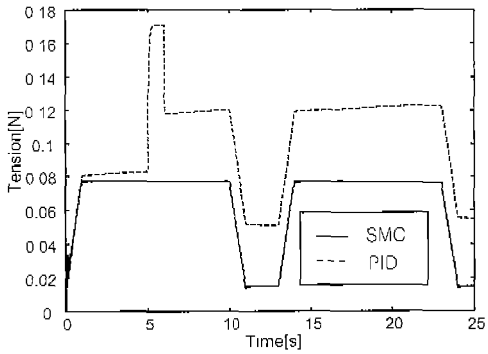

Fig. 18 Tension variation for the external disturbance

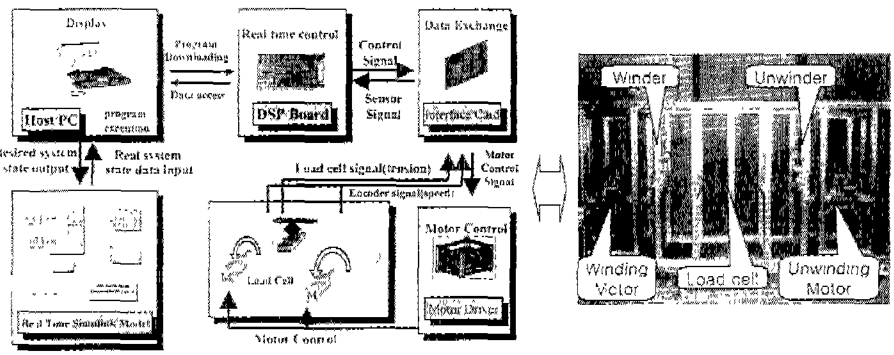

Fig. 19 Experimental device setup

shows the experimental device setup for the real-time tension control. DSP (Digital Signal Processor) board is used for real time control. AC motors are used to drive the winding and unwinding roll. An Interface card is made for connecting the DSP board, motor driver, and sensors. The motors are located at the winder roll and unwinder roll. The moving web is passing through three idlers. The load cell, which is located at the center of the idlers, indirectly monitors the tension of the film

5.1 Reference speed tracking test

In order to investigate reference speed tracking performance, the trapezoidal speed input is applied. Fig. 20 and Fig. 21 show the transferring speed and the tension of the film. The dotted line indicates the reference speed and the continuous line indicates the experiment result of the sliding mode controller. In these figures, time delay and

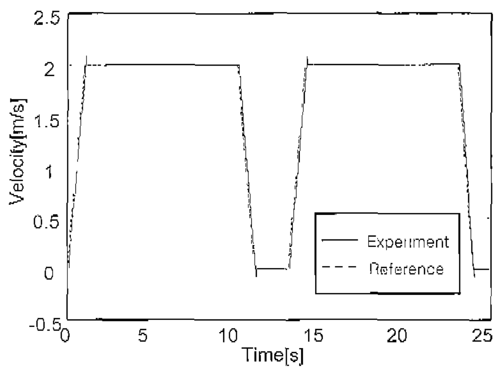

Fig. 20 Transferring speed to the trapezoidal input test

overshoot exist in the speed graph, however, these values are small. The tension in Fig. 21 maintains small value. The performance of the sliding controller is satisfactory.

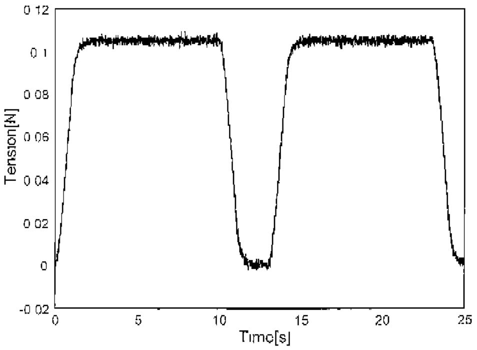

Fig. 21 Tension variation to the trapezoidal input test

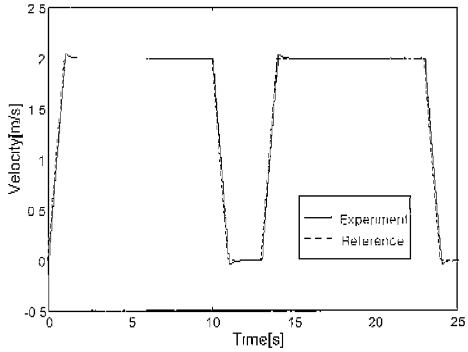

Fig. 22 Transferring speed to the parameter change test

5.2 Robust performance for the system parameter change test

In this test, the film stiffness in the sliding mode controller is increased by 10 percent, and real value is not changed. All gains were the same as in the previous test. The tension in Fig. 23 shows somewhat fluctuating characteristics, however, tension value remains small. The sliding mode controller, which is insensitive to system parameter variations, overcomes undesirable disturbances.

6. Conclusions

In this study, a non-linear model of the film transferring system and a new control method are introduced. Using the sliding mode control schemes, a speed and tension control system can be designed. The results of this study can be

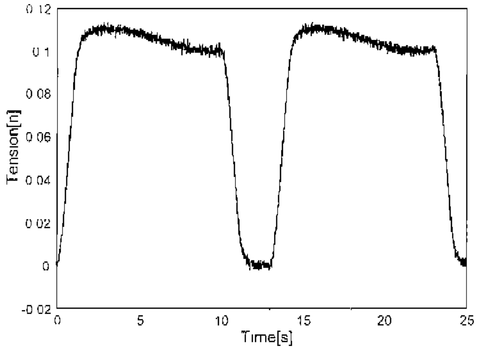

Fig. 23 Tension variation to the parameter change test

summarized as follows:

(1) By using sliding mode controller, transferring speed and tension of film are controlled for a given speed profile.

(2) The sliding mode controller is not sensitive to parameter variations, so the film transferring system maintains its stability.

(3) When the system experiences unexpected disturbances, it has the disturbance rejection tendency to overcome it.

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