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Fuzzy Subsystems of A Fuzzy Finite State Machine

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ABSTRACT

In this paper we define fuzzy subsystems of a fuzzy finite state machine by using maps S^{α} of each state subset to its all α-successors, which is a natural generalization of crisp submachines as fuzzy. And the corresponding concepts are also examined.

Key Words : Fuzzy finite state machine, fuzzy subsystem, and strong homomorphism.

1. Introduction

Automata theory is one of basic and important theorics in computer science. Since theory of fuzzy sets was introduced by Zadeh [6, 7], Wee [5] introduced the idea of fuzzy automata. There has been considerable growth in the area of fuzzy automata [1]. Malik, Mordeson and Sen [2, 3, 4] used algebraic techniques to the study of fuzzy automata, and also in [3] they introduced the notions of subsystems of a fuzzy finite state machine in order to consider state membership as fuzzy.

In this paper we introduce the notions of fuzzy subsystems of a fuzzy finite state machine by using maps S^{α} of a state subset to its all α-successors, which are equivalent to the form of subsystems and strong subsystems defined by Malik, Mordeson and Sen[3]. These are a natural generalizations of crisp submachines as fuzzy. The notions of fuzzy homomorphism and strong fuzzy homomorphism also are introduced. And it is proved that the fuzzy homomorphic inverse image of a fuzzy subsystem is a fuzzy subsystem. Moreover, strong fuzzy homomorphic image of a fuzzy subsystem is shown to be a fuzzy subsystem.

Before we go further, wc introduce the following definitions and notations. Let A be a fuzzy subset of Q, with thc membership $\mu_A: Q \to [0,1]$. The closure of $\{x \in Q \mid \mu_A(x) > 0\}$ is called the support of A, denoted by $suppA$. If B is also a fuzzy subset of Q, then the fuzzy sets $A \cup B$, $A \cap B$ are defined as

$$\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x), \quad \forall x \in Q,$$
$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x), \quad \forall x \in Q.$$

When we want to exhibit an element $x \in Q$ that typically belongs to a fuzzy set A, we may demand its membership value to be greater than some threshold $\alpha \in (0,1)$. The ordinary set of such elements is the α-cut A_{α} of A, $A_{\alpha} = \{x \in Q \mid \mu_A(x) \geq \alpha\}$. It is easily checked that the following properties hold :

$$(A \cup B)_{\alpha} = A_{\alpha} \cup B_{\alpha}, \ (A \cap B)_{\alpha} = A_{\alpha} \cap B_{\alpha}.$$

A is said to be included in B, denoted by $A \subset B$, if for each x, $\mu_A(x) \leq \mu_B(x)$. A fuzzy finite state machine is a triple $M = (Q, X, \mu)$, where Q and X are finite nonempty sets and μ is a membership of some fuzzy subsets of $Q \times X \times Q$, i.e., $\mu: Q \times X \times Q \to [0, 1]$. Let X^* denote the set of all words of elements of X of finite length. Q is called the set of states and X is called the set of input symbols. Let λ denote the cmpty word in X^* and $|x|$ denote the length of x, $\forall x \in X^*$.

Define $\mu^*: Q \times X^* \times Q \to [0, 1]$ by

$$\mu^*(q, \lambda, p) := \begin{cases} 1 & \text{if } q = p \\ 0 & \text{if } q \neq p \end{cases}$$

and

$$\mu^*(q, xa, p) = \bigvee \{\mu^*(q, x, r) \wedge \mu(r, a, p) \mid r \in Q\},$$
$$\forall p, q \in Q, \ \forall x \in X^*, \ \forall a \in X.$$

Thcn

$$\mu^*(q, xy, p) = \bigvee \{\mu^*(q, x, r) \wedge \mu^*(r, y, p) \mid r \in Q\},$$
$$\forall p, q \in Q, \ \forall x, y \in X^*.$$

This means that a fuzzy subset μ of $Q \times X \times Q$ can be naturally extended to a fuzzy subset μ^* of $Q \times X^* \times Q$ under max-min operation.

Definition 1.1. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let $p, q \in Q$, p is called an immediate α-successor of q if there exists $a \in X$ such that $\mu(q, a, p) \geq \alpha$, $0 < \alpha \leq 1$. And p is an α-successor of p if there exists $x \in X^*$ such that $\mu^*(q, x, p) \geq \alpha$,

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$0 < \alpha \leq 1$.

Remark 1.2. If $M = (Q, X, \mu)$ is a fuzzy finite state machine, μ is a fuzzy subset of $Q \times X \times Q$. From the definition 1.1, we note that "p is an immediate α-successor of q" means that there exists $a \in X$ such that $(q, a, p) \in Q \times X \times Q$ is an element of α-cut of μ, $0 < \alpha \leq 1$.

In fuzzy set theory, we usually consider the support of μ as 0-cut. From the meaning, we naturally define 0-successor as follows : p is said to be an immediate 0-successor of q if there exists $a \in X$ such that $\mu(q, a, p) > 0$. In the same ways, we have the definition of 0-successors. 0-successors are simply called successors [3,4].

Lemma 1.3. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine, and let $q, p, r \in Q$.
Then

(1) q is an α-successor of q,
(2) If p is an α-successor of q and r is an α-successor of p, then r is an α-successor of q.

Proof. Let $\alpha > 0$. Since $\mu^*(q, \lambda, q) = 1 \geq \alpha$, (1) holds. Let p be an α-successor of q, and let r be an α-successor of p. Then there exist $x, y \in X$ such that $\mu^*(q, x, p) \geq \alpha$ and $\mu^*(p, y, r) \geq \alpha$. So

$$\mu^*(q, xy, r) \geq \mu^*(q, x, p) \wedge \mu^*(p, y, r) \geq \alpha.$$

Hence r is an α-successor of q.
For $\alpha = 0$, see [4, Proposition 5].

Let $M = (Q, X, \mu)$ be a fuzzy finite state machine, and let $q \in Q$. We denote by $S^\alpha(q)$ the set of all α-successors of q. And if $T \subset Q$, then the set of all α-successors of T in Q, denoted by $S^\alpha_Q(T)$, is defined to be the set $S^\alpha_Q(T) = \bigcup\{S^\alpha(q) \mid q \in T\}$. This is in fact a mapping of the set of all subsets of Q into itself. If no confusion arises, then we write $S^\alpha(T)$ for $S^\alpha_Q(T)$.

Lemma 1.4. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine, and let $A, B \subset Q$. Then the following assertions hold for each $\alpha \in [0, 1]$.

(1) If $A \subset B$, then $S^\alpha(A) \subset S^\alpha(B)$.
(2) $A \subset S^\alpha(A)$.
(3) $S^\alpha(S^\alpha(A)) = S^\alpha(A)$.
(4) $S^\alpha(A \cup B) = S^\alpha(A) \cup S^\alpha(B)$.
(5) $S^\alpha(A \cap B) \subset S^\alpha(A) \cap S^\alpha(B)$.
(6) If $\alpha \geq \beta$, then $S^\alpha(A) \subset S^\beta(A)$.

Proof. We only prove (3). The others can be proved by the similar arguments. By (2), $S^\alpha(A) \subset S^\alpha(S^\alpha(A))$. Let $q \in S^\alpha(S^\alpha(A))$, then $q \in S^\alpha(p)$ for some $p \in S^\alpha(A)$.

And also $p \in S^\alpha(r)$ for some $r \in A$. Thus q is an α-successor of p and p is an α-successor of r, which implies by Lemma 1.3(2) that $q \in S^\alpha(r) \subset S^\alpha(A)$. Hence $S^\alpha(S^\alpha(A)) = S^\alpha(A)$.

The above theorem is in fact the generalized formula of Theorem 8 [4], which is exactly in the particular case of $\alpha = 0$.

2. Fuzzy Subsystems

Mappings S^α on the set of all sebsets of Q act an important role to analyze the structure of a fuzzy finite state machine.

By using these mappings S^α, we now define fuzzy subsystems of a fuzzy finite state machine as follows.

Definition 2.1. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let δ be a fuzzy subset of Q. Then $N = (Q, \delta, X, \mu)$ is said to be an α-subsystem of M if $S^\alpha(\delta_\alpha) \subset \delta_\alpha$, for α-cut δ_α of δ. And if N is an α-subsystem of M for each α, then N is simply called a fuzzy subsystem of M.

From the following theorem, we can find that the definition of fuzzy subsystem is equivalent to that of a subsystem given by D. S. Malik *et al.* [3].

Theorem 2.2. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine, and let δ be a fuzzy subset of Q. Then $N = (Q, \delta, X, \mu)$ is a fuzzy subsystem of M if and only if $\forall p, q \in Q$, $\forall x \in X^*$, $\delta(q) \geq \delta(p) \wedge \mu^*(p, x, q)$.

Proof. Let $N = (Q, \delta, X, \mu)$ be a fuzzy subsystem of M. And let $q \in Q$. We assume that $\delta(q) < \delta(p) \wedge \mu^*(p, x, q)$ for some $p \in Q$ and $x \in X^*$. Set $\delta(p) \wedge \mu^*(p, x, q) = \alpha$. If $\alpha = 0$, then it is clear. Suppose $\alpha > 0$, then $p \in \delta_\alpha$ and $\mu^*(p, x, q) \geq \alpha$. Since N is a fuzzy subsystem of M, $q \in S^\alpha(\delta_\alpha) \subset \delta_\alpha$, contradicting $\delta(q) < \alpha$. To prove the converse, if we let $q \in S^\alpha(\delta_\alpha)$ for every α, then there exist $p \in \delta_\alpha$ and $x \in X^*$ such that $\mu^*(p, x, q) \geq \alpha$. By the hypothesis $\delta(q) \geq \delta(p) \wedge \mu^*(p, x, q) \geq \alpha$ (>0 if $\alpha = 0$), which implies that $q \in \delta_\alpha$. This completes the proof.

Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Now we consider the crisp subset T of Q. Let ν be a fuzzy subset of $T \times X \times T$. The fuzzy finite state machine $N = (T, X, \nu)$ is called a submachine of M [4] if (i) $\mu|_{T \times X \times T} = \nu$, and (ii) $S(T) \subset T$.

If we consider fuzzy subsystems $N = (Q, T, X, \mu)$ for crisp subsets T of Q, $S^\alpha(T_\alpha) = S^\alpha(T) \subset T$ for all α. Since $S^\alpha(T) = S^0(T) = S(T)$, N is a fuzzy subsystem if and only if $S(T) \subset T$. Thus for crisp subsets T of Q, (Q, T, X, μ) is a fuzzy subsystem of $M = (Q, X, \mu)$ if and only if (T, X, ν) is a submachine of M, where $\nu = \mu|_{T \times X \times T}$. This implies that the notion of fuzzy subsystems is the generalization of that of submachines. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let $I^{\alpha}(q)$ denote the set of all immediate α-successors of $q \in A$. If $T \subset Q$, then we define $I^{\alpha}(T) = \bigcup\{I^{\alpha}(q) | q \in T\}$. Clearly, $I^{\alpha}(T)$ is a subset of $S^{\alpha}(T)$. In the following lemma we know that if we only check for $I^{\alpha}(\delta_{\alpha})$ to be a subset of δ_{α}, (Q, δ, X, μ) is a fuzzy subsystem of $M = (Q, X, \mu)$.

Lemma 2.3. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let δ be a fuzzy subset of Q. Then $N = (Q, \delta, X, \mu)$ is a fuzzy subsystem of M if and only if $I^{\alpha}(\delta_{\alpha}) \subset \delta_{\alpha}$, $\forall \alpha \in [0, 1]$.

Proof. If $N = (Q, \delta, X, \mu)$ is a fuzzy subsystem of M, $S^{\alpha}(\delta_{\alpha}) \subset \delta_{\alpha}$ for $\forall \alpha \in [0, 1]$. But since $I^{\alpha}(\delta_{\alpha})$ is a subset of $S^{\alpha}(\delta_{\alpha})$, $I^{\alpha}(\delta_{\alpha}) \subset \delta_{\alpha}$. To prove the converse, let δ be a fuzzy subset of Q and assume $I^{\alpha}(\delta_{\alpha}) \subset \delta_{\alpha}$, $\forall \alpha \in [0, 1]$. Let $q \in S^{\alpha}(\delta_{\alpha})$, then there exist $p \in \delta_{\alpha}$ and $x \in X^*$ such that

$$\mu^*(p, x, q) \geq \alpha (\mu^*(p, x, q) > 0 \text{ if } \alpha = 0).$$

If $x = \lambda$, then

$$\mu^*(p, \lambda, q) \geq \alpha (\mu^*(p, \lambda, q) > 0 \text{ if } \alpha = 0).$$

By the definition of empty word λ, $p = q$. Hence $q \in \delta_{\alpha}$. Suppose that x is not empty word, say $x = a_1 a_2 \cdots a_n$, where $a_i \in X$, $i = 1, \cdots, n$. Then

$$\begin{aligned} \mu^*(p, x, q) &= \mu^*(p, a_1 \cdots a_n, q) \\ &= \bigvee_{r_1 \in Q} \cdots \bigvee_{r_{n-1} \in Q} \mu(p, a_1, r_1) \wedge \mu(r_1, a_2, r_2) \wedge \cdots \\ &\quad \wedge \mu(r_{n-2}, a_{n-1}, r_{n-1}) \wedge \mu(r_{n-1}, a_n, q) \\ &\geq \alpha \quad (>0 \text{ if } \alpha = 0), \end{aligned}$$

which follows that there exist $s_1, \cdots, s_{n-1} \in Q$ such that

$$\mu(p, a_1, s_1), \mu(s_1, a_2, s_2), \cdots, \mu(s_{n-2}, a_{n-1}, s_{n-1}), \mu(s_{n-1}, a_n, q) \geq \alpha \quad (>0 \text{ if } \alpha = 0).$$

Since $p \in \delta_{\alpha}$ and $a_1 \in X, s_1 \in I^{\alpha}(\delta_{\alpha})$. Hence by hypothesis, $s_1 \in \delta_{\alpha}$. Also $a_2 \in X$ implies $s_2 \in I^{\alpha}(\delta_{\alpha}) \subset \delta_{\alpha}$. If we continue the process, then we obtain $q \in \delta_{\alpha}$. Hence $S^{\alpha}(\delta_{\alpha})$ is a subset of δ_{α}, which proves that N is a fuzzy subsystem of M.

If we follows the proof of Theorem 2.2 after replacing X^*, μ^* and S^{α} with X, μ and I^{α}, respectively, then by Lemma 2.3 we obtain the following result.

Theorem 2.4. $M = (Q, X, \mu)$ be a fuzzy finite state machine and let δ be a fuzzy subset of Q. Then $N = (Q, \delta, X, \mu)$ is a fuzzy subsystem of M if and only if $\forall p, q \in Q, \forall a \in X, \delta(q) \geq \delta(p) \wedge \mu(p, a, q)$.

Theorem 2.5. Let $M = (Q, X, \mu)$ be a fuzzy finite state machine. Let $N_1 = (Q, \delta_1, X, \mu)$, $N_2 = (Q, \delta_2, X, \mu)$ be fuzzy subsystems of M. Then the following assertions hold.

(1) $N_1 \cup N_2 = (Q, \delta_1 \cup \delta_2, X, \mu)$ is a fuzzy subsystem of M,

(2) $N_1 \cap N_2 = (Q, \delta_1 \cap \delta_2, X, \mu)$ is a fuzzy subsystem of M.

Proof. (1) For each α,

$$\begin{aligned} S^{\alpha}((\delta_1 \cup \delta_2)_{\alpha}) &= S^{\alpha}((\delta_1)_{\alpha} \cup (\delta_2)_{\alpha}) \\ &= S^{\alpha}((\delta_1)_{\alpha}) \cup S^{\alpha}((\delta_2)_{\alpha}) \\ &\subset (\delta_1)_{\alpha} \cup (\delta_2)_{\alpha} \\ &= (\delta_1 \cup \delta_2)_{\alpha}. \end{aligned}$$

Hence $N_1 \cup N_2$ is a fuzzy subsystem of M.

(2) can be proved similarly.

Let $M = (Q, X, \mu)$ be a fuzzy finite state machine and let A be a subset of Q. For each $x \in X^*$, define a subset $S_x^{\alpha}(A)$ of Q by

$$S_x^{\alpha}(A) = \{q \in Q | \mu(p, x, q) \geq \alpha \ (>0 \text{ if } \alpha = 0) \text{ for } p \in A\}.$$

For each $x \in X^*$, S_x^{α} is in fact a map on the power set of Q, and $\bigcup_{x \in X^*} S_x^{\alpha} = S^{\alpha}$. The composition of these maps $S_x^{\alpha}, x \in X^*$, is given by the following lemma.

Lemma 2.6 Let $M = (Q, X, \mu)$ be a fuzzy finite state machine, and let $x, y \in X^*$. Then $S_y^{\alpha} \circ S_x^{\alpha} = S_{xy}^{\alpha}$.

Proof. Let A be a subset of Q and let $r \in S_y^{\alpha} \circ S_x^{\alpha}(A)$. Then there exists $q \in S_x^{\alpha}(A)$ such that

$$\mu^*(q, y, r) \geq \alpha \quad (>0 \text{ if } \alpha = 0).$$

And $q \in S_x^{\alpha}(A)$ if and only if there exists $p \in A$ such that

$$\mu^*(p, x, q) \geq \alpha \quad (>0 \text{ if } \alpha = 0).$$

Thus

$$\mu^*(p, xy, r) \geq \mu^*(p, x, q) \wedge \mu^*(q, y, r) \geq \alpha \quad (>0 \text{ if } \alpha = 0),$$

which implies that $S_y^{\alpha} \circ S_x^{\alpha}(A) \subset S_{xy}^{\alpha}(A)$ for $A \subset Q$. Conversely, if $r \in S_{xy}^{\alpha}(A)$ then there exists $p \in A$ such that

$$\mu^*(p, xy, r) \geq \alpha \quad (>0 \text{ if } \alpha = 0),$$

and

$$\mu^*(p, xy, r) = \bigvee\{\mu^*(p, x, q) \wedge \mu^*(q, y, r) | q \in Q\}.$$

Thus for some $q \in Q$,

$$\mu^*(p, x, q) \wedge \mu^*(q, y, r) \geq \alpha \quad (>0 \text{ if } \alpha = 0).$$

It follows that $r \in S_y^\alpha(S_x^\alpha(A))$. Therefore $S_y^\alpha(S_x^\alpha(A)) = S_{xy}^\alpha(A)$.

Definition 2.7. Let $M_1 = (Q_1, X_1, \mu_1)$ and $M_2 = (Q_2, X_2, \mu_2)$ be two fuzzy finite state machines. A pair (f, g) of mappings, $f: Q_1 \to Q_2$ and $g: X_1 \to X_2$, is called a fuzzy homomorphism, written by $(f, g): M_1 \to M_2$, if $f(S^{\alpha_x}(p)) \subset S^\alpha_{g(x)}(f(p))$, i.e., for every $q \in S_x^\alpha(p)$, $f(q) \in S^\alpha_{g(x)}(f(p)), p, q \in Q_1, x \in X_1, 0 \le \alpha \le 1$. And this fuzzy homomorphism $(f, g): M_1 \to M_2$ is called strong if for every $f(q) \in S^\alpha_{g(x)}(f(p))$, there exists $q' \in f^{-1}(f(q))$ such that $q' \in S_x^\alpha(p)$, $p, q, q' \in Q_1$, $x \in X_1$, $\forall \alpha \in [0, 1]$. In particular, if $X_1 = X_2$ and g is the identity map, then we simply write $f: M_1 \to M_2$ and say that f is a (strong) fuzzy homomorphism accordingly.

This definition by the maps S_x^α is in fact the equivalent form of those defined in [4] by the following proposition.

Proposition 2.8. Let $M_1 = (Q_1, X_1, \mu_1)$ and $M_2 = (Q_2, X_2, \mu_2)$ be fuzzy finite state machines and let $(f, g): M_1 \to M_2$. Then

(1) (f, g) is a fuzzy homomorphism if and only if $\mu_1(p, x, q) \le \mu_2(f(p), g(x), f(q))$, $\forall p, q \in Q_1$ and $\forall x \in X_1$.

(2) (f, g) is a strong fuzzy homomorphism if and only if $\mu_2(f(p), g(x), f(q)) = \bigvee\{\mu_1(p, x, t) | t \in Q_1, f(t) = f(q)\}$, $\forall p, q \in Q_1, \forall x \in X_1$.

Proof. (1) can be easily obtained from the definition of fuzzy homomorphism.

(2) Suppose that $(f, g): M_1 \to M_2$ is a strong fuzzy homomorphism. Let $p, q \in Q_1$ and $x \in X_1$, and set $\alpha = \mu_2(f(p), g(x), f(q))$. Then $f(q) \in S^\alpha_{g(x)}(f(p))$, which implies by hypothesis that there exists $q' \in f^{-1}(f(q))$ such that $q' \in S_x^\alpha(p)$. Thus $\mu_1(p, x, q') \ge \alpha$ (>0 if $\alpha = 0$), $f(q) = f(q')$. But since (f, g) is a fuzzy homomorphism, $\mu_1(p, x, r)$ must be less than or equal to α for $\forall r \in f^{-1}(f(q))$. Hence

$$\mu_2(f(p), g(x), f(q)) = \bigvee\{\mu_1(p, x, t) \mid t \in Q_1, f(t) = f(q)\}.$$

The converse can be easily derived.

Let $f: M_1 \to M_2$, and let δ be a fuzzy subset of Q_1. Then the fuzzy subset $f(\delta)$ of Q_2 is defined by $(f(\delta))_\alpha = f(\delta_\alpha)$, $0 \le \alpha \le 1$. We note that if $q \in Q_2$, then

$$f(\delta)(q) = \begin{cases} \bigvee\{\delta(q') | q' \in Q_1, f(q') = q\} & \text{if } f^{-1}(q) \ne \phi, \\ 0 & \text{if } f^{-1}(q) = \phi. \end{cases}$$

On the other hand, let λ be a fuzzy subset of Q_2. We define a fuzzy subset $f^{-1}(\lambda)$ of Q_1 by $(f^{-1}(\lambda))_\alpha = f^{-1}(\lambda_\alpha)$, $0 \le \alpha \le 1$. Then $f^{-1}(\lambda)(p) = \lambda(f(p))$, $\forall p \in Q_1$.

Theorem 2.9. Let $M_1 = (Q_1, X_1, \mu_1)$ and $M_2 = (Q_2, X_2, \mu_2)$ be fuzzy finite state machines and let $(f, g): M_1 \to M_2$ be a fuzzy homomorphism. If $N_2 = (Q_2, \lambda, X_2, \mu_2)$ is a fuzzy subsystem of M_2, then $N_1 = (Q_1, f^{-1}(\lambda), X_1, \mu_1)$ is a fuzzy subsystem of M_1.

Proof. Let λ be a fuzzy subset of Q_2. If $f^{-1}(\lambda) = \phi$, then (Q_1, ϕ, X_1, μ_1) is a trivial fuzzy subsystem. Suppose $f^{-1}(\lambda) \ne \phi$, and let $p \in S^\alpha(f^{-1}(\lambda_\alpha))$, $0 \le \alpha \le 1$. Then there exists $p' \in Q_1$ such that $p \in S_x^\alpha(p')$ and $f(p') \in \lambda_\alpha$ for $x \in X_1^*$. Since $(f, g): M_1 \to M_2$ is a fuzzy homomorphism, $f(p) \in S^\alpha_{g(x)}(f(p')) \subset S^\alpha_{g(x)}(\lambda_\alpha)$. The fact that N_2 is a fuzzy subsystem of M_2 implies $f(p) \in \lambda_\alpha$, that is, $p \in f^{-1}(\lambda_\alpha)$. Hence $N_1 = (Q_1, f^{-1}(\lambda), X_1, \mu_1)$ is a fuzzy subsystem of M_1.

Theorem 2.10 Let $M_1 = (Q_1, X_1, \mu_1)$ and $M_2 = (Q_2, X_2, \mu_2)$ be fuzzy finite state machines and let $(f, g): M_1 \to M_2$ be a strong fuzzy homomorphism, where f, g are onto. If $N_1 = (Q_1, \delta, X_1, \mu_1)$ is a fuzzy subsystem of M_1, then $N_2 = (Q_2, f(\delta), X_2, \mu_2)$ is a fuzzy subsystem of M_2.

Proof. Suppose δ is a nonempty subset of Q_1, and let $r \in S^\alpha(f(\delta_\alpha))$. Then there exists some $p \in \delta_\alpha$ such that $r \in S_y^\alpha(f(p))$ for $y \in X_2^*$. Since f and g are onto, there exist $q \in Q_1$, $x \in X_1^*$ such that $f(q) = r$ and $y = f(x)$, thus $f(q) \in S^\alpha_{f(x)}(f(p))$. Strong fuzzy homomorphism of (f, g) implies $q' \in S_x^\alpha(p)$, $f(q') \in f(q)$ for some $q' \in Q_1$. By the definition of a fuzzy subsystem N_1 of M_1, $q' \in \delta_\alpha$, which implies $r = f(q) = f(q') \in f(\delta_\alpha)$. Hence $N_2 = (Q_2, f(\delta), X_2, \mu_2)$ is a fuzzy subsystem of M_2.

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