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Some Fuzzy Continuous Mappings and Fuzzy Mildly Normal Spaces

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Abstract

We introduce the new concepts of some fuzzy continuous and closed mappings and study their properties. Also we investigate the properties of fuzzy mildly normal spaces.

Key Words : fr-open, fr-closed, fg-closed, fg-open, frg-closed, frg-open, falg-continuous, falrg-continuous, fal-continuous, fg-continuous, fuzzy normal space, fuzzy mildly normal space.

0. Introduction

The study of fuzzy sets was initiated with the famous paper of Zadeh[17] in 1965, and thereafter Chang[5] paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Almost all the central concepts of general topology have been tried for extension to the fuzzy situation by different mathematicians in a more or less satisfactory manner, sometimes in different and independent ways. One way of the recent directions is the generalization of various continuous mappings to fuzzy setting, e.g. the papers[1-4,9-15] may be referred in this connection.

In this paper we introduce the new concepts of some fuzzy continuous and closed mappings and study their properties. Also we investigate the properties of fuzzy mildy normal spaces introduced by Singal and Rajvanshi[7,13].

1. Preliminaries

For a set X, I^X denotes the collection of all the mappings from X into I, where $I=[0,1]$. Each member of I^X is called a fuzzy subset of X or a fuzzy set in X (See[17]). The concept of fuzzy points and their properties refer to [8,15]. We will denote the collection of all fuzzy points in a set X as $F_p(X)$.

Now we will list some concepts and their properties :

Result 1.A[11]. Let $f: X \to Y$ be a mapping and let $A \in I^X, B \in I^Y$. Then $f^{-1}(B) \subset A$ if and only if $B \subset [f(A^c)]^c$.

Result 1.B[5,16]. Let $f: X \to Y$ be a mapping. Then :

(1) $f^{-1}(B^c) = [f^{-1}(B)]^c$ for each $B \in I^Y$.

(2) $f(A^c) \supset [f(A)]^c$ for each $A \in I^X$.
In particular, if f is bijective, then $[f(A)]^c = f(A^c)$.

(3) If $B_1 \subset B_2$, then $f^{-1}(B_1) \subset f^{-1}(B_2)$, where $B_1, B_2 \in I^Y$.

(4) If $A_1 \subset A_2$, then $f(A_1) \subset f(A_2)$, where $A_1, A_2 \in I^X$.

(5) $B \supset f(f^{-1}(B))$ for each $B \in I^Y$.
In particular, if f is surjective, then $f(f^{-1}(B)) = B$.

(6) $A \subset f^{-1}(f(A))$ for each $A \in I^X$.
In particular, if f is injective, then $f^{-1}(f(A)) = A$.

(7) Let $g: Y \to Z$ be any mapping. Then $(g \circ f)^{-1}(C) = f^{-1}(g^{-1}(C))$ for each $C \in I^Z$.

(8) $f^{-1}(\bigcup_{\alpha \in \Lambda} B_\alpha) = \bigcup_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ and $f^{-1}(\bigcap_{\alpha \in \Lambda} B_\alpha) = \bigcap_{\alpha \in \Lambda} f^{-1}(B_\alpha)$ where $\{B_\alpha\}_{\alpha \in \Lambda} \subset I^Y$.

The concepts of a fuzzy topological space (briefly, fts), a fuzzy continuous mapping(briefly, F-continuous mapping), a fuzzy closed mapping(briefly, F-closed mapping) and a fuzzy open mapping(briefly, F-open mapping) refer to [1,3].

For a fts X, $FO(X)$[resp. $FC(X)$] denotes the collection of all fuzzy open[resp. fuzzy closed] sets in X.

Let X be a fts and let $A \in I^X$. Then clA and $intA$ denote the closure and interior of A in X, respectively. The properties of clA and $intA$ refer to [8].

Definition 1.1. Let X be a fts and let $A \in I^X$. Then A is called :

(1) a fuzzy regularly open(briefly, fr-open) set in X [2] if $A = int(clA)$.

(2) a fuzzy regularly closed(briefly, fr-losed)set in X [2] if $A = cl(intA)$.

(3) a fuzzy generalized closed(briefly, fg-closed)set in X[3] if for each $O \in FO(X)$ with $A \subset O$, $clA \subset O$.

(4) a fuzzy generalized open(briefly, fg-open)set in X

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[3] if A^c is fg-closed in X.
(5) a fuzzy regular generalized closed(briefly, frg-closed)set in X[1] if for each fr-open set U in X with $A \subset U$, $clA \subset U$.
(6) a fuzzy regular generalized open(briefly, frg-open)set in X[1] if A^c is frg-closed in X.

We will denote the family of all the fr-open(resp. fr-closed, fg-closed, fr-open, frg-closed and frg-open] sets in X as $FRO(X)$(resp. $FRC(X)$, $FGC(X)$, $FGO(X)$, $FRGC(X)$, and $FRGO(X)$).

Result 1.C[3]. $A \in FGO(X)$ if and only if for each $F \in FC(X)$ such that $F \subset A$, $F \subset intA$.

Result 1.D[1]. Let X be a fts and let $A \in I^X$. Then $A \in FRGO(X)$ if and only if for each $F \in FRC(X)$ with $F \subset A$, $F \subset intA$.

Result 1.E[1]. Let X be a fts. Then :
(1) $FRC(X) \subset FC(X) \subset FGC(X) \subset FRGC(X)$.
(2) $FRO(X) \subset FO(X) \subset FGO(X). \subset FRGO(X)$

2. Some fuzzy continuous and closed Mappings

Definition 2.1. A mapping $f: X \to Y$ is said to be :
(1) fuzzy regular generalized continuous(briefly, frg-continuous) if for each $V \in FO(Y), f^{-1}(V) \in FRGO(X)$.
(2) fuzzy regular continuous(briefly, fr-continuous) or fuzzy completely continuous[9] if for each $V \in FO(Y)$, $f^{-1}(V) \in FRO(X)$.
(3) fuzzy regular generalized irresolute(briefly, frg-irresolute) if for each $V \in FRGO(Y)$, $f^{-1}(V) \in FRGO(X)$.
(4) fuzzy regular irresolute(briefly, fr-irresolute) or a fuzzy R-map [4] if for each $V \in FRO(Y)$, $f^{-1}(V) \in FRO(X)$.

We will denote the family of all the frg-continuous[resp. fr-continuous, frg-irresolute, fr-irresolute] mappings of a fts X into another fts Y as $FRGC(Y^X)$ [resp. $FRC(Y^X)$, $FRGI(Y^X)$, $FRI(Y^X)$].

Proposition 2.2. $FRC(Y^X) \subset FRGC(Y^X)$. But the converse inclusion does not hold as shown by Example 2.3.

(proof) Let $f \in FRC(Y^X)$ and let $V \in FC(Y)$. Then clearly $f^{-1}(V) \in FRC(X)$. By Result 1.E, $FRC(X) \subset FRGC(X)$ and thus $f^{-1}(V) \in FRGC(X)$. So f is frg-continuous. Hence $f \in FRGC(Y^X)$.

Example 2.3. From Proposition 2.9 and Example 2.13, we can find the example that $f: X \to Y$ is frg-continuous but not fr-continuous.

From Definitions 1.1 and 2.1, we can easily obtain the following results :

Proposition 2.4. (1) $f \in FRGC(Y^X)$ if and only if for each $F \in FC(Y)$, $f^{-1}(F) \in FRGC(X)$.
(2) $f \in FRC(Y^X)$ if and only if for each $F \in FC(Y)$, $f^{-1}(F) \in FRC(X)$.
(3) $f \in FRGI(Y^X)$ if and only if for each $F \in FRGC(Y)$, $f^{-1}(F) \in FRGC(X)$.
(4) $f \in FRI(Y^X)$ if and only if for each $F \in FRC(Y)$, $f^{-1}(F) \in FRC(X)$.

We can easily obtain the following result from Definition 2.1 :

Proposition 2.5. Every frg-irresolute mapping is frg-continuous, but not conversely as shown by the following example.

Example 2.6. Let $X = \{a, b, c\}$, $T = \{\emptyset, O_1, O_2, O_3, X\}$, $T^* = \{\emptyset, O_1, X\}$, where $O_1 = \{(a, \lambda), (b, 0), (c, 0)\}$, $O_2 = \{(a, 0), (b, \mu), (c, 0)\}$, $O_3 = \{(a, \lambda), (b, \mu), (c, 0)\}$, $\lambda, \mu \in (0, 1]$. Consider the identity mapping $id: (X, T) \to (X, T^*)$. Then id is fr-continuous and hence frg-continuous, but not frg-irresolute.

Definition 2.7. A mapping $f: X \to Y$ is said to be :
(1) **fuzzy almost generalized continuous**(briefly, **falg-continuous**] if for each $V \in FRO(Y)$, $f^{-1}(V) \in FGO(X)$
(2) **fuzzy almost regular generalized continuous**(briefly, falrg-continuous) if for each $V \in FRO(Y)$, $f^{-1}(V) \in FRGO(X)$.
(3) **fuzzy almost continuous**(briefly, **fal-continuous**) [2,12] if for each $V \in FRO(Y)$, $f^{-1}(V) \in FO(X)$.
(4) **fuzzy generalized continuous**(briefly, **fg-continuous**)[3] if for each $V \in FO(Y)$, $f^{-1}(V) \in FGO(X)$.

From Definition 1.1 and 2.7, we obtain the following results :

Proposition 2.8. Let $f: X \to Y$ be a mapping. Then :
(1) f is falg-continuous if and only if for each $F \in FRC(Y)$, $f^{-1}(F) \in FGC(X)$.
(2) f is falrg-continuous if and only if for each $F \in FRC(Y)$, $f^{-1}(F) \in FRGC(X)$.
(3) f is fal-continuous if and only if for each $F \in FRC(Y)$, $f^{-1}(F) \in FC(X)$.
(4) f is fg-continuous if and only if for each $F \in FC(Y)$, $f^{-1}(F) \in FGC(X)$.

From the Definitions 2.1 and 2.7, we can obtain the following implications :

Proposition 2.9. We have the following diagram Ⅰ :

fr-continuity ⇒ F-continuity ⇒ fg-continuity ⇒ frg-continuity
⇓ ⇓ ⇓ ⇓
fr-irresolute ⇒ fal-continuity ⇒ falg-continuity ⇒ falrg-continuity

None of the implications in Proposition 2.9, is reversible as shown by the following examples.

Example 2.10. Let $X = \{ ㄱ, ㄴ, ㄷ, ㄹ \}$, $Y = \{ a, b \}$, $T = \{ \varnothing, O, X \}$ and $T^* = \{ \varnothing, O^*, Y \}$, where $O = \{ (ㄱ, \lambda), (ㄴ, \mu), (ㄷ, 0), (ㄹ, 0) \}$, $O^* = \{ (a, \gamma), (b, 0) \}$, $\lambda, \mu, \gamma \in (0, 1]$. Consider the mapping $f: (X, T) \to (Y, T^*)$ defined by $f(ㄱ) = a = f(ㄴ)$ and $f(ㄷ) = b = f(ㄹ)$. Then f is F-continuous but not fr-continuous.

Example 2.11. Let $X = \{ a, b, c \}$, $T = \{ \varnothing, O_1, O_2, X \}$ and $T^* = \{ \varnothing, O_1, O_2, O_3, X \}$, where $O_1 = \{ (a, \lambda), (b, 0), (c, 0) \}$, $O_2 = \{ (a, \lambda), (b, \mu), (c, 0) \}$, $O_3 = \{ (a, 0), (b, \mu), (c, 0) \}$, $\lambda, \mu \in (0, 1]$. Consider the identity mapping $id: (X, T) \to (X, T^*)$. Then id is fg-continuous but not fal-continuous.

Example 2.12. Let $X = \{ a, b, c \}$, $T = \{ \varnothing, O_1, O_2, O_3, X \}$ and $T^* = \{ \varnothing, O_1, O_4, O_5, X \}$, where $O_1 = \{ (a, \lambda), (b, 0), (c, 0) \}$, $O_2 = \{ (a, \lambda), (b, 0), (c, \gamma) \}$, $O_3 = \{ (a, \lambda), (b, 0), (c, \gamma) \}$, $O_4 = \{ (a, 0), (b, \mu), (c, 0) \}$, $O_5 = \{ (a, \lambda), (b, 0), (c, 0) \}$, $\lambda, \mu, \gamma \in (0, 1]$. Consider the identity mapping $id: (X, T) \to (X, T^*)$. Then id is frg-continuous but not falg-continuous.

Example 2.13. Let $X = \{ a, b, c \}$, $T = \{ \varnothing, O_1, O_2, X \}$ and $T^* = \{ \varnothing, O_1, O_3, X \}$, where $O_1 = \{ (a, \lambda), (b, 0), (c, 0) \}$, $O_2 = \{ (a, \lambda), (b, \mu), (c, 0) \}$, $O_3 = \{ (a, \lambda), (b, 0), (c, \gamma) \}$, $\lambda, \mu, \gamma \in (0, 1]$ Consider the identity mapping $id: (X, T) \to (X, T^*)$. Then id is fr-irresolute and frg-continuous but not fg-continuous.

Now we will introduce some fuzzy closed mappings :

Definition 2.14. A mapping $f: X \to Y$ is said to be :

(1) **fuzzy regular closed**(briefly, **fr-closed**) if for each $F \in FC(X)$, $f(F) \in FRC(Y)$.

(2) **fuzzy generalized closed**(briefly, **fg-closed**) if for each $F \in FC(X)$, $f(F) \in FGC(Y)$.

(3) **fuzzy regular generalized closed**(briefly, **frg-closed**) if for each $F \in FC(X)$, $f(F) \in FRGC(Y)$.

(4) **fuzzy regular closed preserving**(briefly, **frc-preserving**) if for each $F \in FRC(X)$, $f(F) \in FRC(Y)$.

(5) **fuzzy almost closed**(briefly, **fal-closed**)[12,13] if for each $F \in FRC(X)$, $f(F) \in FC(Y)$.

(6) **fuzzy almost generalized closed**(briefly, **falg-closed**) if for each $F \in FRC(X)$, $f(F) \in FGC(Y)$.

(7) **fuzzy almost regular generalized closed**(briefly, **falrg-closed**) if for each $F \in FRC(X)$, $f(F) \in FRGC(Y)$.

From Definition 2.14, we obtain the following diagram :

Proposition 2.15. We have the following diagram Ⅱ :

fr-closed ⇒ F-closed ⇒ fg-closed ⇒ frg-closed
⇓ ⇓ ⇓ ⇓
frc-preserving ⇒ fal-closed ⇒ falg-closed ⇒ falrg-closed.

Remark 2.16. The following example and the inverse mapping $id^{-1}: (X, T^*) \to (X, T)$ in Examples 2.11, 2.12 and 2.13, enable as to realize that none of the implication in Proposition 2.5, is reversible.

Example 2.17. $f: X \to Y$ is F-closed but not frc-preserving.
Let $X = \{ a, b, c \}$, $T = \{ \varnothing, O_1, O_2, O_3, X \}$ and $T^* = \{ \varnothing, O_1, O_2, O_3, O_4, X \}$, where $O_1 = \{ (a, 1), (b, 0), (c, 0) \}$, $O_2 = \{ (a, 0), (b, 1), (c, 0) \}$, $O_3 = \{ (a, 1), (b, 1), (c, 0) \}$, $O_4 = \{ (a, 1), (b, 0), (c, 1) \}$. Consider the identity mapping $id: (X, T) \to (X, T^*)$. Then id is F-closed but not frc-preserving.

Finaly we will introduce some fuzzy open mappings :

Definition 2.18. A mapping $f: X \to Y$ is said to be :

(1) fuzzy regular open(in short, fr-open) if and only if for each $V \in FO(X)$, $f(V) \in FRO(Y)$.

(2) fuzzy generalized open (in short, fg-open) if for each $V \in FO(X)$, $f(V) \in FGO(Y)$.

(3) fuzzy regular generalized open (in short, frg-open) if for each $V \in FO(X)$, $f(V) \in FRGO(Y)$.

(4) fuzzy regular open preserving(in short, fro-preserving) if for each $V \in FRO(X)$, $f(V) \in FRO(Y)$.

(5) fuzzy almost open (in short, fal-open)[12,13] if for each $V \in FRO(X)$, $f(V) \in FO(Y)$.

(6) fuzzy almost generalized open (in short, falg-open) if for each $V \in FRO(X)$, $f(V) \in FGO(Y)$.

(7) fuzzy almost regular generalized open (in short, falrg-open) if for each $V \in FRO(X)$, $f(V) \in FRGO(Y)$.

From Definition 2.18, we obtain the following diagram Ⅲ :

fr-open ⇒	F-open ⇒	fg-open ⇒	frg-open
⇓	⇓	⇓	⇓
fro-preserving ⇒	fal-open ⇒	falg-open ⇒	falrg-open

Note that none of the implications in diagram Ⅲ is revesible.

Theorem 2.19. Let $f: X \to Y$ be a mapping.

(1) If f is frg-continuous, frc-preserving then it is frg-irresolute.

(2) If f is fr-irresolute and frg-closed, then $f(A) \in FRGC(Y)$ for each $A \in FRGC(X)$.

(proof) (1) Let $B \in FRGC(Y)$ and let $U \in FRO(X)$ such that $f^{-1}(B) \subset U$. Let $V = [f(U^c)]^c$. Then, by Result 1.A and 1.B, $B \subset V$ and $f^{-1}(V) \subset U$. Since f is frc-preserving, $V \in FRO(Y)$. Since $B \in FRGC(Y)$ and $B \subset V$, $clB \subset V$. Thus $f^{-1}(clB) \subset U$. But, since f is frg-continuous, $f^{-1}(clB) \in FRGC(X)$. Thus $clf^{-1}(B) \subset clf^{-1}(clB) \subset U$. So $f^{-1}(B) \in FRGC(X)$. Hence f is frg-irresolute.

(2) Let $A \in FRGC(X)$ and let such that $f(A) \subset V$. $f^{-1}(V) \in FRO(X)$ $clA \subset f^{-1}(V)$ Since f is fr-irresolute, $V \in FRO(Y)$ and $A \subset f^{-1}(V)$. Since $A \in FRGC(X)$, and hence $f(clA) \subset V$.Since f is frg-closed, $f(clA) \in FRGC(Y)$.So $clf(A) \subset clf(clA) \subset V$. Hence $f(A) \in FRGC(Y)$. ■

The following is an immediate consequence of Proposition 2.9, Proposition 2.15 and Theorem 2.19 :

Corollary 2.19. Let $f: X \to Y$ be a mapping.

(1) If f is F-continuous, fr-closed, then f is frg-irresolute.

(2) If f is fr-irresolute and F-closed, then $f(A) \in FRGC(Y)$ for each $A \in FRGC(X)$.

Theorem 2.20. A mapping $f: X \to Y$ is falrg-closed[resp. falg-closed] if and only if for each $S \in I^Y$ and each $U \in FRO(X)$ containing $f^{-1}(S)$, there exists a $V \in FRGO(Y)$[resp. $FGO(Y)$] such that $S \subset V$ and $f^{-1}(V) \subset U$.

(proof) We prove only that the first case, the proof of the second is entirely analogous.

(⇒): Suppose f is falrg-closed. Let $S \in I^Y$ and let $U \in FRO(X)$ such that $f^{-1}(S) \subset U$. Let $V = [f(U^c)]^c$. Then, by Results 1.A and 1.B, $S \subset V$ and $f^{-1}(V) \subset U$. Since f is fal-closed and $U \in FRO(X)$, $V \in FRGC(Y)$. This completes the proof.

(⇐): Suppose the necessary condition holds. Let $F \in FRC(X)$. Let $W = [f(F)]^c$ and let $U = F^c$. Then $U \in FRO(X)$ and $f^{-1}(W) = f^{-1}[(f(F))^c] = [f^{-1}(f(F))]^c \subset F^c = U$ (by Result 1.B).

By the hypothesis, there exists a $V \in FRGO(Y)$ such that $W \subset V$ and $f^{-1}(V) \subset U = F^c$. Thus $F \subset [f^{-1}(V)]^c = f^{-1}(V^c)$. So $f(F) \subset V^c$. Since $W \subset V$, $f(F) = W^c \supset V^c$. Thus $f(F) = V^c \in FRGC(Y)$. Hence f is falrg-closed. ■

The following are proved by the similar argument of Theorem 2.20 :

Theorem 2.21. A mapping $f: X \to Y$ is falrg-open[resp. falg-open] if and only if for each $S \in I^Y$ and each $F \in FRC(X)$ containing $f^{-1}(S)$, there exists a $C \in FRGC(Y)$[resp. $C \in FGC(Y)$] such that $S \subset C$ and $f^{-1}(C) \subset F$.

Theorem 2.22. A mapping $f: X \to Y$ is fal-closed if and only if for each $S \in I^Y$ and each $U \in FRO(X)$ such that $f^{-1}(S) \subset U$, there exists a $V \in FO(Y)$ such that $S \subset V$ and $f^{-1}(V) \subset U$.

Result 2.A[11]. A mapping $f: X \to Y$ is fal-open if and only if for each $S \in I^Y$ and each $F \in FRC(X)$ such that $f^{-1}(S) \subset F$, there exists a $C \in FC(Y)$ such that $S \subset C$ and $f^{-1}(V) \subset U$.

3. Fuzzy mildly normal spaces

Two fuzzy sets A and B in a set X are said to be disjoint[7] if $A \odot B = \emptyset$, where $(A \odot B)(x) = \max[0, A(x) + B(x) - 1]$ for all $x \in X$.

It is clear that $A \odot B = \emptyset$ if and only if $A \subset B^c$.

Definition 3.1[6]. A fts X is **fuzzy normal** if for each $A, B \in FC(X)$ such that $A \odot B = \emptyset$, there exists $U, V \in FO(X)$ such that $A \subset U$, $B \subset V$ and $U \odot V = \emptyset$.

Result A[6]. For a fts X, the following are equivalent :

(1) X is fuzzy normal.

(2) For each $F \in FC(X)$ and each $U \in FO(X)$ such that $F \subset U$, there exists a $V \in FO(X)$ such that $F \subset V \subset clV \subset U$.

(3) For each $F_1, F_2 \in FC(X)$ and each $F_1 \odot F_2 = \emptyset$, there exists $U, V \in FO(X)$ such that $F_1 \subset U$, $F_2 \subset V$ and $clU \odot clV = \emptyset$.

(4) For each $F \in FC(X)$ and each $U \in FO(X)$ such that $F \subset U$, there exists a $V \in I^X$ such that $F \subset intV \subset clV \subset U$.

Definition 3.2[7,13]. A fts X is said to be **fuzzy mildly normal** if for each $F_1, F_2 \in FRC(X)$ such that $F_1 \odot F_2 = \emptyset$, there exists $U, V \in FO(X)$ such that $F \subset U$, $F_2 \subset V$ and $U \odot V = \emptyset$.

Result 3.B[13]. For a fts X, the following are equivalent :

(1) X is fuzzy mildly normal.

(2) For each $F \in FRC(X)$ and each $U \in FRO(X)$ such that $F \subset U$, there exists a $V \in FO(X)$ such that $F \subset V \subset clV \subset U$.

(3) For each $U \in FRO(X)$ such that $F \subset U$ and $F \in FRC(X)$, there exists a $V \in FRO(X)$ such that $F \subset V \subset clV \subset U$.

(4) For each $F_1, F_2 \in FRC(X)$ and each $F_1 \odot F_2 = \emptyset$, there exists $U, V \in FO(X)$ such that $F_1 \subset U, F_2 \subset V$ and $clU \odot clV = \emptyset$.

Theorem 3.3. Let X be a fuzzy normal space(not neccessarily T_1) and let Y a fts. If $f: X \to Y$ is fal-continuous, fal-closed and surjective, then Y is fuzzy mildly normal.

(proof) Let $F_1, F_2 \in FRC(Y)$ such that $F_1 \odot F_2 = \emptyset$. Since f is fal-continuous, $f^{-1}(F_1), f^{-1}(F_2) \in FC(X)$ and $f^{-1}(F_1) \odot f^{-1}(F_2) = \emptyset$. Since X is fuzzy normal, there exist $U_1, U_2 \in FO(X)$ such that $f^{-1}(F_i) \subset U_i$ for each $i = 1, 2$ and $U_1 \odot U_2 = \emptyset$. Consider $int(clU_1)$ and $int(clU_2)$. Then clearly $int(clU_i) \in FRO(X)$ for $i = 1, 2$ and $int(clU_1) \odot int(clU_2) = \emptyset$. Furthermore $f^{-1}(F_i) \subset U_i \subset int(clU_i)$ for $i = 1, 2$. Since f is fal-closed, by Result 2.A, there exist $V_i \in FO(Y)$ such that $F_i \subset V_i$ and $f^{-1}(V_i) \subset int(clU_i)$ for $i = 1, 2$. Moreover $V_1 \odot V_2 = \emptyset$. Hence Y is fuzzy mildly normal.

Corollary 3.3(Theorem 6.6 in [13]). A fal-continuous, F-closed image of a fuzzy normal space is fuzzy mildly normal.

(proof) It is clear from Proposition 2.15 and Theorem 3.3.

Result 3.C[13]. Every fal-continuous, fal-closed and F-open image of a fuzzy mildly normal space is fuzzy mildly normal.

Corollary 3.C. Every F-continuous, F-closed and F-open image of a fuzzy mildly normal space is fuzzy mildly normal.

(proof) It is clear from Propositions 2.9 and 2.15 and Result 3.C.

Theorem 3.4. The following are equivalent for an fts X:

(1) X is fuzzy mildly normal.

(2) For any disjoint $H, K \in FRC(X)$, there exist disjoint $U, V \in FGO(X)$ such that $H \subset U$ and $K \subset V$.

(3) For any disjoint $H, K \in FRC(X)$, there exist disjoint $U, V \in FRGO(X)$ such that $H \subset U$ and $K \subset V$.

(4) For any $H \in FRC(X)$ and any $V \in FRO(X)$ containing H, there exists a $U \in FRGO(X)$ such that $H \subset U \subset clU \subset V$.

(proof) (1)⇒(2) and (2)⇒(3) are obvious from Result 1.E.

(3)⇒(4): Suppose the condition (3) holds. Let $H \in FRC(X)$ and let $V \in FRO(X)$ such that $H \subset V$. Let $K = V^c$. Then $K \in FRC(X)$ and $H \odot K = \emptyset$. By the hypothesis, there exist disjoint $U, W \in FRGO(X)$ such that $H \subset U$ and $K \subset W$. By Result 1.D, $K \subset intW$ and $U \odot intW = \emptyset$. Furthermore $clU \odot intW = \emptyset$. Hence $H \subset U \subset clU \subset (intW)^c \subset V$.

(4)⇒(1): Suppose the condition (4) holds and let $H, K \in FRC(X)$ be disjoint. Then $H \subset K^c \in FRO(X)$. By the hypothesis, there exists $G \in FRGO(X)$ such that $H \subset G \subset clG \subset K^c$. Let $U = intG$ and let $V = (clG)^c$. Then clearly $U, V \in FO(X)$, $H \subset U$, $K \subset V$ and $U \odot V = \emptyset$. Hence X is fuzzy mildly normal. ■

Theorem 3.5. Let $f: X \to Y$ be falrg-continuous, frc-preserving[resp. fal-closed) injective. If Y is fuzzy mildly normal[resp. fuzzy normal], then X is fuzzy mildly normal.

(proof) Let $A, B \in FRC(X)$ be disjoint. Since f is frc-preserving[resp. fal-closed] injective, $f(A)$ and $f(B)$ are disjoint fr-closed[resp. F-closed] sets in Y. By the fuzzy mild normality[resp. fuzzy normality] of Y, there exist disjoint $U, V \in FO(Y)$ such that $f(A) \subset U$ and $f(B) \subset V$. Now let $G = int(clU)$ and let $H = int(clV)$. Then $G, H \in FRO(Y)$ such that $f(A) \subset G, f(B) \subset H$ and $G \odot H = \emptyset$. Since f is falg-continuous, $f^{-1}(G)$, $f^{-1}(H) \in FRGO(X)$. Furthermore $A \subset f^{-1}(G)$, $B \subset f^{-1}(H)$ and $f^{-1}(G) \odot f^{-1}(H) = \emptyset$. Hence by Theorem 3.4, X is fuzzy mildly normal.

Theorem 3.6. Let $f: X \to Y$ be fr-continuous, falg-closed and surjective. If X is fuzzy mildly normal, then Y is fuzzy normal.

(proof) Let $A, B \in FC(Y)$ be disjoint. Since f is fr-continuous, $f^{-1}(A)$, $f^{-1}(B) \in FRC(X)$ and $f^{-1}(A) \odot f^{-1}(B) = \emptyset$. Since X is fuzzy mildly normal, there exist $U, V \in FO(X)$ such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and $U \odot V = \emptyset$. Let $G = int(clU)$ and let $H = int(clV)$. Then clearly $G, H \in FRO(X)$ such that $f^{-1}(A) \subset G$, $f^{-1}(B) \subset H$ and $G \odot H = \emptyset$. Since f is falg-closed, by Theorem 2.20, there exist $K, L \in FGO(Y)$ such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since $G \odot H = \emptyset$, $K \odot L = \emptyset$. Since $K, L \in FGO(Y)$, $A \subset intK$ and $B \subset intL$. Furthermore $intK \odot intL = \emptyset$. Hence Y is fuzzy normal.

Corollary 3.6. Let $f: X \to Y$ be fr-continuous, F-closed and surjective. If X is fuzzy mildly normal, then Y is fuzzy normal.

(proof) It is clear from Theorem 3.6 and Proposition 2.15.

Theorem 3.7. Let $f: X \to Y$ be a fr-irresolute[resp. fal-continuous], falrg-closed and surjective. If X is fuzzy mildly normal[resp. fuzzy normal], then Y is fuzzy mildly normal.

(proof) Let $A, B \in FRC(Y)$ be disjoint. Since f is a fuzzy fr-irresolute[resp. fal-closed], $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint fr-closed[resp. F-closed] sets in X. Since X is fuzzy mildly normal[resp. fuzzy normal-closed], there exist $U, V \in FO(X)$ such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and $U \odot V = \emptyset$. Let $G = int(clU)$ and let $H = int(clV)$. Then clearly $G, H \in FRO(X)$ such that $f^{-1}(A) \subset G$, $f^{-1}(B) \subset H$ and $G \odot H = \emptyset$. Since f is falrg-closed, by Theorem 2.20, there exist $K, L \in FRGO(Y)$ such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since $G \odot H = \emptyset$, $K \odot L = \emptyset$. Hence, by Theorem 3.4, Y is fuzzy mildly normal.

Corollary 3.7. Let $f: X \to Y$ be fal-continuous, fal-closed and surjective. If X is fuzzy normal, then Y is fuzzy mildly normal.

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