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Some Fuzzy Closed Sets and Fuzzy Approximately Continuous Mappings

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ABSTRACT

First, we find the characterization of fg-closure of a fuzzy set. Second, we study some properties of frg-closeds and fg-continuous mappings. Finaly, we introduce the concept of a fuzzy approximately continuous mapping and study its properties.

Key Words : fg-closed set, fg-open set, frg-closed set, frg-open set, fg-continuous, fa-continuous.

0. Introduction

The concept of fuzzy sets and fuzzy set operations was first introduced by Zadeh in his classic paper[8]. Subsequently, several authors including Zadeh have discussed various application of the theory and aspects of fuzzy sets. Chang [3], Pu Pao-Ming and Liu Yin-Ming [4], Wong[5,6] and others applied some basic concepts of general topology to fuzzy sets and developed a theory of fuzzy topological spaces. In particular, Azad[1] generalized the concept of regularly open sets to fuzzy topological space and also generalized almost continuous mappings. G. Balasubramanian and P. Sundaram introduced the concepts of fuzzy generalized closed sets and fuzzy generalized continuous mappings and investigated their properties.

The purpose of this paper is to introduce and study fuzzy approximately continuous mappings.

In section 1, we list some concept and their results used in later sections. Section 2 is devoted to fuzzy generalized closed sets and their properties.

In section 3, we introduce the concept of a fuzzy regular generalized closed set and study its property. Section 4 is devoted to fuzzy generalized continuous mappings and their properties. In section 5, we introduce the concept of a fuzzy approximately continuous mapping and study its properties.

1. Preliminaries

Throughout this paper, we will denote the unit interval $[0,1]$ of the real line as I. X, Y and $Z, \cdots$ will denote sets. For X, I^X denotes the collection of all the mappings from X into I. Each member of I^X is called a **fuzzy subset** of X or a **fuzzy set** in X (See [8]). The concepts of fuzzy points and their properties refer to [4,6]. We will denote the collection of all fuzzy points in a set X as $F_p(X)$.

Now in order to use in later sections, we will list some concepts and their properties :

Definition 1.1[4]. Let $A, B \in I^X$ and let $x_\lambda \in F_p(X)$. Then :

(1) A is said to be **quasi-coincident with** B, denoted by AqB, if there exists $x \in X$ such that $A(x) > B^c(x)$ or $A(x) + B(x) > 1$.

(2) x_λ is said to be **quasi-coincident with** A, denoted by $x_\lambda qA$, if $\lambda > A^c(x)$ or $\lambda + A(x) > 1$.

If A is not quasi-coincident with B, then we write as $A\bar{q}B$.

Result 1.A[4]. Let $A, B \in I^X$. Then $A \subset B$ if and only if $A\bar{q}B^c$. Particularly, $x_\lambda \in A$ if and only if $x_\lambda \bar{q} A^c$.

Result 1.B[3,7]. Let $f: X \to Y$ be a mapping. Then :

(1) $f^{-1}(B^c) = [f^{-1}(B)]^c$ for each $B \in I^Y$.

(2) $f(A^c) \supset [f(A)]^c$ for each $A \in I^X$.
In particular, if f is bijective, then $[f(A)]^c = f(A^c)$.

(3) If $B_1 \subset B_2$, then $f^{-1}(B_1) \subset f^{-1}(B_2)$, where $B_1, B_2 \in I^Y$.

(4) If $A_1 \subset A_2$, then $f(A_1) \subset f(A_2)$, where $A_1, A_2 \in I^X$.

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(5) $B \supset f(f^{-1}(B))$ for each $B \in I^Y$.
In particular, if f is surjective, then $f(f^{-1}(B)) = B$.

(6) $A \subset f^{-1}(f(A))$ for each $A \in I^X$.
In particular, if f is injective, then $f^{-1}(f(A)) = A$.

(7) Let $g: Y \to Z$ be any mapping. Then $(g \circ f)^{-1}(C) = f^{-1}(g^{-1}(C))$ for each $C \in I^Z$.

(8) $f^{-1}(\bigcup_{\alpha \in \Lambda} B_\alpha) = \bigcup_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ and $f^{-1}(\bigcap_{\alpha \in \Lambda} B_\alpha) = \bigcap_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ where $\{B_\alpha\}_{\alpha \in \Lambda} \subset I^Y$.

The concepts of a fuzzy topological space (briefly, fts), a fuzzy continuous mapping(briefly, F-continuous mapping), a fuzzy closed mapping(briefly, F-closed mapping) and a fuzzy open mapping(briefly, F-open mapping) refer to [1,3].

For a fts X, $FO(X)$ [resp. $FC(X)$] denotes the collection of all fuzzy open[resp. fuzzy closed] sets in X.

Let X be a fts and let $A \in I^X$. Then clA and $intA$ denote the closure and interior of A in X, respectively.

Result 1.C[4]. Let X be a fts, let $A \in I^X$ and let $x_\lambda \in F_p(X)$. Then :

(1) $x_\lambda \in intA$ if and only if x_λ has a fuzzy neighborhood contained in A.

(2) $x_\lambda \in clA$ if and only if each q-neighborhood of x_λ is quasi-coincident with A.

(3) $(intA)^c = clA^c$ and $(clA)^c = intA^c$.

Definition 1.2[5]. Let $\{(x_\alpha, T_\alpha)\}_{\alpha \in \Lambda}$ be a family of fts's, let $X = \prod_{\alpha \in \Lambda} X_\alpha$, let π_α the projection from X into X_α for each $\alpha \in \Lambda$, and let $\rho = \{\pi_\alpha^{-1}(B) : B \in T_\alpha, \alpha \in \Lambda\}$. Let $\mathcal{B}$ be the family of all the finite intersection of members of ρ and let T the family of all the unions of member of $\mathcal{B}$. Then T is a fuzzy topology on X with $\mathcal{B}$ as a base and ρ a subbase.

In this case, T is called the **fuzzy product topology** on X and (X, T) is called the **product fuzzy topological space**(briefly, **product fts**).

Result 1.D[5]. Let (X, T) be the product fts of the family $\{(X_\alpha, T_\alpha)\}_{\alpha \in \Lambda}$ of fts's. Then :

(1) For each $\alpha \in \Lambda$, the projection π_α is F-continuous.

(2) T is the smallest fuzzy topology on X such that (1) is true.

(3) Let (Y, T_Y) be a fts and let $f: Y \to X$ a mapping. Then f is F-continuous if and only if $\pi_\alpha \circ f$ is F-continuous for each $\alpha \in \Lambda$.

Definition 1.3[1]. Let X be a fts and let $A \in I^X$. Then A is called :

(1) **a fuzzy regularly open**(briefly, **fr-open**) set in X if $A = int(clA)$.

(2) **a fuzzy regularly closed**(briefly, **fr-closed**)set in X if $A = cl(intA)$.

We will denote the family of all the fr-open(resp. fr-closed) sets in X as $FRO(X)$[resp. $FRC(X)$].

It is clear that $\emptyset \in FRC(X) \cap FRO(X)$ and $X \in FRC(X) \cap FRO(X)$.

Result 1.E[1]. We have the following results :

(1) $A \in FRC(X)$ if and only if $A^c \in FRO(X)$.

(2) $FRO(X) \subset FO(X)$ and $FRC(X) \subset FC(X)$.

But the converse inclusions are not true, as shown by Example 1.4.

Example 1.4. Let $X = \{ㄱ, ㄴ\}$ and let $T = \{\emptyset, O_1, O_2, O_3, O_4, X\}$, where

$$O_1 = \{(ㄱ, 0.3), (ㄴ, 0.6)\},$$
$$O_2 = \{(ㄱ, 0.5), (ㄴ, 0.4)\},$$
$$O_3 = \{(ㄱ, 0.3), (ㄴ, 0.4)\},$$
$$O_4 = \{(ㄱ, 0.5), (ㄴ, 0.6)\}.$$

Then clearly, $O_1 \in T$ but $O_1 \notin FRO(X)$ and $O_1^c \in FC(X)$ but $O_1^c \notin FRC(X)$.

2. Generalized closed and open sets.

Definition 2.1[2]. A fuzzy A in a fts $(X, \mathfrak{T})$ is said to be a **fuzzy generalized closed**(briefly, **fg-closed**) set if for each $O \in \mathfrak{T}$ with $A \subset O$, $clA \subset O$.

We will denote the family of all the fg-closed sets in X as $FGC(X)$. Then it is clear that $\emptyset \in FGC(X)$ and $X \in FGC(X)$.

Example 2.2. Let $X = \{ㄱ, ㄴ, ㄷ\}$ and let $T = \{\emptyset, O_1, O_2, O_3, O_4, X\}$, where

$$O_1 = \{(ㄱ, 0.5), (ㄴ, 0.6), (ㄷ, 0.8)\},$$
$$O_2 = \{(ㄱ, 0.4), (ㄴ, 0.7), (ㄷ, 0.2)\},$$
$$O_3 = \{(ㄱ, 0.4), (ㄴ, 0.6), (ㄷ, 0.2)\},$$
$$O_4 = \{(ㄱ, 0.5), (ㄴ, 0.7), (ㄷ, 0.8)\}.$$

Let $A = \{(ㄱ, 0.5), (ㄴ, 0.2), (ㄷ, 0.1)\}$. Then (X, T) is a fuzzy toplogical space and A is a fg-closed fuzzy set in X.

From Definition 2.1, we can easily obtain the following result :

Proposition 2.3. Let $FC(X) \subset FGC(X)$. But the converse inclusion is not true, as shown by Example 2.4.

Example 2.4. Let $X = \{ㄱ, ㄴ\}$ and let $T = \{\emptyset, O, X\}$, where $O = \{(ㄱ, 0.4), (ㄴ, 0.6)\}$. Let $A = \{(ㄱ, 0.2),$

(ㄴ, 0.8)}. Then clearly $A \in FGC(X)$, but $A \notin FC(X)$.

Result 2.A[2]. If $A, B \in FGC(X)$, then $A \cup B \in FGC(X)$.

But the intersection of two fg-closed sets is generally not an fg-closed set(See Example 2.3 in [2]).

Definition 2.5[2]. A fuzzy set A in a fts X is called a **fuzzy generalized open**(briefly, **fg-open**) set if $A^c \in FGC(X)$.

We will denote the family of all the fg-open sets in X as $FGO(X)$. It is clear that $\emptyset, X \in FGO(X)$.

Result 2.B[2]. $A \in FGO(X)$ if and only if for each $F \in FC(X)$ such that $F \subset A$, $F \subset intA$.

From Definition 2.5, we can easily obtain the following result :

Proposition 2.6. $FO(X) \subset FGO(X)$. However, the converse inclusion is not true, as shown by Example 2.7.

Example 2.7. Let $X = \{ㄱ, ㄴ\}$ and let $T = \{\emptyset, O, X\}$, where $O = \{(ㄱ, 0.7), (ㄴ, 0.8)\}$. Let $A = \{(ㄱ, 0.8), (ㄴ, 0.9)\}$. Then clearly $A \in FGO(X)$, but $A \notin T$.

Result 2.C[2]. In a fts (X, T), $T = T^c$(the fuzzy closed sets) if and only if $A \in FGC(X)$ for each $A \in I^X$.

Corollary 2.8. In a fts (X, T), $T = T^c$ if and only if $A \in FGO(X)$ for each $A \in I^X$.

Theorem 2.9. Let X be an fts and let A a crisp subset of X. If $B \in FGC(A)$ and $A \in FGC(X)$, then $B \in FGC(X)$.

(proof) Let $O \in FO(X)$ such that $B \subset O$. Then $B \subset A \cap O$. Since $B \in FGC(A)$, $cl_A(B) \subset A \cap O$, where $cl_A(B)$ denotes the closure of B in A. Thus $A \cap clB \subset A \cap B$. So $A \subset O \cup (clB)^c$. Since $A \in FGC(X)$, $clA \subset O \cup (clB)^c$. Thus $clB \subset clA \subset O \cup (clB)^c$. So $clB \subset O$. Hence $B \in FGC(X)$.

Definition 2.10[2]. Let A be a fuzzy set in a fts X. Then the intersection of all fg-closed sets containing A is called the **fuzzy generalization closure** (briefly, **fg-closure**)of A and is denoted by cl^*A. Hence

$$cl^*A = \cap\{F \in I^X : F \in FGC(X) \text{ and } A \subset F\}.$$

It is clear that $A \subset cl^*(A) \subset clA$. We shall assume that $cl^*A \in FGC(X)$. It is easily proved that $A \in FGC(X)$ if and only if $cl^*A \in FGC(X)$.

Definition 2.11. Let x_λ be a fuzzy point of X and let $N \in I^X$. Then N is a called a **fuzzy generalizad quasi-neighborhood**(in short, **fgq-nbd**)of x_λ in X if there exists $O \in FGO(X)$ such that $x_\lambda q O \subset N$.

Theorem 2.12. Let $A \in I^X$. Then $x_\lambda \in cl^*A$ if and only if for each fgq-nbd N_{x_λ} of x_λ in X, $N_{x_\lambda} q A$.

(proof) ($\Rightarrow$) : Suppose $x_\lambda \in cl^*A$. Assume that necessary condition does not hold. Then there exists an fgq-nbd N of x_λ in X such that $N \bar{q} A$. Since N is an fgq-nbd of x_λ, by Definition 2.11, there exists $O \in FGO(X)$ such that $x_\lambda q O \subset N$. Thus $\lambda + O(x) > 1$ and $O(x) \le N(x)$. Since $N \bar{q} A$, by Result 1.A, $N \subset A^c$ and thus $N(x) + A(x) \le 1$. Thus $O(x) + A(x) \le N(x) + A(x)$. So $O(x) + A(x) \le 1$ and $A \bar{q} O$. Hence, by Result 1.A, $A \subset O^c$. By Definition 2.5, $O^c \in FGC(X)$. Thus $cl^*A \subset O^c$. Since $\lambda + O(x) > 1$, $x_\lambda \notin O^c$. So $x_\lambda \notin cl^*A$. This is contrary to the hypothesis.

($\Leftarrow$) : Suppose the necessary condition holds. Assume that $x_\lambda \notin cl^*A$. By Definition 2.10, there exists $F \in FGC(X)$ such that $A \subset F$ and $x_\lambda \notin F$. Thus, by Result 1.A, $x_\lambda q F^c$ and $F^c \in FGO(X)$. So F^c is an fgq-nbd of x_λ in X. By the hypothesis, $F^c q A$. But $F^c \bar{q} A$. This is a contradiction.

3. Fuzzy regular generalized closed and open sets

Definition 3.1. Let X be a fts and let $A \in I^X$. Then A is called :

(1) a **fuzzy regular generalized closed**(briefly, **frg-closed**) set in X if for each $U \in FRO(X)$ with $A \subset U$, $clA \subset U$.

(2) a **fuzzy regular generalized open**(briefly, **frg-open**) **set** in X if A^c is frg-closed set in X.

We will denote the family of all the frg-closed [resp. open] set as $FRGC(X)$ [resp. $FRGO(X)$].

It is clear that $\emptyset$, $X \in FRGC(X)$ [resp. $FRGO(X)$]

From Definitions 2.1, 2.5 and 3.1, we can easily obtain the following result :

Proposition 3.2. $FGC(X) \subset FRGC(X)$ and $FGO(X) \subset FRGO(X)$. But the converse inclusions are not true, as shown by Example 3.3.

Example 3.3. In Example 1.4, let $A = \{(ㄱ, 0.3), (ㄴ, 0.4)\}$. Then clearly $A \in FRGC(X)$ but $A \notin FGC(X)$ and $A^c \in FRGC(X)$ but $A^c \notin FGO(X)$.

The following are easily obtained from Propositions 2.3,2.6,3.2 and Result 1.E :

Corollarly 3.4. Let X be a fts. Then :

(1) $FRC(X) \subset FC(X) \subset FGC(X) \subset FRGC(X)$.

(2) $FRO(X) \subset FO(X) \subset FGO(X) \subset FRGO(X)$.

Theorem 3.5. Let X be a fts and let $A \in I^X$. Then $A \in FRGO(X)$ if and only if for each $F \in FRC(X)$ with $F \subset A$, $F \subset intA$.

(proof)($\Rightarrow$) : Suppose $A \in FRGO(X)$ and let $F \in FRC(X)$ such that $F \subset A$. Then clearly $F^c \in FRO(X)$ and $A^c \subset F^c$. By Definition 3.1, $clA^c \subset F^c$. Thus $F \subset (clA^c)^c$. But, by Result 1.C(3), $(cl A^c)^c = int(A^c)^c = intA$. Hence $F \subset intA$.

($\Leftarrow$): Suppose the necessary condition holds. Let $U \in FRO(X)$ such that $A^c \subset U$. Then clearly $U^c \subset A$ and $U^c \in FRC(X)$. By the hypothesis, $U^c \subset intA$. Thus $(intA)^c \subset U$. But, by Result 1.C(3), $(intA)^c = clA^c$. So $clA^c \subset U$ and thus $A^c \in FRGC(X)$. Hence $A \in FRGO(X)$.

From Corollary 3.4 we obtain the following result immediately :

Theorem 3.6. Let $A, B \in FRGC(X)$. Then $A \cup B \in FRGC(X)$.

(proof) Let $U \in FRO(X)$ such that $A \cup B \subset U$. Then $A \subset U$ and $B \subset U$. Since $A, B \in FRGC(X)$, $clA \subset U$ and $clB \subset U$. But $clA \cup clB = cl(A \cup B)$. So $cl(A \cup B) \subset U$. Hence $A \cup B \in FRGC(X)$.

4. Fuzzy generalized continuous mappings.

Definition 4.1[2]. Let X and Y be fts's. Then a mapping $f: X \to Y$ is said to be **fuzzy generalized continuous** (briefly, **fg-continuous**) if the inverse image of every fuzzy closed set in Y is an fg-closed set in X.

Result 4.A[2]. Every F-continuous mapping is fg-continuous. However, the converse is not true(See Example 3.3 in [2]).

Result 4.B[2]. A mapping $f: X \to Y$ is fg-continuous if and only if $f^{-1}(U) \in FGO(X)$ for each $U \in FO(Y)$.

Theorem 4.2. Let $\{X_\alpha : \alpha \in \Lambda\}$ be a family of fts's and let X be a fts. If $f: X \to \prod_{\alpha \in \Lambda} X_\alpha$ is an fg-continuous mapping, then $\pi_\alpha \circ f: X \to X_\alpha$ is an fg-continuous for each $\alpha \in \Lambda$, where π_α is the projection of $\prod_{\alpha \in \Lambda} X_\alpha$ onto X_α.

(proof) For each $\alpha \in \Lambda$, let U_α be an arbitrary fuzzy open set in X_α. Since π_α is F-continuous by Result 1.D(1), $\pi_\alpha^{-1}(U_\alpha) \in FO(\prod_{\alpha \in \Lambda} X_\alpha)$. By the hypothesis and Result 4.B, $f^{-1}(\pi_\alpha^{-1}(U_\alpha)) = (\pi_\alpha \circ f)^{-1}(U_\alpha) \in FGO(X)$. Therefore $\pi_\alpha \circ f$ is fg-continuous.

Theorem 4.3. Let X and Y be fts's and let $f: X \to Y$ be fg-continuous and F-closed. If $G \in FGO(Y)$[resp. $FGC(Y)$], then $f^{-1}(G) \in FGO(X)$[resp. $FGC(X)$].

(proof) Suppose $G \in FGO(Y)$. Let $F \subset f^{-1}(G)$, where $F \in FC(X)$. Then $f(F) \subset G$ and $f(F) \in FC(Y)$, since f is F-closed. Thus, by Result 2.B, $f(F) \subset intG$. So $F \subset f^{-1}(intG)$. Since f is fg-continuous and $intG \in FO(Y)$, $F \subset intf^{-1}(intG) \subset intf^{-1}(G)$. Hence $f^{-1}(G) \in FGO(X)$.

By taking complements, we can show that if G is fg-closed in Y, then $f^{-1}(G) \in FGC(X)$.

From Theorem 4.3, we optain the following result immediately :

Corollary 4.4. Let X, Y and Z be fts's. If $f: X \to Y$ is an F-closed and fg-continuous mappoing, and $g: Y \to Z$ is an fg-continuous mapping, then $g \circ f: X \to Z$ is fg-continuous.

5. Fuzzy approximately continuous mappings.

Definition 5.1. Let X and Y be fts's. Then a mapping $f: X \to Y$ is said to be:

(1) **fuzzy approximately closed**(or briefly **fa-closed**) if $f(F) \subset intA$ whenever F is a fuzzy closed set in X, A is an fg-open set in Y and $f(F) \subset A$.

(2) **fuzzy approximately continuous**(or briefly **fa-continuous**) if $clA \subset f^{-1}(V)$ whenever V is a fuzzy open set in Y, A is an fg-closed in X and $A \subset f^{-1}(V)$.

From Definitions of F-continuous, F-closed and Definition 5.1, we can easily obtain the following results :

Proposition 5.2. Every F-continuous mapping is fa-continuous. Also every F-closed mapping is fa-closed. However the following Example 5.3 shows that converses do not hold.

Example 5.3. Let $X = \{a, b\}$ and let $T = \{\emptyset, \{(a, \lambda), (b, 0)\}, \{(a, 0), (b, \mu)\}, \{(a, \lambda), (b, \mu)\}, X\}$, where $\lambda \neq \mu$ and $0 < \lambda, \mu < 1$. Then clearly T is a fuzzy topology on X. Let $f: X \to X$ be the mapping defined by $f(a) = b$, and $f(b) = a$. Then f is neither F-closed nor F-continuous. However, since the image of each fuzzy closed set is a fuzzy open set in X, f is fa-closed. Also since the inverse image of each fuzzy open set is fuzzy closed in X, f is fa-continuous.

The proof of the following result is a straightforward argument using complements and is omitted.

Theorem 5.4. Let $f: X \to Y$ be bijective. Then f is fa-closed if and only if f is fa-continuous.

Theorem 5.5. If $f: X \to Y$ is fg-continuous and fa-closed, then $f^{-1}(A)$ is an fg-closed[resp. fg-open] set in X whenever A is an fg-closed[resp. fg-open] set in Y.

(proof) Let A be an fg-closed set in Y and let U be any fuzzy open set in X such that $f^{-1}(A) \subset U$. Then $U^c \subset [f^{-1}(A)]^c = f^{-1}(A^c)$ or $f(U^c) \subset A^c$. Since f is fa-closed, $f(U^c) \subset int(A^c) = (clA)^c$. Thus $U^c \subset f^{-1}((clA)^c) = [f^{-1}(clA)]^c$ and hence $f^{-1}(clA) \subset U$. Since f is fg-continuous and $clA \in FC(Y)$, $f^{-1}(clA) \in FGC(X)$. Thus $clf^{-1}(A) \subset clf^{-1}(clA) \subset U$. Hence $f^{-1}(A)$ is an fg-closed set in X.

A similar argument shows that the inverse images of fg-open sets are an fg-open set.

Theorem 5.6. If $f: X \to Y$ is fa-continuous and F-closed, then $f(A)$ is an fg-closed set in Y whenever A is an fg-closed set in X.

(proof) Let A be any fg-closed set in X and let V be any fuzzy open set in Y such that $f(A) \subset V$. Then $A \subset f^{-1}(V)$. Since f is fa-continuous and A is an fg-closed set in X, $clA \subset f^{-1}(V)$. Thus $f(clA) \subset V$. Since f is F-closed, $f(clA) \in FC(Y)$. So $clf(A) \subset clf(clA) = f(clA) \subset V$, and hence $clf(A) \subset V$. Therefore $f(A)$ is an fg-closed fuzzy set in Y.

Theorem 5.7. Let $f: X \to Y$ be a mapping for which $f(F) \in FO(Y)$ for each $F \in FC(X)$. Then f is fa-closed.

(proof) It is clear by Definition 5.1.

Theorem 5.8. Let $f: X \to Y$ be a mapping and let $FO(Y) = FC(Y)$. Then f is fa-closed if and only if $f(F) \in FO(Y)$ for each $F \in FC(X)$.

(proof)(⇒): Suppose f is fa-closed and let $F \in FC(X)$. Then, by Corollary 2.8, $f(F) \in FGO(X)$. Since f is fa-closed, $f(F) \subset intf(F)$. Hence $f(F) \in FO(Y)$.

(⇐): Suppose the necessary condition holds. Let $F \in FC(X)$ and let $A \in FGO(Y)$ such that $f(F) \subset A$. By the hypothesis, $f(F) \in FO(Y)$. Thus $f(F) = intf(F) \subset intA$. Hence f is fa-closed.

Corollary 5.9. Let $FO(Y) = FC(Y)$. Then a mapping $f: X \to Y$ is fa-closed if and only if it is F-closed.

The proofs of the following results for fa-continuous mappings are alalogous and are omitted.

Theorem 5.10. Let $FO(Y) = FC(Y)$. Then a mapping $f: X \to Y$ is fa-continuous if and only if $f^{-1}(V) \in FC(X)$ for each $V \in FO(Y)$.

Corollary 5.11. Let $FO(Y) = FC(Y)$. Then a mapping $f: X \to Y$ is fa-continuous if and only if it is F-continuous.

Theorem 5.12. Let $f: X \to Y$ be a mapping such that $f^{-1}(V) \in FC(X)$ for each $V \in FO(Y)$. Then f is fa-continuous.

(proof) It is clear by Definition 5.1.

Since the identity mapping on any fuzzy topological space is both fa-continuous and fa-closed, it is clear that the converse of Theorems 5.7, and 5.12, do not hold.

Example 5.13. Let X be an indiscrete fts and let B a crisp nonempty proper subset. Then the identity mapping $id: X \to X$ is fa-closed.

Since $FO(X) = FC(X)$, by Corollary 2.8, $id(B) \in FGO(X)$. On the other hand, B is F-closed in B and $intid(B) = intB = \emptyset$. Thus $id(B) \subset intid(B)$. Hence $id|_B: B \to X$ is not fa-closed.

It follows easily from the definition that the restriction of an fa-closed mapping to a crisp closed set is fa-closed.

Theorem 5.14. Let (X, T) be an fts and let B a crisp subset of X. If $f: X \to Y$ is fa-continuous and $B \in FGC(X)$, then $f|_B: B \to Y$ is fa-continuous.

(proof) Let $V \in FO(Y)$ and let $A \in FGC(B)$ such that $A \subset (f|_B)^{-1}(V)$. Then $A \subset f^{-1}(V) \cap B$. By Theorem 2.9, $A \in FGC(X)$. Since f is fa-continuous, $clA \subset f^{-1}(V)$. Thus $clA \cap B \subset f^{-1}(V) \cap B$. So $cl_B A \subset (f|_B)^{-1}(V)$. Hence $f|_B: B \to Y$ is fa-continuous.

Compositions of fa-continuous(or fa-closed) mappings are not in general fa-continuous (or fa-closed). However the following results hold:

Theorem 5.15. If $f: X \to Y$ is F-closed and $g: Y \to Z$ is fa-closed, then $g \circ f: X \to Z$ is fa-closed.

(proof) Let F be any fuzzy closed set in X and let A be an fg-open set in Z such that $g \circ f(F) \subset A$. Since f is F-closed, $f(F) \in FC(Y)$. Since g is fa-closed, $g(f(F)) \subset intA$. Hence $g \circ f$ is fa-closed.

Theorem 5.16. If $f: X \to Y$ is fa-closed and $g: Y \to Z$ is F-open and inversely preserves fg-open sets, then $g \circ f: X \to Z$ is fa-closed.

(proof) Let F be a fuzzy closed set in X and let A be an fg-open set in Z such that $g \circ f(F) \subset A$. Then $f(F) \subset g^{-1}(A)$. Since $g^{-1}(A) \in FGO(Y)$ and f is fa-closed, $f(F) \subset intg^{-1}(A)$. Then $g \circ f(F) = g(f(F)) \subset g(intg^{-1}(A)) \subset intg(g^{-1}(A)) \subset intA$. Hence $g \circ f$ is fa-closed.

Theorem 5.17. If $f: X \to Y$ is fa-continuous and $g: Y \to Z$ is F-continuous, then $g \circ f: X \to Z$ is fa-continuous.

(proof) Let A be any fg-closed set in X and let V be a fuzzy open set in Z such that $A \subset (g \circ f)^{-1}(V)$. Since g is F-continuous, $g^{-1}(V) \in FO(Y)$. Since f is fa-continuous, $clA \subset f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Hence $g \circ f$ is fa-continuous.

Corollary 5.18. Let $f_\alpha : X \to Y_\alpha$ be a mapping for each $\alpha \in \Lambda$ and let $f: X \to \prod_{\alpha \in \Lambda} Y_\alpha$ be the product mapping given by $f(x) = (f_\alpha(x))$. If f is fa- continuous, then f_α is fa-continuous for each $\alpha \in \Lambda$.

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