經營科學
第18卷 第1號
2001年 5月

기회비용개념을 이용한 실물옵션가치분석*

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Pricing Real Options Value Based On The Opportunity Cost Concept*

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■ Abstract ■

Traditionally, companies have been concerned with making an investment decision either to go now or never to go forever. However, owing to the development of the theory of options pricing in a financial investment field and its introduction to the appraisal of real investments in these days, we are now partially allowed to derive the value of a managerial flexibility of real investment projects. In this paper, we derived a general mathematical model to price the option value of real investment projects assuming that they have only one-period of time under which uncertainty exists. This mathematical model was developed based on the opportunity cost concept. We will show a simple numerical example to illustrate how the mathematical model works comparing it with the existing models.

1. Introduction

The term of "investment" is defined as an activity of committing resources made now with the expectation of realizing profits which will occur over a reasonably long period of time in the future. It can be then inferred from the definition that all investing activities involve to some extent a degree of uncertainty inevitably. Especially, the installation of the manufacturing

* 이 논문은 1999년도 조선대학교 학술연구비의 지원을 받아 연구되었음.
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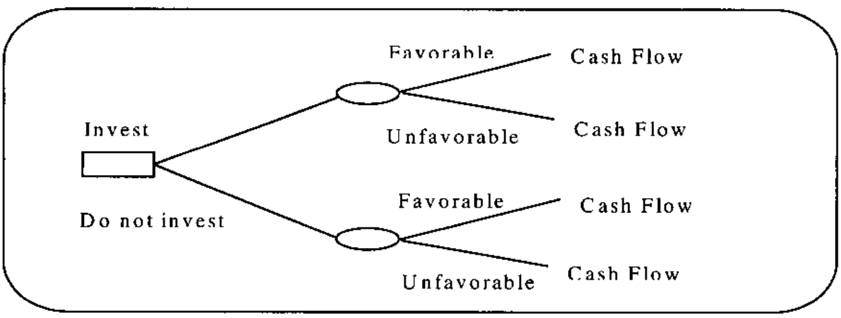
facility of today requires a huge amount of the initial investment and the benefits from the investments in it generally take a long time to attain. Another characteristic of investment in the manufacturing facility in these days is one which once investment is made, a company can hardly recover it either fully or partially. In addition, due to the development of the information technology a product market gets globalized and thus becomes widely open to everybody all over the world. Taken all together, the investment environment becomes more uncertain to accomplish the expected profits. Therefore, when making a decision on the investment project, the question arises about "how to achieve a targeted (expected) profit effectively dealing with uncertainty which lies in the underlying investment project?"

Traditionally, uncertainty has been incorporated in the process of calculating the discounted cash flow (DCF) techniques for the investment project with the aid of a decision tree analysis (DTA), a simulation analysis, or a sensitivity analysis. These three analyses are performed based on the "what-if" scenario concept. That is, a company makes a decision to invest and waits to see what will happen in the future as depicted in <Figure 1>. Like the deterministic DCF techniques, basically they do not also allow a decision-maker to delay implementing the investment project at a later time. It means that he/she must take the investment project now or reject it forever. There are, however, many cases in which the investment projects whose profitability turns out to be economically infeasible at the initial evaluation date will become profitable at a later date if one waits a period of time to get some amount of uncertainty resolved.

Another way to cope with uncertainty is to apply a minimum rate of return (MARR), conceived as a risk-adjusted rate of return, for the expected cash flows to compensate for uncertainty. Depending on a degree of uncertainty for the projects, a company uses a different level of an MARR value. The less uncertainty the project involves, the lower the corresponding MARR value is, and vice versa. It is reported that the appropriate MARR value ranges from 8% to 30%, with a median of 15% and a mean of 17% [5]. However, the problem with this practice arises in that a constant MARR value is always kept over the investment project life. In this line, Hodder and Riggs argue in [7] that the DCF techniques lack in providing the ability to apply the proper MARR value adjusted to the varying degrees of uncertainty in different phases of the



〈Figure 1〉 "What-if" scenario to deal with uncertainty investment project. They also point out that risk often declines in later phases of the investment project while excessive risk-adjust ments are maintained.

In summary for the shortfalls of the DCF techniques briefly discussed above, firstly they do not provide an operating flexibility such as the ability to delay, expand, contract, or abandon operations for a decision-maker. Secondly, they have a problem of selecting a proper MARR value at the different stages of the investment project having a different degree of uncertainty.

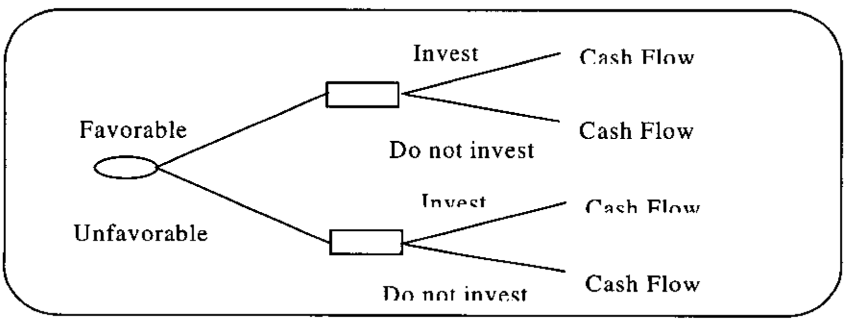
To remedy the shortfalls of the DCF techniques, many researchers have proposed the application of the financial option pricing theory to the valuation of the real assets since Black and Scholes published their research work on how to value the financial options in 1973 [2]. This valuation approach is now popularly named a "real options pricing" method.

Since the main purpose of this paper is not to explain the option pricing theory in detail, we will shortly describe only a conceptual work of the option pricing theory, focusing on the development of the mathematical model, in valuing the real assets in contrast with the traditional valuation approach. Readers who are interested in the real options pricing theory can refer to the related-books in the list of references[1, 4, 10].

Black and Scholes define that an option is a security giving the right to buy or sell an asset, subject to certain conditions, within a specified period of time. Dixit and Pindyck [4], and Sharp [9] also defines an option as the ability, but not the obligation, to take advantage of opportunities available at a later date that would not have been possible without the earlier investment. We can infer from the definition how a decision is made in an option-thinking frame. Contrary to the traditional approach as depicted in <Figure 1>, in an option thinking approach a decision-maker needs to wait and see what will happen, and then makes a decision to invest or not. This process can be graphically depicted in <Figure 2>.

In this paper, we will present three methods to calculate an option value of a real investment project. The first method is one that is performed based on the quasi decision analysis concept and the second one based on the traditional binomial lattice model. The last one is one that was developed as a mathematical model based on the opportunity concept in this paper.

There are two major advantages that we can take from the mathematical model developed in this paper. Firstly, we do not have to make an effort to find out the replicating portfolio to price



〈Figure 2〉 An option thinking approach to deal with uncertainty the option value of the real investment projects. Secondly, it allows us to understand the real options value in more analytical way and analyze it from the various aspects. One thing worthwhile to note is that all of these three models provide us the same result. However, care must be exercised that the model developed in this paper can be applied to the investment projects that have only one period of an uncertain time to expand.

2. Pricing Real Options Value in Three Different Ways

Let's consider the following simple hypothetical example. The investment project under consideration is assumed to have only one-period of time to expand and to generate the same amount of cash flows after one-period of time to expand. Before showing the ways to calculate the option value, we will first present how to make an investment decision in a traditional investment appraisal practice. Thereafter, we will go on to the next step.

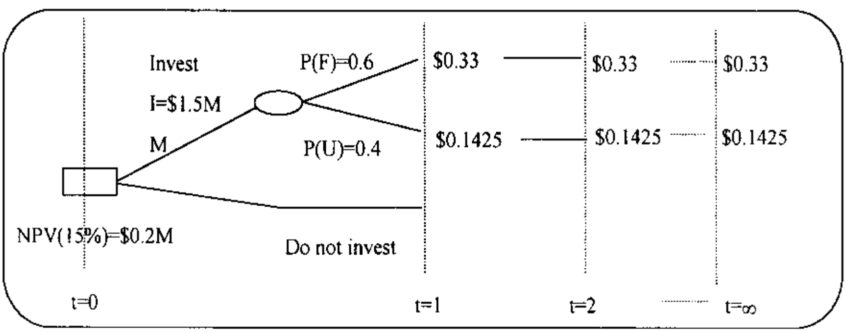
Suppose that we wish to value an investment project that requires an initial investment of $1.5 million and generates an expected net cash flow of $0.33 million with a probability of 60% if an economic situation will become favorable one year later. If the economic situation turns out to be unfavorable, the expected net cash flow comes up with $0.1425 million with a probability of 40%. In this paper, it is assumed that the expected net cash flow generated after one year will be kept forever. For instance, if the economic situation turns out to be favorable at the end of the first year, the expected net cash flow will become $0.33 million and thereafter this amount will be kept forever and vice versa (See <Figure 3>). The MARR value for this investment project is assumed to be 15%.

Calculating the net present value (NPV) of this investment in traditional investment appraisal practice, we obtain

$$NPV(15\%) = -\$1.5M + \sum_{t=1}^{\infty}(\$0.33\times 0.6 + \$0.1425\times 0.4)/(1+0.15)^t = \$0.2M$$

It appears that the NPV(15%) of this investment project is greater than zero. Hence, it would seem that the project could be undertaken to implement.

Let's look at the value of the investment project if it is delayed by one year. If its NPV(15%) is greater than the NPV(15%) we just derived in the traditional way, we'd better delay implemen-



〈Figure 3〉 The traditional thinking approach ting the investment project. Otherwise, it's better to make an investment decision right now. As a rule, the NPV derived by considering the options related to the investment is called the strategic or expanded NPV (SNPV or ENPV) [6, 7]. We need to employ the option thinking to calculate the SNPV/ENPV. By doing so, strategic concerns or a flexibility of an investment can be captured.

2.1 Calculating Real Options Value Based on The Quasi Decision Analysis Concept

Suppose that a company wants to delay implementing the investment project at one year later and by doing so it collects a perfect information on the economic outcome of the next year. If the economic outcome of the next year comes up with a favorable condition, then the company will definitely make a decision to invest in the investment project because we get the positive NPV (15%) of \$0.7million at t = 1. Otherwise, it will give up investing in it because we get the negative NPV(15%) of \$-0.55 million at t = 1. This is a way to evaluate the investment project in the option thinking approach as depicted in <Figure 4>. Calculating the SNPV(15%) of this investment project, we get

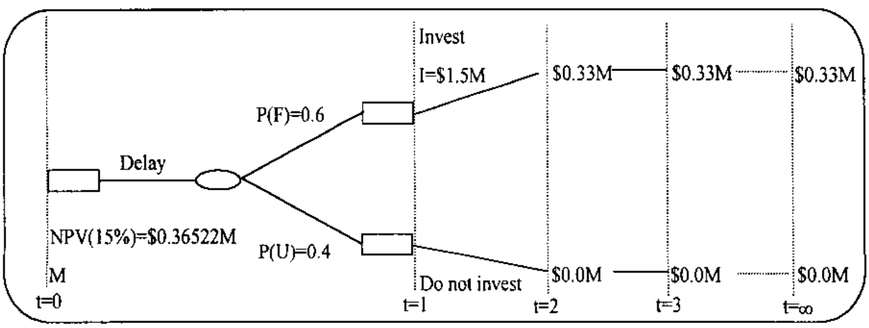
$$SNPV(15\%) = \left\{-\$1.5M + \sum_{t=2}^{\infty} \$0.33M/(1+0.15)^t\right\} \cdot 0.6 \,/\, (1+0.15) = \$0.36522M$$

This positive resulting SNPV(15%) of \$0.36522 million says that the company can invest in the current project at one year later because it is greater than the NPV(15%) of \$0.2 million. How much more money does the company make by delaying one more year? It earns by \$0.16522 million more money than the case for which the company tries to make a decision with the traditional investment appraisal practice. The value of \$0.16522 million is usually called a "value of flexibility" or "real option premium(ROP)"[5, 7]. It is simply the difference between the *SNPV* and the conventional NPV. That is, it is given by

$$ROP = SNPV(15\%) - Conventional\ NPV(15\%)$$

2.2 Calculating Real Options Value Based on the Traditional Binomial Lattice Model

This process is a little bit more complicated than Case I to calculate the option value. Readers who are interested in the presentation of this process can refer to Dixit and Pindyck [4]. To calculate the option value we need to introduce



<Figure 4> An option thinking approach

two variables. Let $SNPV_t$ be the strategic net present value of the investment project at time "t". For this problem we have $SNPV_0$ and $SNPV_1$ because it is assumed that the project will be delayed by one year only. Then, $SNPV_1$ is calculated as followings :

$$SNPV_1\,(15\%)$$
$$= \left\{ \sum_{t=2}^{\infty} \$0.33M/(1+0.15)^t \right\} - \$1.5M$$
$$= \$0.7M \qquad \text{if price moves up}$$
$$= 0 \qquad \text{if price moves down}$$

We found all possible values for $SNPV_1$. The next thing to do is to find the $SNPV_0$. To do this, we need to take several steps as follows :

1st step : Construct a portfolio consisting of two components : the investment project itself and a certain number of products and find the value of this portfolio at each time denoted by Φ_t. For the problem we are considering now for this paper, the value of the portfolio at t = 0 and 1 are given as the following :

$$\Phi_0 = F_0 - nP_0 = F_0 - n(0.255)$$

Note that the P_0 value of \$0.255M is the average of two cash flows (0.6*0.33+0.4*0.1425).

$$\Phi_1 = F_1 - nP_1$$
$$= 0.7 - 0.33n \quad \text{if price moves up}$$
$$= 0.0 - 0.1425n \quad \text{if price moves down}$$

2nd step : Find the value of a certain number of products so that the value of the portfolio at t = 1 is independent of whether the price of the product goes up or down assuming that "n" products can be sold short. For this problem, solving the two equations for Φ_1 then yield n = 3.7333

3rd step : Plug "n" value into the equations for Φ_0 and Φ_1 to find the value of the portfolio at t = 0 and 1. Then, the values are calculated as the followings :

$$\Phi_1 = -0.142(3.7333) = -\$0.532M$$
$$\Phi_0 = F_0 - (3.7333)(0.255) = F_0 - \$0.952M$$

4th step : Calculate the return from holding this portfolio. The return is the difference between Φ_1 and Φ_0 minus the interest payment.

$$\Phi_1 - \Phi_0 - \text{Payment} = -0.532 - (F_0 - 0.952) - (0.255)(0.15)(3.7333) = \$0.2772M - F_0$$

Since this return must be equivalent to the risk-free rate times the value of Φ_0, we can have F_0 = \$0.36522M

The resulting value of F_0 is the same as SNPV (15%) derived in the first case.

2.3 Calculating Real Options Value Based on the Opportunity Cost Concept

If we decide to delay making an investment decision by some period of time, we certainly expect to obtain more useful information on making the decision and eventually to get a higher return from the investment. We have to sacrifice some amount of profit as a compensation for getting the higher return from the investment, though. For this matter, we can calculate the option value by taking the opportunity profit/cost into account.

In the first case, we calculated the SNPV in a lump-sum style, which can be actually separated into two terms as follows :

$$\text{SNPV}(15\%) = \text{Conventional NPV}(15\%) + \text{ROP} = 0.2M + 0.16522M = \$0.36522M$$

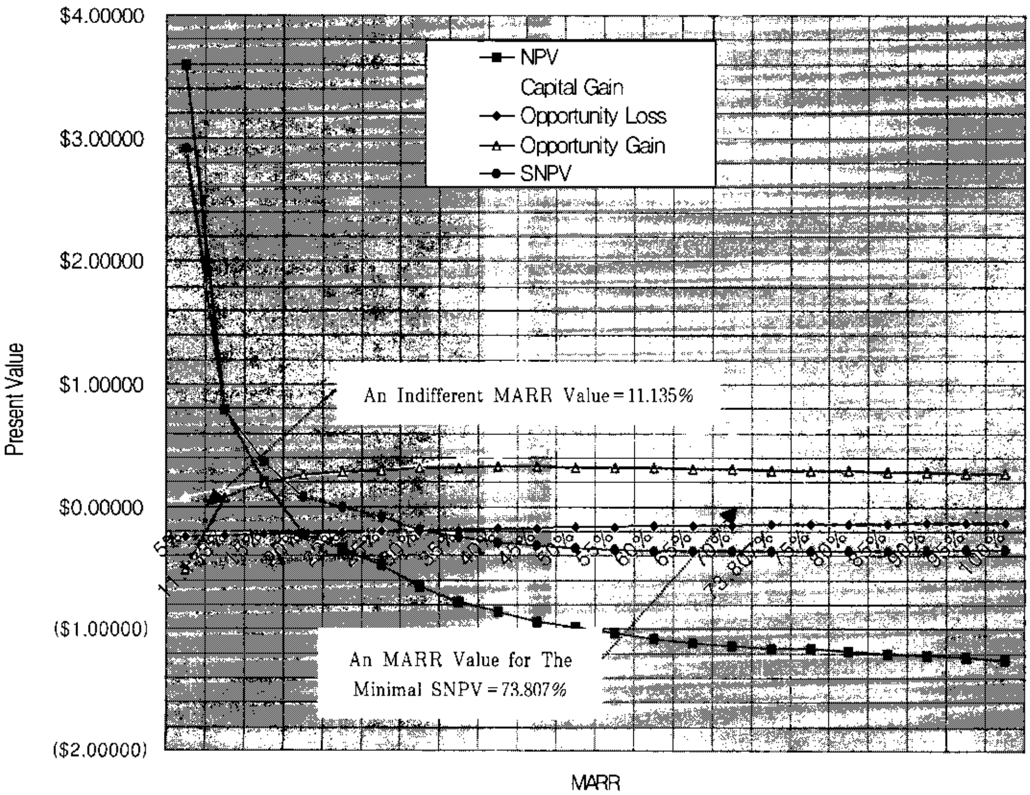
The term of ROP generally consists of the following three terms : the capital gain and the opportunity loss/ gain incurred due to delaying the investment. It is then given by

$$\begin{aligned} SNPV &= Conventional\ NPV + ROP \\ &= Conventional\ NPV + Capital\ Gain \\ &\quad - Opportunity\ Loss \\ &\quad + Opportunity\ Gain \qquad (1) \end{aligned}$$

The first term at the right side in the equation above is the net present value calculated in the traditional thinking way. The second term is the present value of interest earned on the initial investment cost by delaying the investment project. The third term is the present value of the net cash flows lost by delaying the investment project, which is calculated based on the expected cash flows. The last term is the present value of the net cash flows gained by making a decision not to invest in a future time when the economic situation becomes unfavorable.

The way to divide the ROP in three terms as in the Equation (1) provides us with more insight into an option value and makes us more easily analyze the investment project in various aspects. The mathematical expression for Equation (1) is then given in more detail by

$$\begin{aligned} SNPV &= NPV + ROP \\ &= NPV + \{I \cdot (1+MARR)^{n} - I\}(1+MARR)^{-n} \\ &\quad - ACF \cdot \sum_{t=1}^{n} (1+MARR)^{-t} \\ &\quad + [\{I - SUCF\} \cdot q] \cdot (1+MARR)^{-n} \end{aligned}$$



〈Figure 5〉 Present Value of Each Component of SNPV and ROP to Varying MARR Value where,

I : initial investment

$MARR$: a minimum attractive rate of return

FCF : a cash flow when the economic situation becomes favorable.

UCF : a cash flow when the economic situation becomes unfavorable

p : probability that the favorable economic situation occurs.

q : probability that the unfavorable economic situation occurs

ACF : an average cash flow

$ACF = (FCF \cdot p + UCF \cdot q)$

n : time to delay the investment

$SUCF$: the present value of the sum of the net cash flows at $t = n$ when the economic situation becomes unfavorable.

$$SUCF = \sum_{t=n+1}^{\infty} UCF \cdot (1 + MARR)^{-t}$$

Based on the equation above, we can calculate the SNPV in a more analytical way when the investment project is delayed by one year as the following.

$$\begin{aligned} SNPV(15\%) &= 0.2 + (1.5 * 0.15)/1.15 \\ &\quad -[\{(0.33 * 0.6 + 0.1425 * 0.4)/(1.15)\} \\ &\quad + \{(1.5 - 0.95)(0.4)\}/(1.15)] \\ &= 0.2 + 0.19565 - 0.22174 + 0.1913 \\ &= \$0.36521M \end{aligned}$$

Here, we also have the same SNPV as derived in the first and second cases. One of the advantages for decomposing the SNPV and ROP as described above allows us to better understand how the value of each component of the SNPV and ROP behaves as the MARR value varies.

<Table 1> and <Figure 5> show the present value behavior of each component of the SNPV and ROP when the amount of the initial investment is $1.5 million. As the MARR value increases, the conventional NPV constantly decreases and ultimately approach the initial investment of -$1.5 million, whereas the capital gain increases and finally goes to the initial investment of +$1.5 million. The opportunity loss converges to zero, but the opportunity gain steadily increases until when the MARR value reaches at 40% and thereafter decreases and comes to zero. To put together all of these facts lead the SNPV to have a minimum value at a certain value of an MARR. For this example, the SNPV decreases before an MARR value reaches 73.807% and thereafter gradually increases and goes to zero.

One more important fact that should be noted regarding to <Table 1> is that the SNPV is less than the conventional NPV when the MARR value is less than or equal to 10%. Exactly speaking, the MARR value making both the SNPV and the conventional NPV indifferent is 11.135%. As shown in <Table 1>, the conventional NPV becomes greater than the SNPV for the range of the MARR value between 0% and 11.135%. It implies that the investment project under consideration should be undertaken right now rather than delayed if the MARR value lies between this range. If the MARR value is greater than 11.135%, it is better decision to delay the investment project because for this range the SNPV is always greater than the conventional NPV. <Table 2> shows the relationship between the conventional NPV and SNPV at the MARR value that provides the minimal SNPV.

Now, we need to derive the equations to calculate the MARR value making both the SNPV and conventional NPV indifferent and the minimal SNPV. To do this work, what we have to do is to rewrite and simplify Equation (2) such as Equation (3) by employing the capitalized equivalent method for convenience. Then it be-

〈Table 1〉 Present Value of Each Component of SNPV and ROP To Different MARR Values

MARR	NPV	Capital Gain	Opportunity Loss	Opportunity Gain	SNPV
1%	24.00000	$0.0149	-$0.2525	-$5.0495	$18.7129
5%	3.60000	$0.0714	-$0.2429	-$0.5143	$2.9143
10%	1.05000	$0.1364	-$0.2318	$0.0273	$0.9818
11.135%	0.79008	$0.1503	-$0.2295	$0.0793	$0.79019
15%	0.20000	$0.1957	-$0.2217	$0.1913	$0.3652
20%	(0.22500)	$0.2500	-$0.2125	$0.2625	$0.0750
25%	(0.48000)	$0.3000	-$0.2040	$0.2976	($0.0864)
30%	(0.65000)	$0.3462	-$0.1962	$0.3154	($0.1846)
35%	(0.77143)	$0.3889	-$0.1889	$0.3238	($0.2476)
40%	(0.86250)	$0.4286	-$0.1821	$0.3268	($0.2893)
45%	(0.93333)	$0.4655	-$0.1759	$0.3264	($0.3172)
50%	(0.99000)	$0.5000	-$0.1700	$0.3240	($0.3360)
55%	(1.03636)	$0.5323	-$0.1645	$0.3202	($0.3484)
60%	(1.07500)	$0.5625	-$0.1594	$0.3156	($0.3563)
65%	(1.10769)	$0.5909	-$0.1545	$0.3105	($0.3608)
70%	(1.13571)	$0.6176	-$0.1500	$0.3050	($0.3630)
73.807%	(1.15450)	$0.6370	-$0.1467	$0.3008	($0.3635)
75%	(1.16000)	$0.6429	-$0.1457	$0.2994	($0.3634)
80%	(1.18125)	$0.6667	-$0.1417	$0.2938	($0.3625)
85%	(1.20000)	$0.6892	-$0.1378	$0.2881	($0.3606)
90%	(1.21667)	$0.7105	-$0.1342	$0.2825	($0.3579)
95%	(1.23158)	$0.7308	-$0.1308	$0.2769	($0.3547)
100%	(1.24500)	$0.7500	-$0.1275	$0.2715	($0.3510)

comes as follows :

$$\begin{aligned} SNPV =& \ Traditional\ NPV + Capital\ Gain \\ &+ Opportunity\ Loss + Opportnity\ Gain \qquad (3) \\ =& \{-I + (FCF\cdot p + UCF\cdot q)\cdot MARR^{-1}\} \\ &+ [\{I\cdot(1+MARR)^n - I\}\cdot(1+MARR)^{-n}] \\ &- \left\{(FCF\cdot p + UCF\cdot q)\cdot \sum_{t=1}^{n}(1+MARR)^{-t}\right\} \\ &+ \{q\cdot(I\cdot MARR - UCF)\cdot MARR^{-1}\cdot(1+MARR)^{-n}\} \end{aligned}$$

First, we will derive the equation used to find out the MARR value making both the SNPV and conventional NPV indifferent. Since the SNPV consists of the conventional NPV and the ROP, we need to push the terms related to the ROP in Equation (3) to become zero. By rearranging those terms, we finally obtained the following Equation (4) denoted by f(MARR).

$$\begin{aligned} f(MARR) = \{&(1+MARR)^n \cdot \\ &(I\cdot MARR - FCF\cdot p - UCF\cdot q) \\ &+ p\cdot(FCF - I\cdot MARR)\} \\ &\{MARR^{-1}\cdot(1+MARR)^{-n}\} = 0 \quad (4) \end{aligned}$$

Since Equation (4) is of the polynomial function, it is not easy to solve the equation. So, in this paper, we tried to find out the MARR value of interest simply using the trial and error technique. One of the strategies to effectively use the trial and error technique is to take advantage of the fact that we have to decrease the MARR value if the conventional NPV is less than the SNPV, or increase it otherwise. <Table 2> shows the indifferent MARR values for the different initial investment.

〈Table 2〉 The Indifferent MARR Values with the Different Initial Investment

Amount of Initial Investment	MARR	NPV&SNPV
$1.5M	11.135%	$0.7900M
$3.0M	5.180%	$1.9228M
$4.5M	3.36%	$3.0800M

Second, we need to take the first order of a partial derivative of Equation (3) with respect to an MARR to derive the minimal SNPV, and set it to zero and solve it for the MARR. It is then given by the following formula.

$$\begin{aligned}\frac{\partial SNPV}{\partial MARR} = &-(FCF\cdot p+UCF\cdot q)\cdot MARR^{-2}\\ &+(n\cdot I)(1+MARR)^{-(n+1)}\\ &+(FCF\cdot p+UCF\cdot q)\\ &\quad(\sum_{t=1}^{n} t\cdot(1+MARR)^{(t+1)})\\ &+q\cdot\{UCF\cdot(1+MARR+MARR\cdot n)\\ &-I\cdot MARR^2\cdot n\}\\ &\quad\{MARR^2\cdot(1+MARR)^{-(n+1)}\} \qquad (5)\end{aligned}$$

By applying Equation (5) to the example presented in this paper by changing the initial amount of investment, we obtained the results as shown in <Table 3>.

〈Table 3〉 The Relationship Between The SNPV And Conventional NPV For The Different Amount of Initial Investment

Amount of Initial Investment	MARR	NPV	SNPV
$1.5M	73.87%	-$1.1545M	-$0.3635M
$3.0M	45.9%	-$2.4450M	-$0.9381M
$4.5M	35.39%	-$3.7795M	-$1.5810M

We have seen that an option thinking approach provides a managerial flexibility for a decision-maker. Therefore, it has received much attention from an academic and industry for the valuation of the investment projects such as R&D, natural resource development, real estate development projects, and so on. However, many researchers note that an option-based valuation approach is not an absolute alternative for the traditional DCF techniques. Instead they recommend that it is better to use the option-based valuation approach in parallel with the traditional DCF techniques.

3. Concluding Remarks

This paper was concerned with a brief discussion about difference in the investment value between the traditional investment practice and the option thinking practice and about how to value the flexibility of the investment timing using the option thinking concept in two different ways. Although two methods provide the identical result, using the concept of opportunity losses and gains gives us more freedom to analyze the value of the investment flexibility in many different aspects. For example, managers probably want to know how much the initial investment would be if the investment project under consideration is undertaken today. To this end, the mathematical model suggested may be more helpful.

A real option pricing method has received much attention from academics and practitioners since the last several years ago. Thus, research on this topic is widely open to everybody. As a further research regarding to the work done in this paper, we have been developing a more general mathematical model which is supposed to cope with the investment projects having more than two-period of time to expand in the option-thinking frame.

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