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TSK 모델 시간 지연 퍼지제어기의 안정성

Stability of TSK-type Time-Delay FLC

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요 약

본 논문은 기존의 시간 지연 제어 기법을 이용하여 TSK 모델 퍼지 제어 시스템의 안정화에 적용하는 방법을 제시한다. TSK 모델 퍼지 시스템이 기존의 PDC 방법에 의하여 안정화되어지는 경우 발생하게 되는 LMI 문제를 피하기 위하여 시간 지연 방법에 의한 새로운 안정화 조건이 다루어진다.. 제시되어진 방법은 mass-spring- damper 시스템에 적용되어 적용가능성이 평가된다.

Abstract

A stable TSK-type FLC can be designed by the method of Parallel Distributed Compensation (PDC) [2], but in this case, solving the LMI problem is not a trivial task. To overcome such a difficulty, a Time-Delay based FLC (TDFLC) is proposed. TSK-type TDFLC consists of Time-Delay Control (TDC) and Sliding Mode Control (SMC) schemes, which result in a robust controller based upon an integral sliding surface. Finally, simulation study is conducted for a mass-spring-damper system.

Key Words : TSK-type FLC, Time-delay, Sliding-mode Control

1. Introduction

The TSK (Tagaki-Sugeno-Kang) fuzzy model [1] has been widely used for a stable fuzzy control system [2], because the linear control theory can be applied in the design of the stable TSK-type FLC. One of the notable design methods to stabilize a TSK fuzzy system is to apply the Parallel Distributed Compensation (PDC) suggested by Wang et. al [2]. In the system described by the TSK fuzzy model combined with PDC scheme, however, it is often difficult to find a positive-definite matrix that satisfies a set of linear matrix inequalities simultaneously.

As another approach, the robust control theory [3] has been presented for the stability of TSK-fuzzy systems. In this method, the nonlinear time-varying fuzzy system is considered as a linear time-invariant system with a norm-bounded model uncertainty: in this case, the LMI problem embedded in the PDC becomes a problem of finding a positive definite solution of an algebraic Riccati equation that stems from the defined Lyapunov function. It is remarked that, since robustness is desirable in consideration of the model uncertainty, various efforts have been given for robust stabilization of a TSK fuzzy model [6]. There is no assurance, however, that the algebraic Riccati equation is always solvable. If unsolvable, the related matrices including the control gains in all the rules must be changed, which affects the norm-bounds of uncertainties. As a result, the LMI problem is indirectly embedded in solving the algebraic Riccati equation. Moreover, no analytical method is known to construct the solvable algebraic Riccati equation.

For uncertain systems, a TDC (Time-Delay Control) technique is also known to be successfully applied for controller design [4]. The TDC algorithm is simple and requires little priori knowledge of the system dynamics. In this paper, we shall use the TDC scheme for a new approach, called a TSK-type TDFLC (Time-Delay FLC), to avoid such difficulties of designing a stable TSK-type FLC: the LMI problem or the problem of selecting appropriate interdependent matrices in the algebraic Riccati equation. The proposed TSK-type TDFLC can be designed to overcome disadvantages of the TDC while ensuring the stability by means of a modified sufficient condition for stability.

In Section 2, the concept of TDC is briefly reviewed. In Section 3, we introduce a robust TSK-type TDFLC and derive its stability. Finally, in Section 4, some simulations are given to illustrate effectiveness of the proposed methods.

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2. Time-Delay Control [4]: Brief Review

Consider a class of nonlinear plants described by

$$x^{(n)}(t)=f(\underline{x},t)+b(\underline{x},t)u(t),\quad y(t)=x_1(t),$$

or

$$\underline{\dot{x}}(t)=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}\underline{x}(t)+\underline{f}(\underline{x},t)+\underline{b}(\underline{x},t)u(t),$$

$$y(t)=x_1(t), \tag{1}$$

where $\underline{x}\in R^n$, $u\in R$, $\underline{f}(\underline{x},t)=[0,\cdots,0,f(\underline{x},t)]\in R^n$ with $f(\underline{x},t)$: $R^n\to R\in C^\infty$, and $\underline{b}(\underline{x},t)=[0,\cdots,-,b(\underline{x},t)]^T\in R^n$, with $b(\underline{x},t)$: $R^n\to R\in C^\infty$. The desired performance is defined by the response of a given stable linear time-invariant reference model:

$$\underline{\dot{x}}_m(t)=A_m\,\underline{x}_m(t)+\ \underline{b}_m r_m. \tag{2}$$

where $\underline{x}_m\in R^n$, $r_m\in R$ (a reference input), $A_m\in R^{n\times n}$ (a canonical Hurwitz matrix), $\underline{b}_m=[0,\cdots,0,b_m]\in R^n$. Let $\underline{e}(t)=\ \underline{x}_m(t)-\underline{x}(t)$. Then, from (1) and (2), the error dynamics is written as

$$\underline{\dot{e}}(t)=A_m\underline{e}(t)+[-\underline{\hat{b}}\,\underline{\hat{b}}^+\,\underline{\hat{f}}(\underline{x},t)+\underline{\hat{b}}\,\underline{\hat{b}}^+\,\underline{b}_m r_m\ ,\ -\underline{\hat{b}}u(t)], \tag{3}$$

where

$$\underline{\hat{f}}(\underline{x},t)=\underline{f}(\underline{x},t)+[\underline{b}(\underline{x},t)-\underline{\hat{b}}]u(t)\in R^n, \tag{4}$$

and

$\underline{\hat{b}}=[0,\cdots,0,\hat{b}]^T\in R^n$ and $\underline{b}^+=(\ \underline{b}^T\underline{b})^{-1}\in R^{1\times n}\ \underline{b}^T$ is a pseudoinverse of . Applying the control input

$$u^*_d(t)=\ \underline{\hat{b}}^+[-\underline{\hat{f}}(\underline{x},t)+A_m\underline{x}(t)+\ \underline{b}_m r_m], \tag{5}$$

to (3) sets the bracket in (3) to be zero. Here $\underline{\hat{b}}^+=(\ \underline{\hat{b}}^T\underline{\hat{b}})^{-1}\ \underline{\hat{b}}^T\in R^{1\times n}$ is a pseudoinverse of $\underline{\hat{b}}$. Under the assumption that $\underline{\hat{f}}(\underline{x},t)$ is a continuous function of its arguments, we can write

$$\underline{\hat{f}}(\underline{x},t)=\underline{\hat{f}}(\underline{x},t-L)+\varepsilon_{error}, \tag{6}$$

and , for small L , we have

$$\underline{\hat{f}}(\underline{x},t)\cong\underline{\hat{f}}(\underline{x},t-L), \tag{7}$$

By using (1) and (4), (7) is rewritten as

$$\underline{\hat{f}}(\underline{x},t)=\underline{\dot{x}}(t)-\underline{\hat{b}}u(t)\cong\underline{\dot{x}}(t-L)-\underline{\hat{b}}u(t-L), \tag{8}$$

Accordingly, combination of (5) and (8) results in the TDC control input

$$\begin{aligned}u_d(t)=&\ \underline{\hat{b}}^+[-\underline{\dot{x}}(t-L))\underline{\hat{b}}u_d(t-L)+A_m\underline{x}(t-L)\\&+\ \underline{b}_m r_m]\\=&\ u_d(t-L)+\ \underline{\hat{b}}^+[-\underline{\dot{x}}(t-L))+A_m\underline{x}(t-L)\\&+\ \underline{b}_m r_m].\end{aligned} \tag{9}$$

For stability of TDC-based control system, recall the following Lemma [8].

Lemma 1 [4] : Given the system in (1), assume that there exists a positive number N such that

$$|b\ \hat{b}^{-1}-I_1|<1\ \text{ for }\ t>N, \tag{10}$$

where is an identity matrix. Then the TDC in (9) guarantee $I_1\in R$s that $y(t)\to x_{m1}(t)$ for sufficiently small time-delay L and sufficiently large t.

The MIMO case ($B\in R^{n\times r}$ and $\hat{B}\in R^{n\times r}$ for b and $\hat{b}$, respectively) can be proved by the result of [8], without loss of generality, if the number of the inputs is identical to that of the outputs.

3. Stability of TSK-type TDFLC

Let there be given the TSK fuzzy model whose ith model is of the following form:

$$R_i:\ \begin{aligned}&IF\ x_1(t)\ is\ M_{i1}\text{ and},\ \cdots\ ,\text{and }x_n(t)\ is\ M_{in},\\&THEN\ \underline{\dot{x}}(t)=\ \tilde{A}_i\underline{x}(t)+\ \underline{\tilde{b}}_i u(t),\end{aligned} \tag{11}$$

where M_{ij} is a fuzzy set, $\underline{x}(t)\in R^n$, $u(t)\in R$, while

$$\tilde{A}_i=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&1\\\tilde{a}_{i1}&\tilde{a}_{i2}&\tilde{a}_{i3}&\cdots&\tilde{a}_{i1n}\end{bmatrix}\in R^{n\times n}, \tag{12}$$

and $\underline{\tilde{b}}_i=[0,\cdots,0,\ \tilde{b}_i]^T\in R^n$ are uncertain matrices with $\tilde{b}_i>0$ for $i=1,\cdots,r$, and $j=1,\cdots,n$. Given a pair of $(\underline{x}(t),u(t))$, the final output of the fuzzy system is inferred as follows [14]:

$$\begin{aligned}\underline{\dot{x}}(t)&=\frac{\sum_{i=1}^r w_i(\underline{x}(t))[\ \tilde{A}_i\underline{x}(t)+\ \underline{\tilde{b}}_i u(t)]}{\sum_{i=1}^r w_i(\underline{x}(t))}\\&=\ \sum_{i=1}^r h_{mi}(\underline{x}(t))[\ \tilde{A}_i\underline{x}(t)+\ \underline{\tilde{b}}_i u(t)],\end{aligned} \tag{13}$$

where

$$w_i(\underline{x}(t))=\ \prod{}_{j=1}^{n}M_{ij}(\underline{x}(t)),\quad h_{mi}(\underline{x}(t))=\frac{w_i(\underline{x}(t))}{\sum_{i=1}^r w_i(\underline{x}(t))}. \tag{14}$$

For stability of a TSK fuzzy model in (13), the TDC input in (9) is replaced with

$$\begin{aligned}u_d(t)=&\ u_d(t-L)+\ \underline{\hat{b}}^+[-\ \sum_{i=1}^r h_{mi}(\underline{x}(t-L))[\ \tilde{A}_i\underline{x}(t-L)\\&+\ \underline{\tilde{b}}_i u_d(t-L)]+A_m\underline{x}(t-L)+\ \underline{b}_m r_m],\end{aligned} \tag{15}$$

where

$$\begin{aligned}\underline{\dot{x}}(t-L)=&\ \sum_{i=1}^r h_{mi}(\underline{x}(t-L))[\ \tilde{A}_i\underline{x}(t-L)\\&+\ \underline{\tilde{b}}_i u_d(t-L)],\end{aligned} \tag{16}$$

and

$$\underline{\hat{b}}=[0,\cdots,0,\ \hat{b}]^T\in R^n,\ \hat{b}>0. \tag{17}$$

Then, Lemma 2 guarantees stability in the same manner as Lemma 1 in Section 2.

Lemma 2 [4]: Given the system in (13), assume that there exists a positive number N such that

$$|\hat{b}(\underline{x},t)\ \hat{b}^{-1}-I_1|<1 \text{ for } t>N,$$

where

$$\hat{b}(\underline{x},t)=\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\ \hat{b}_i. \tag{18}$$

Then, the TDC input in (15) guarantees that $y(t)\rightarrow x_{m1}(t)$ for sufficiently small time-delay L and sufficiently large t.

According to Lemma 2, the error can be made to converge to zero as the time goes to infinity with $\hat{b}$ and a sufficiently small time-delay L being properly selected. It is noted, however, that a large time-delay or a value of $\hat{b}(\underline{x},t)\ \hat{b}^{-1}$ being almost unity may deteriorate performance. Accordingly, in order to overcome such difficulties, a different scheme is proposed to stabilize a TSK fuzzy model.

Theorem 1: Given the system in (13), assume that

$$|\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\ \underline{\hat{b}}^+\ \underline{\hat{b}}-1|<1,\forall t\geq 0, \tag{19}$$

and that the time-delay error is

$$\begin{aligned}\Delta\ \underline{\hat{f}}_d(\underline{x},t)=&\underline{\hat{b}}\ \underline{\hat{b}}^+[\ \underline{\hat{f}}_d(\underline{x},t)-\underline{\hat{f}}_d(\underline{x},t-L)]\\ =&\underline{\hat{b}}\ \underline{\hat{b}}^+[\ \sum_{i=1}^{r}h_{mi}(\underline{x}(t))(\ \hat{A}_i-A_m)\underline{x}(t)\\ &-\sum_{i=1}^{r}h_{mi}(\underline{x}(t-L))(\ \hat{A}_i-A_m)\underline{x}(t-L)\\ &+(\ \sum_{i=1}^{r}h_{mi}(\underline{x}(t))\ \underline{\hat{b}}_i-\underline{\hat{b}})(u_{ds}(t)-u_{ds}(t-L))\\ &-\underline{\varepsilon}_d(t)],\end{aligned} \tag{20}$$

where

$$\underline{\varepsilon}_d(t)=\sum_{i=1}^{r}[h_{mi}(\underline{x}(t-L))\ \underline{\hat{b}}_i-h_{mi}(\underline{x}(t-L))\ \underline{\hat{b}}_i]\ u_{ds}(t-L). \tag{21}$$

Let the sliding surface value and the SMC gain K be defined as

$$s(t)=\underline{\hat{b}}^T[\underline{e}(t)-A_m\int_0^t\underline{e}(\tau)d\tau], \tag{22}$$

where A_m is a Hurwitz matrix of a canonical form, and

$$K>|\ \underline{\hat{b}}^+\Delta\ \underline{\hat{f}}_d(\underline{x},t)|_\infty, \tag{23}$$

respectively. Then, the proposed TDC combined with SMC

$$u_{ds}(t)=u_d(t)+u_s(t), \tag{24}$$

asymptotically stabilizes the TSK-type fuzzy model

$$\dot{\underline{x}}(t)=\sum_{i=1}^{r}h_{mi}(\underline{x}(t))[\ \hat{A}_i\underline{x}(t)+\ \underline{\hat{b}}_iu(t)], \tag{25}$$

if $u_s(t)$ is defined as

$$u_s(t)=Ksgn(s(t)), \tag{26}$$

Proof : Define the Lyapunov function

$$V(t)=s^2(t)/2. \tag{27}$$

The differentiation of (27) with respect to time is given by

$$\begin{aligned}\dot{V}(t)=&s(t)[\ \underline{\hat{b}}^T(\ \dot{\underline{e}}(t)-A_m\underline{e}(t))]\\ =&s(t)[\ \underline{\hat{b}}^T(-\Delta\ \underline{\hat{f}}_d(\underline{x},t)-\underline{\hat{b}}Ksgn(s(t)))].\end{aligned} \tag{28}$$

By (23), $\dot{V}(t)$ becomes negative-definite, which means that the switching condition is always satisfied, hence the asymptotic stability is guaranteed by a stable dynamics on the sliding manifold. *Q.E.D.*

Remark 1 : The condition in (19) is derived from

$$Ksgn(s(t))>|\ \sum_{i=1}^{r}h_{mi}(\underline{x}(t))\ \underline{\hat{b}}^+\ \underline{\hat{b}}_i-1|Ksgn(s(t)), \tag{29}$$

considering

$$\begin{aligned}&u_{ds}(t)-u_{ds}(t-L)\\ =&\ \underline{\hat{b}}^+[-\sum_{i=1}^{r}h_{mi}(\underline{x}(t-L))[\ \hat{A}_i\underline{x}(t-L)+\ \underline{\hat{b}}_{iu_d}(t-L)]\\ &+A_m\underline{x}(t)+\ \underline{b}_mr_m+Ksgn(s(t)),\end{aligned} \tag{30}$$

in (20). Compared with the condition in (18), it is shown that $|\sum_{i=1}^{r}h_{mi}(\underline{x}(t))\ \underline{\hat{b}}^+\ \underline{\hat{b}}_i-1|<1$ given in Theorem 1 should be satisfied from the initial time ($\forall t\geq 0$).

Remark 2 : When the sliding surface value remains zero as the time goes to infinite, its derivative

$$\dot{s}(t)=\ \underline{\hat{b}}^T[\ \dot{\underline{e}}(t)-A_m\underline{e}(t)], \tag{31}$$

also becomes zero, which means that the desired error dynamics of the SMC is equal to that of the TDC.

In terms of the generalized TDC, consider TSK-type TDFLC as follows:

$$\begin{aligned}R_i:\ & IF\ x_1(t)\ is\ M_{i1}\text{ and},\ \cdots\ ,\text{and }x_n(t)\ is\ M_{in},\\ & THEN\ u_{ti}(t)=u_{ti}(t-L)+\ \underline{\hat{b}}_i^+[-\dot{\underline{x}}(t-L)\\ &+A_m\underline{x}(t-L)+\ \underline{b}_mr_m]+K_isgn(s(t)),\end{aligned} \tag{32}$$

where $\underline{\hat{b}}_i=[0,\cdots,0,\ \hat{b}_i]^T\in R^n,\ \hat{b}_i>0$ for $i=1,\cdots,r$.

The final output of this fuzzy controller in (32) is

$$\begin{aligned}u_f(t)=&\sum_{i=1}^{r}h_{ci}(\underline{x}(t))u_{ti}(t)\\ =&\sum_{i=1}^{r}h_{ci}(\underline{x}(t))[u_{ti}(t-L)+\ \underline{\hat{b}}_i^+(-\dot{\underline{x}}(t-L)+A_m\underline{x}(t-L)\\ &+\ \underline{b}_mr_m)+K_isgn(s(t))],\end{aligned} \tag{33}$$

where

$$h_{ci}(\underline{x}(t)) = \frac{w_i(\underline{x}(t))}{\sum_{i=1}^{r} w_i(\underline{x}(t))}. \tag{34}$$

Applying (33) ($u(t)=u_t(t)$) to (25) leads to

$$\dot{\underline{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{A}_i\underline{x}(t) + \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{\underline{b}}_i u_i(t), \tag{35}$$

and the error dynamics is given by

$$\dot{\underline{e}}(t) = A_m\underline{e}(t) + [-\bar{B}(t)\hat{\underline{B}}^{+}(t)(\ \hat{\underline{f}}_t(\underline{x},t) - A_m\underline{x}(t), - \underline{b}_m r_m) - \bar{B}_m(t)u_t(t)], \tag{36}$$

where

$$\dot{\underline{f}}_t(\underline{x},t) = \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{A}_i\underline{x}(t) + [\ \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{\underline{b}}_i - \hat{B}(t)]u_t(t), \tag{37}$$

and

$$\hat{B}(t) = (\ \sum_{i=1}^{r} h_{ci}(\underline{x}(t))\ \hat{\underline{b}}_i^{\ T})^{T}. \tag{38}$$

Theorem 2 : Given the system in (13), assume that

$$\left|\sum_{i=1}^{r}\sum_{j=1}^{r} h_{ci}(\underline{x}(t))h_{mj}(\underline{x}(t))\ \bar{\underline{b}}_i\hat{\underline{b}}_j^{-1} - 1\right| < 1,\ \ \forall t \ge 0, \tag{39}$$

and that the time-delay error is

$$\begin{aligned}\Delta\hat{\underline{f}}_t(\underline{x},t) &= \bar{B}\hat{B}^{+}[\ \hat{\underline{f}}_t(\underline{x},t) - \hat{\underline{f}}_t(\underline{x},t-L)] \\ &= \bar{B}\hat{B}^{+}[\ \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{A}_i\underline{x}(t) - \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{A}_i\underline{x}(t-L) \\ &+ (\ \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{\underline{b}}_i - \hat{B})(u_t(t) - u_t(t-L)) - \underline{\varepsilon}_t(t)],\end{aligned} \tag{40}$$

where

$$\underline{\varepsilon}_t(t) = \sum_{i=1}^{r}[h_{mi}(\underline{x}(t-L))\ \bar{\underline{b}}_i - h_{mi}(\underline{x}(t-L))\ \bar{\underline{b}}_i]\ u_t(t-L). \tag{41}$$

Let the sliding surface be defined as

$$s(t) = \underline{i}_n^T[\underline{e}(t) - A_m\int_0^t \underline{e}(\tau)d\tau],\ \ \underline{i}_n = [0,\cdots,0,1]^T \in R^n, \tag{42}$$

and the SMC gain be given by

$$\begin{aligned}K(t) &= \sum_{i=1}^{r} h_{ci}(\underline{x}(t))K_i > \sum_{i=1}^{r} h_{ci}(\underline{x}(t))\,\hat{b}_i^{-1}|\underline{i}_n^T\Delta\hat{\underline{f}}_t|_\infty \\ &= |\ \hat{B}^{+}\Delta\hat{\underline{f}}_t(\underline{x},t)|_\infty,\end{aligned} \tag{43}$$

or

$$K_i > \hat{b}_i^{-1}|\underline{i}_n^T\Delta\hat{\underline{f}}_t|_\infty = |\ \hat{\underline{b}}^{+}\Delta\hat{\underline{f}}_t(\underline{x},t)|_\infty. \tag{44}$$

Then, the proposed TSK-type TDFLC

$$u_t(t) = \sum_{i=1}^{r} h_{ci}(\underline{x}(t))[u_{ti}(t-L) + \hat{\underline{b}}_i^{+}(-\dot{\underline{x}}(t-L) + A_m\underline{x}(t-L) + \underline{b}_m r_m) + K_i sgn(s(t))], \tag{45}$$

asymptotically stabilizes

$$\dot{\underline{x}}(t) = \sum_{i=1}^{r} h_{mi}(\underline{x}(t))[\ \bar{A}_i\underline{x}(t) + \bar{\underline{b}}_i u(t)]. \tag{46}$$

Theorem 2 can be easily proved in the same way as in Theorem 1 and the proof is omitted.

Remark 3 : Assuming that there exists a positive number N such that

$$\left|\sum_{i=1}^{r}\sum_{j=1}^{r} h_{ci}(\underline{x}(t))h_{mj}(\underline{x}(t))\ \bar{\underline{b}}_i\hat{\underline{b}}_j^{-1} - 1\right| < 1,\ \text{for}\ \ t > N, \tag{47}$$

for sufficiently small time-delay L and sufficiently large t, the TSK-type TDFLC proposed in (45) can also be changed into

$$u_{ti}(t) = u_{ti}(t-L) + \hat{\underline{b}}^{+}[-\dot{\underline{x}}(t-L)) + A_m\underline{x}(t-L) + \underline{b}_m r_m], \tag{48}$$

for each rule. In this case, stability is guaranteed based on Lemma 2.

Remark 4 : While TDC with SMC uses a constant value, K (as the upper bound of the error defined in (23)), the proposed general robust TSK-type TDFLC can adopt a time-varying value $K(t)$ in (43). The time-varying SMC gain $K(t)$ consists of piecewise constant gain K_i in (44) that depends on the local region of each rule (and $|\hat{\underline{b}}_i^{+}\Delta\hat{\underline{f}}_t(\underline{x},t)|_\infty$ also varies depending on each rule). Therefore, as a state varies, the TSK-type TDFLC with SMC has an advantage to use less SMC gain than TDC with SMC. Moreover, $\hat{\underline{b}}_i$ can also be selected, depending on the local region of each rule. This means that the general robust TSK-type TDFLC can be known to satisfy the assumption for stability in (39), more generally and efficiently in terms of the control input.

Remark 5 : As a continuous approximation of SMC, a saturation function

$$sat(s(t)/\Phi) = \begin{cases} sgn(s(t)/\Phi) & \text{if}\ |s(t)| \ge \Phi \\ s(t)/\Phi & \text{if}\ |s(t)| < \Phi \end{cases},\ \Phi > 0, \tag{49}$$

can replace for and have an advantage in attenuating a chattering problem [16]. In general, the convergence within a guaranteed precision ε_Φ is obtained rather than a perfect convergence and ε_Φ is reduced to be the smaller value in proportion to Φ [17]. In TSK-type TDFLC, however, a guaranteed precision ε_Φ depends on not Φ but a time-delay and the value of $\sum_{i=1}^{r}\sum_{j=1}^{r} h_{ci}(\underline{x}(t))$ $h_{mj}(\underline{x}(t))\ \bar{\underline{b}}_i\hat{\underline{b}}_j^{-1}$,which is explained in Theorem 3.

Theorem 3 : Given the system in (13), assume that there exists a positive number N such that

$$\left|\sum_{i=1}^{r}\sum_{j=1}^{r} h_{ci}(\underline{x}(t))h_{mj}(\underline{x}(t))\ \bar{\underline{b}}_i\hat{\underline{b}}_j^{-1} - 1\right| < 1,\ \text{for}\ \ t > N, \tag{50}$$

and let the sliding surface be defined as

$$s(t) = \underline{i}_n^T[\underline{e}(t) - A_m\int_0^t \underline{e}(\tau)d\tau], \tag{51}$$

Then, the TSK-type TDFLC input

$$u_i(t) = \sum_{i=1}^{r} h_{ci}(\underline{x}(t))[u_{ti}(t-L) + \hat{\underline{b}}_i^{+}(-\underline{\dot{x}}(t-L) + A_m\underline{x}(t-L) + \underline{b}_m r_m) + K_i s(t)/\Phi], \quad (52)$$

guarantees that $s(t) \to 0$ for sufficiently small time-delay L and sufficiently large t.

Proof : The TSK-type TDFLC input is rewritten as

$$u_i(t) = \sum_{i=1}^{r} h_{ci}(\underline{x}(t))[u_{ti}(t-L) + \hat{\underline{b}}_i^{+}(\underline{\dot{e}}(t-L) - A_m\underline{e}(t-L)) + K_i s(t)\ \hat{\underline{b}}_i/\Phi]$$
$$= \sum_{i=1}^{r} h_{ci}(\underline{x}(t))[u_{ti}(t-L) + \hat{b}_i^{-1}(\dot{s}(t-L) + K_i s(t)\ \hat{b}_i/\Phi]. \quad (53)$$

We have from (36) and (51)-(52) that

$$\dot{s}(t) = -\underline{i}_n^T \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{A}_i\underline{x}(t) + \underline{i}_n^T A_m\underline{x}(t) + \underline{i}_n^T \underline{b}_m r_m - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1}(\dot{s}(t-L) + \hat{b}_j K_j s(t)/\Phi) - \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{b}_i u_i(t-L). \quad (54)$$

Let $a(\underline{x}(t)) = -\underline{i}_n^T \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{A}_i\underline{x}(t) + \underline{i}_n^T A_m\underline{x}(t) + \underline{i}_n^T \underline{b}_m r_m$, then we have

$$\dot{s}(t-L) = a(\underline{x}(t-L)) - \sum_{i=1}^{r} h_{mi}(\underline{x}(t-L))\ \bar{b}_i u_i(t-L). \quad (55)$$

Applying (55) to (54) results in

$$\dot{s}(t) = (1 - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1})\dot{s}(t-L) - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1} K_j s(t)\ \hat{b}_j/\Phi + a(\underline{x}(t)) - a(\underline{x}(t-L)) + \varepsilon_1, \quad (56)$$

where

$$\varepsilon_1 = (\sum_{i=1}^{r} h_{mi}(\underline{x}(t-L))\ \bar{b}_i - \sum_{i=1}^{r} h_{mi}(\underline{x}(t))\ \bar{b}_i u_i(t-L). \quad (57)$$

In the same manner as (56), we get

$$s(t-L) = (1 - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1})\dot{s}(t-2L) - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1} K_j s(t-L)\ \hat{b}_j/\Phi + a(\underline{x}(t-L)) - a(\underline{x}(t-2L)) + \varepsilon_2. \quad (58)$$

By subtracting (58) from (56), we obtain that

$$\dot{s}(t) - \dot{s}(t-L) = (1 - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1})(\dot{s}(t-L) - \dot{s}(t-2L)) + \varepsilon_3, \quad (59)$$

where $|\varepsilon_3| \le \beta_1 L$ and $\beta_1 > 0$. Taking the norm on both sides of the above equation leads to

$$|\dot{s}(t) - \dot{s}(t-L)| \le |(1 - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1})|\ |\dot{s}(t-L) - \dot{s}(t-2L)| + |\varepsilon_3|$$
$$\le \alpha|\dot{s}(t-L) - \dot{s}(t-2L)| + |\varepsilon_3|$$
$$\le \alpha^{t-N}|\dot{s}(t-L) - \dot{s}(t-2L)| + |\varepsilon_4|. \quad (60)$$

where $|\varepsilon_4| \le \beta_2 L + \beta_3 \alpha^{t-L}$. Accordingly, we can say from $\alpha < 1$ that

$$|\dot{s}(t) - \dot{s}(t-L)| \to 0 \ \ as\ \ t \to \infty \ \text{and}\ L \to 0, \quad (61)$$

or

$$\dot{s}(t) \to \dot{s}(t-L) \ \ as\ \ t \to \infty \ \text{and}\ L \to 0, \quad (62)$$

By applying the above result to

$$\dot{s}(t) - \dot{s}(t-L) = -\sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1} s(t-L) - \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1} K_j s(t)\ \hat{b}_j/\Phi + a(\underline{x}(t)) - a(\underline{x}(t-L)) + \varepsilon_1, \quad (63)$$

we conclude that

$$s(t-L) \to -\frac{\sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1} K_j}{\Phi \sum_{i=1}^{r}\sum_{j=1}^{r} h_{mi}(\underline{x}(t))h_{cj}(\underline{x}(t))\ \bar{b}_i \hat{b}_j^{-1}} s(t) \quad as\ t \to \infty \ \text{and}\ L \to 0, \quad (64)$$

or

$$s(t) \to 0 \ \ as\ \ t \to \infty \ \text{and}\ L \to 0, \quad (65)$$

Q.E.D.

To guarantee a stable TSK-type fuzzy system, the LMI problem should be solved, which is to find a common matrix 'P' satisfying the Lyapunov equations caused by the rules of the fuzzy system. In addition, the LMI problem is not easily solvable analytically, which makes the matter worse for the normal TSK-type fuzzy system. Also, it is remarked that the LMI problem becomes very complicated as the number of rules increases. But, compared with the normal case, the proposed general robust TSK-type TDFLC shows its stability depending on the related assumptions, without considering a common matrix 'P'. Moreover, the proposed method can also overcome disadvantages which the normal TDC has: heavy dependence on a time-delay and a value of $\bar{b}(\underline{x}, t)\ \hat{b}^{-1}$ for performance. Meanwhile, the replacement of $sgn(s(t))$ with $sat(s(t))$ can solve a chattering problem which the general robust TSK-type TDFLC causes.

4. Simulations

In order to demonstrate the effectiveness of the proposed method, a mass-spring-damper system is simulated for TSK-type TDFLC. simulations are given for the case of the general robust TSK-type TDFLC. The mass-spring-damper system is described as follows:

$$\dot{x}_1(t)=x_2(t)$$
$$\dot{x}_2(t)=-0.2x_2^3(t)-2x_1(t)-0.1x_1^3-c_1(t)x_2(t)-c_2(t)x_1(t)+(2.44-0.5x_2^2(t)+c_3(t)\cos(5x_2(t))u(t), \quad (66)$$

where

$$c_1(t)=-0.3\sin(10t),\ c_2(t)=-0.3\cos(5t),$$
$$c_3(t)=-0.2\cos(5t). \quad (67)$$

For the TSK fuzzy model $(i=1,\cdots,4)$,

$$R_i:\ \text{IF } x_1(t) \text{ is } M_{1i} \text{ and } x_2(t) \text{ is } M_{2i},$$
$$\text{THEN } \dot{\underline{x}}(t)=(\bar{A}_i+\Delta A_i)\underline{x}(t)+(\underline{\bar{b}}_i+\Delta\underline{b}_i)u(t), \quad (68)$$

where

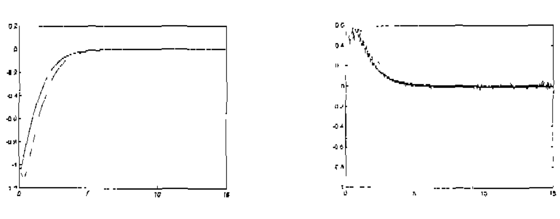
$$A_1=\begin{bmatrix}0 & 1\\ -2 & 0\end{bmatrix}, A_2=\begin{bmatrix}0 & 1\\ -2 & -1.35\end{bmatrix},$$
$$A_3=\begin{bmatrix}0 & 1\\ -2.68 & 0\end{bmatrix}, A_4=\begin{bmatrix}0 & 1\\ -2.68 & -1.35\end{bmatrix},$$
$$\underline{b}_1=\begin{bmatrix}0\\ 2.44\end{bmatrix}, \underline{b}_2=\begin{bmatrix}0\\ 1.31\end{bmatrix}, \underline{b}_3=\begin{bmatrix}0\\ 2.44\end{bmatrix}, \underline{b}_4=\begin{bmatrix}0\\ 1.31\end{bmatrix},$$
$$\Delta A_1=\Delta A_2=\Delta A_3=\Delta A_4=\begin{bmatrix}0 & 0\\ c_1(t) & c_2(t)\end{bmatrix},$$
$$\Delta\underline{b}_1=\Delta\underline{b}_2=\Delta\underline{b}_3=\Delta\underline{b}_4=\begin{bmatrix}0\\ c_3(t)\end{bmatrix},$$
$$M_{i1}=1-x_i^2(t)/2.25,\ M_{i2}=x_i^2(t)/2.25,\ i=1,2, \quad (69)$$

and, for TSK-type TDFLC,

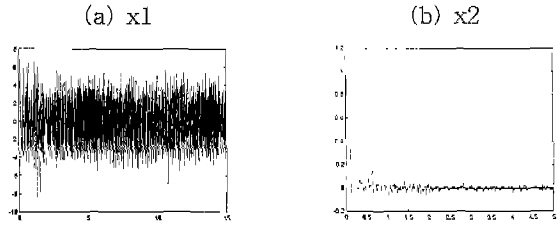
$$\underline{\hat{b}}_1=\underline{b}_1, \underline{\hat{b}}_2=\underline{b}_2, \underline{\hat{b}}_3=\underline{b}_3, \underline{\hat{b}}_4=\underline{b}_4,$$
$$K_1=2.44, K_2=4.80, K_3=2.78, K_4=1.61,$$
$$\underline{x}_{ref}=\begin{bmatrix}0\\ 0\end{bmatrix}, \underline{x}(0)=\begin{bmatrix}-0.5\\ 0\end{bmatrix}, A_m=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}, L=1\text{sec},$$

Here, the SMC gains are calculated based on (43), assuming that

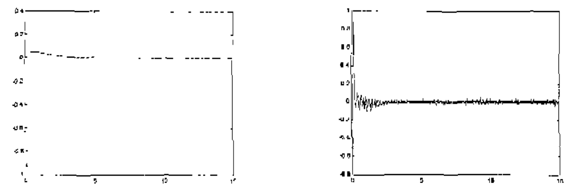
$$|x_i(t)|<1.5 \text{ for } \forall t\geq 0 \text{ and } i=1,2,$$
$$|u(t)|<10, |u(t)-u(t-L)|<10 \text{ for } \forall t\geq 0,$$
$$|x_i(t)-x_i(t-L)|<1 \text{ for } \forall t\geq 0\ i=1,2. \quad (70)$$



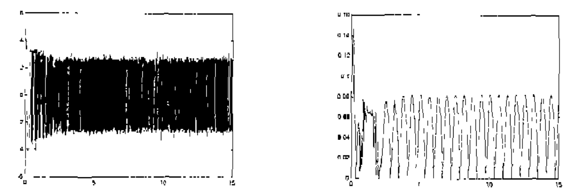
(a) x1 (b) x2



(c) control input (d) sliding surface

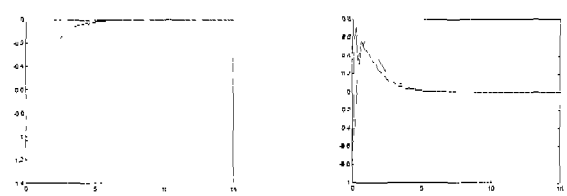


(e) TD of x1 (f) TD of x2

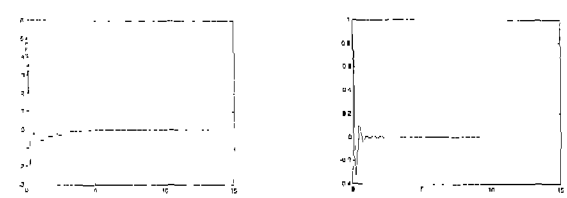


(g) TD of input (h) ratio of (18)

Fig 1. TDFLC



(a) x1 (b) x2



(c) control input (d) sliding surface

Fig 2. Effect of Saturation Function

Fig.1 (a)-(d) show that the proposed TKS-type TDFLC stabilizes the plant under the expected switching action. On the sliding surface, the eigenvalues of the error dynamics are decided by A_m, which are -1 and -2. In Fig.1 (a) and (b), the obtained response is a little bit different from the desired one because of non-zero initial sliding surface value. Fig.1 (e)-(g) insist that all the assumptions in (70) be valid. In Fig.2, SMC is implemented with a saturation function($\Phi=0.1$). As the asymptotic stability is guaranteed in Theorem 3, it is noted that the responses obtained in Fig.2 (a) and (b) converge to zero.

5. Concluding Remarks

Since TDC has characteristics to require little knowledge of the system dynamics, the proposed TSK-type TDFLC overcomes disadvantages that a normal FLC has, such as LMI problem and the robustness. To consider the more general case, weakening the sufficient conditions and constraints for the proposed method needs to be studied.

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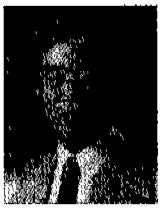

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