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Tests for Panel Regression Model with Unbalanced Data†

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ABSTRACT

This paper consider the testing problem of variance component for the unbalanced two-way error component model. We provide a conditional LM test statistic for testing zero individual(time) effects assuming that the other time-specific(individual) effects are present. This test is extension of Baltagi, Chang and Li (1998, 1992). Monte Carlo experiments are conducted to study the performance of this LM test.

Keywords: Unbalanced Panel Data; LM Tests; Variance Components.

1. INTRODUCTION

Breusch and Pagan (1980) and Godfrey (1989) demonstrated the wide applicability of Lagrange Multiplier(LM) test to various model specifications in econometrics. The LM test is an attractive competitor to the LR and the Wald tests because it requires only the estimation of the model under the null hypothesis and in most cases, the LM test computation requires only ordinary least squares residuals. In the context of the error component regression model, the LM tests for the existence of the random individual and time effects were derived by Breusch and Pagan (1980). Later Honda (1985), Moulton and Randolph (1989) and Baltagi, Chang and Li (1992) extend the work of them to a one-sided tests. For an extensive Monte Carlo study of several tests proposed for the balanced error component model, see Baltagi, Chang and Li (1992). But, the most of econometrics studies focus on the complete or balanced panels, yet the empirical applications face missing observations or incomplete panels. Exceptions are Baltagi and Chang (1994), and Baltagi, Chang and Li (1998).

This paper reconsiders the testing problem of unbalanced two-way error component regression models. In this model, Baltagi, Chang and Li (1998) study the

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joint test for the individual effects and time effects in the unbalanced two-way error component regression models. But, one weakness of these joint tests is that if the null hypothesis is rejected, one can not infer without further testing whether the individual effect, the time effect, or all two effects are absent. Also, this joint test will not be optimal if only one of the two effect does not exist. This is the problem of overtesting discussed in Bera and Jarque (1982). To overcome this problem, we propose the conditional test which tests the presence of individual (time) effects assuming that time (individual) effects are present. This conditional LM test extends the results of Baltagi, Chang and Li (1992, 1998) to the two-way error components model. The outline of this paper is follows : Section 2 describes the model and derives the various tests. The proofs are relegated to the Appendices. Section 3 compares the performance of these tests using Monte Carlo experiments. Section 4 gives a summary and conclusion.

2. THE MODEL AND TEST STATISTICS

We consider the following panel data regression model

$$y_{it} = x'_{it}\beta + u_{it}, \ t = 1, 2, \cdots, T, \quad i = 1, \cdots, N_t, \tag{1}$$

where y_{it} denotes the observation on the dependent variable for the ith individual at the tth time period, x_{it} denotes the it-th observation on k nonstochastic regressors and β is a $k \times 1$ vector of regression coefficients including the intercept. The panel data is incomplete and we observe only N_t individuals in period t $(2 \leq N_t \leq N)$, where N is a number of individuals. The disturbances of (1) are assumed to follow the two-way error component model, see Hsiao (1986),

$$u_{it} = \mu_i + \lambda_t + \nu_{it}, \tag{2}$$

with $\mu_i \sim IIN(0, \sigma^2_\mu)$, $\lambda_t \sim IIN(0, \sigma^2_\lambda)$ and $\nu_{it} \sim IIN(0, \sigma^2_\nu)$. Following Wansbeek and Kapteyn (1989), we order the observations such that all the individuals observed in the first period are stacked on top of those observed in the second period, and so on. In this case, the slower index is t and the faster index is i. In vector form, (2) can be written as

$$u = \Delta_1\mu + \Delta_2\lambda + \nu, \tag{3}$$

where $\Delta_1 = (D'_1, D'_2, \cdots, D'_T)'$, $\Delta_2 = diag(D_t\iota_N) = diag(\iota_{N_t})$, and D_t is the $(N_t \times N)$ matrix obtained from the identity matrix I_N by omitting the rows corresponding to individuals not observed in year t. For complete panels, $\Delta_1 = (\iota_T \otimes I_N)$ and $\Delta_2 = (I_T \otimes \iota_N)$. $\mu' = (\mu_1, \cdots, \mu_N)$, $\lambda' = (\lambda_1, \cdots, \lambda_T)$ and ι_{N_t} is a vector of ones of dimension N_t. The hypotheses under consideration are the following :

a) $H_0^a : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 = 0$), and the one-sided alternative is $H_1^a : \sigma_\mu^2 > 0$ (assuming $\sigma_\lambda^2 = 0$).

b) $H_0^b : \sigma_\lambda^2 = 0$ (assuming $\sigma_\mu^2 = 0$), and the one-sided alternative is $H_1^b : \sigma_\lambda^2 > 0$ (assuming $\sigma_\mu^2 = 0$).

c) $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$), and the one-sided alternative is $H_1^c : \sigma_\mu^2 > 0$ (assuming $\sigma_\lambda^2 > 0$).

d) $H_0^d : \sigma_\lambda^2 = 0$ (assuming $\sigma_\mu^2 > 0$), and the one-sided alternative is $H_1^c : \sigma_\lambda^2 > 0$ (assuming $\sigma_\mu^2 > 0$).

2.1. LM TESTS

a) For Testing H_0^a and H_0^b

We first derive the LM test for the presence of individual (time) effects assuming that the time (individual) effects are absent. In order to construct the LM-type statistics, we need the score vector and information matrix under the null hypothesis. Let $\widetilde{D}$ be the score vector and $\widetilde{J}$ be information matrix evaluated at the restricted MLE, using the results of Baltagi and Li (1990), the partial derivatives and information matrix are given by,

$$\widetilde{D} = (\partial L/\partial\theta)|_{\theta=\widetilde{\theta}} = \frac{n}{2\widetilde{\sigma}_\nu^2}\begin{pmatrix} 0 \\ \dfrac{\widetilde{u}'\Delta_1\Delta_1'\widetilde{u}}{\widetilde{u}'\widetilde{u}} - 1 \\ \dfrac{\widetilde{u}'\Delta_2\Delta_2'\widetilde{u}}{\widetilde{u}'\widetilde{u}} - 1 \end{pmatrix} = \frac{n}{2\widetilde{\sigma}_\nu^2}\begin{pmatrix} 0 \\ A \\ B \end{pmatrix} \tag{4}$$

and

$$\widetilde{J} = \frac{1}{2\widetilde{\sigma}_\nu^4}\begin{pmatrix} n & n & n \\ n & \sum_{i=1}^N T_i^2 & n \\ n & n & \sum_{t=1}^T N_t^2 \end{pmatrix}, \tag{5}$$

where $\theta = (\sigma_\nu^2, \sigma_\mu^2, \sigma_\lambda^2)'$ and $\widetilde{u}$ is a vector of OLS residuals. T_i is the number of time periods observed in individual i and N_t is the number of individuals observed in year t and $n = \sum N_t = \sum T_i$.

Note that if there are no time effects, i.e., assuming that $\sigma_\lambda^2 = 0$, and one is only testing $H_0^a : \sigma_\mu^2 = 0$, versus $H_1^a : \sigma_\mu^2 > 0$ then we ignore the third element of $\widetilde{D}$ in (4) and the corresponding third row and column of $\widetilde{J}$ in (5). In this case, the LM statistic becomes

$$BP_a = \frac{n^2}{2\sum_{i=1}^{N} T_i(T_i - 1)} A^2. \tag{6}$$

Under H_0^a, the LM statistic given by (6) is asymptotically distributed as χ_1^2. Since, σ_λ^2 cannot be negative, the one-sided LM test is

$$LM_a = \sqrt{\frac{n^2}{2\sum_{i=1}^{N} T_i(T_i - 1)}}\, A, \tag{7}$$

which is asymptotically distributed as $N(0,1)$ under H_0^a.

Similarly, if $\sigma_\mu^2 = 0$ and one is only testing $H_0^b : \sigma_\lambda^2 = 0$, versus $H_1^b : \sigma_\lambda^2 > 0$ then we ignore the second element of $\widetilde{D}$ in (4) and the corresponding second row and column of $\widetilde{J}$ in (5). In this case, the LM statistic becomes

$$BP_b = \frac{n^2}{2\sum_{t=1}^{T} N_t(N_t - 1)} B^2, \tag{8}$$

which is asymptotically distributed as χ_1^2. Again, σ_λ^2 cannot be negative. Hence, the one-sided LM test statistic is given by

$$LM_b = \sqrt{\frac{n^2}{2\sum_{t=1}^{T} N_t(N_t - 1)}}\, B. \tag{9}$$

This is asymptotically distributed as $N(0,1)$ under H_0^b. For a similar tests in balanced two-way model, see Baltagi, Chang and Li (1992).

Moulton and Randolph (1989) showed that the asymptotic $N(0,1)$ approximation for testing the random individual effects in the one-way error component model can be poor even in large samples. This occurs when the number of regressors is large or the intra-class correlation of the regressors is high. They suggested an alternative Standardized LM (SLM) test which centers and scales the one-sided LM test so that its mean and variance are zero and one, respectively. The SLM statistics for testing $H_0^a : \sigma_\mu^2 = 0$, and $H_0^b : \sigma_\lambda^2 = 0$ are given by

$$SLM_r = \frac{LM_r - E(LM_r)}{\sqrt{var(LM_r)}} = \frac{d_r - E(d_r)}{\sqrt{var(d_r)}}, \tag{10}$$

for $r = a, b$,where $d_r = \widetilde{u}'U_r\widetilde{u}/\widetilde{u}'\widetilde{u}$ and $U_a = \Delta_1\Delta_1'$ and $U_b = \Delta_2\Delta_2'$. Using the results on moments of quadratic forms in regression residuals (see, for example, Evans and King (1985)), we get

$$E(d_r) = tr(U_r(I_n - P_X))/m, \tag{11}$$

where $m = n - k$ and $P_X = X(X'X)^{-1}X'$ and

$$var(d_r) = 2\{trU_r(I_n - P_X))^2m - [tr(U_r(I_n - P_X)]^2\}/m^2(m+2). \tag{12}$$

Under the respective null hypothesis, SLM_a and SLM_b are asymptotically distributed as $N(0,1)$.

b) For Testing H_0^c and H_0^d

When one uses LM_a to test $H_0^a : \sigma_\mu^2 = 0$, i.e., no individual effects, one implicitly assumes that the time effects do not exist, i.e., that $\sigma_\lambda^2 = 0$. But, when the time effects exist, i.e., $\sigma_\lambda^2 > 0$, LM_a may lead to incorrect decisions. To overcome this problem, we propose the following LM test which tests the presence of individual effects assuming that the time effects are present. The corresponding hypothesis is $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$) vs H_1^c. The detail derivations of test statistic are given in Appendix. From the appendix, the conditional test statistic for testing $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$) is given by

$$LM_c = \sqrt{\frac{\widehat{J}^{\mu\mu}}{det(\widehat{J})}}D(\widehat{\sigma}_\mu^2), \tag{13}$$

where det denotes the determinants, $\widehat{J}^{\mu\mu}$ is the cofactor of (2,2)th element of $\widehat{J}$. $D(\widehat{\sigma}_\mu^2)$ and $\widehat{J}$ are the first derivatives and information matrix evaluated at the null hypothesis ($\sigma_\mu^2 = 0$), and that is given by

$$\frac{\partial L}{\partial \sigma_\mu^2} = D(\widehat{\sigma}_\mu^2) = -\frac{1}{2}\Big[\frac{n-T}{\widehat{\sigma}_\nu^2} + \sum_{t=1}^{T}\frac{1}{N_t\widehat{\sigma}_\lambda^2 + \widehat{\sigma}_\nu^2}\Big] + \frac{1}{2}\widehat{u}'\Big\{\widehat{\Omega}^{-1}\Delta_1\Delta_1'\widehat{\Omega}^{-1}\Big\}\widehat{u} \tag{14}$$

and

$$\widehat{J} = \frac{1}{2}\begin{bmatrix} \sum\frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} + \sum\frac{N_t-1}{\widehat{\sigma}_\nu^4} & \frac{n-T}{\widehat{\sigma}_\nu^4} + \sum_{t=1}^T\frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum\frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \\ \frac{n-T}{\widehat{\sigma}_\nu^4} + \sum_{t=1}^T\frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \widehat{J}_{\mu\mu} & \sum_{t=1}^T\frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \\ \sum\frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum_{t=1}^T\frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum_{t=1}^T\frac{N_t^2}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \end{bmatrix}, \tag{15}$$

where $\widehat{J}_{\mu\mu}=\sum_{i=1}^{N}\frac{T_i^2}{\widehat{\sigma}_\nu^4}-2\sum_{s=1}^{T}\sum_{t=1}^{T}\frac{\widehat{a}_tC_{ts}}{N_t\widehat{\sigma}_\nu^2}+\sum_{s=1}^{T}\sum_{t=1}^{T}\frac{\widehat{a}_t\widehat{a}_s}{N_tN_s}C_{ts}^2$, $\widehat{a}_t=1/\widehat{\sigma}_\nu^2-1/(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)$, and C_{ts} is the number of observations that simultaneously observed at time t and s. Under the null hypothesis, the conditional LM test given in (13) is asymptotically distributed as $N(0,1)$. This extends the work of Baltagi, Chang and Li (1992) to the unbalanced two-way error component model.

Similarly, one can derive the conditional LM test for $H_0^d:\sigma_\lambda^2=0$ assuming that the individual effects are present. In this case, we rearrange the model by individual first and using the same derivation of LM_c, the LM_d is given by (this derivation is simple but tedious and it is available upon request from the author)

$$LM_d=\sqrt{\frac{\widetilde{J}^{\lambda\lambda}}{det(\widetilde{J})}}D(\widetilde{\sigma}_\lambda^2), \tag{16}$$

where $\widetilde{J}^{\lambda\lambda}$ is the cofactor of (3,3)th element of $\widetilde{J}$. $D(\widetilde{\sigma}_\lambda^2)$ and $\widetilde{J}$ are the first derivatives and information matrix evaluated at the null hypothesis ($\sigma_\lambda^2=0$), and that is given by

$$D(\widetilde{\sigma}_\lambda^2)=-\frac{1}{2}\Big[\frac{n-N}{\widetilde{\sigma}_\nu^2}+\sum_{i=1}^{N}\frac{1}{T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2}\Big]+\frac{1}{2}\widetilde{u}'\Big\{\widetilde{\Omega}^{-1}\Delta_2\Delta_2'\widetilde{\Omega}^{-1}\Big\}\widetilde{u} \tag{17}$$

and

$$\widetilde{J}=\frac{1}{2}\begin{bmatrix}\sum\frac{1}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2}+\sum\frac{T_i-1}{\widetilde{\sigma}_\nu^4} & \sum\frac{T_i}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2} & \frac{n-N}{\widetilde{\sigma}_\nu^4}+\sum_{i=1}^{N}\frac{1}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2}\\ \sum\frac{T_i}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2} & \sum_{i=1}^{N}\frac{T_i^2}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2} & \sum_{i=1}^{N}\frac{T_i}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2}\\ \frac{n-N}{\widetilde{\sigma}_\nu^4}+\sum_{i=1}^{N}\frac{1}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2} & \sum_{i=1}^{N}\frac{T_i}{(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)^2} & \widetilde{J}_{\lambda\lambda}\end{bmatrix}, \tag{18}$$

where $\widetilde{J}_{\lambda\lambda}=\sum_{t=1}^{T}\frac{N_t^2}{\widetilde{\sigma}_\nu^4}-2\sum_{k=1}^{N}\sum_{i=1}^{N}\frac{\widetilde{b}_iH_{ki}}{T_i\widetilde{\sigma}_\nu^2}+\sum_{k=1}^{N}\sum_{i=1}^{N}\frac{\widetilde{b}_i\widetilde{b}_k}{T_iT_k}H_{ki}^2$, $\widetilde{b}_i=1/\widetilde{\sigma}_\nu^2-1/(T_i\widetilde{\sigma}_\mu^2+\widetilde{\sigma}_\nu^2)$, and H_{ik} is the number of observations that simultaneously observed at time k and i. LM_d is also distributed as $N(0,1)$ under H_0^d.

2.2. LR TESTS

Following Goureroux, Holly and Monfort (1982), the one-sided LR tests have the following form :

$$LR=-2log\frac{l(res)}{l(unres)}, \tag{19}$$

where $l(res)$ denotes the maximum likelihood value of restricted model(under the null) and $l(unres)$ denotes the maximum likelihood value of unrestricted model(under the alternative). These tests require ML estimators of the one-way and two-way models and are more computationally expensive than their LM counterparts. For example, we consider the testing $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$), then the LR test requires the ML estimator of one-way model and two-way model while the LM test only requires the ML estimator of one-way model. Under the null hypothesis considered, the LR statistic have the same asymptotic distributions as their counterparts, see Goureroux, Holly and Monfort (1982), more specifically, for all hypothesis H_0^a, H_0^b, H_0^c and H_0^d, LR $\sim \frac{1}{2}\chi^2(0) + \frac{1}{2}\chi^2(1)$, where $\chi^2(0)$ equals zero with probability one.

3. MONTE CARLO RESULTS

Monte Carlo studies were carried out to compare the size and power properties in various test statistics described in above section.

3.1. Design of the Monte Carlo Study

We consider the following simple regression equation :

$$y_{it} = \alpha + x_{it}\beta + u_{it}, \quad t = 1, 2, \cdots, T, \quad i = 1, \cdots, N_t, \tag{20}$$

with u_{it} defined by (2). The exogeneous variable x_{it} was generated by a similar method to that of Nerlove (1971). In fact, $x_{it} = 0.3t + 0.8x_{i,t-1} + w_{it}$, where w_{it} is uniformly distributed on the interval $[-0.5, 0.5]$. The initial values x_{i0} were chosen as $(100 + 250w_{i0})$. Throughout the experiment $\alpha = 5$ and $\beta = 2$. For the disturbances u_{it}, we let $\mu_i \sim IIN(0, \sigma_\mu^2)$, $\lambda_t \sim IIN(0, \sigma_\lambda^2)$ and $\nu_{it} \sim IIN(0, \sigma_\nu^2)$. We fix $\sigma^2 = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_\nu^2 = 20$ and let $\gamma_1 = \sigma_\mu^2/\sigma^2$ and $\gamma_2 = \sigma_\lambda^2/\sigma^2$ vary over the set $(0, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8)$ such that $(1 - \gamma_1 - \gamma_2)$ is always positive. We fix N=30 and follow the suggestion given by Swallow and Searle (1978) of selecting T-patterns which intuitively seem to range from slightly to badly unbalanced. Let 5(15) denote the T-pattern with 15 individuals each observed over five periods, then the following unbalanced T-patterns are used: $P_1 = 5(15), 9(15)$; $P_2 = 5(10), 7(10), 9(10)$; $P_3 = 3(6), 5(6), 7(6), 9(6), 11(6)$; $P_4 = 3(9), 5(6), 9(6), 11(9)$; $P_5 = 3(24), 23(6)$; $P_6 = 2(15), 12(15)$. Obviously, pattern P_6 is more unbalanced than P_1. For all patterns considered, the sample size is fixed at 210.

A measure of unbalancedness as given by Ahrens and Pincus (1981) is defined as,

$$r = N/\bar{T}\sum_{i=1}^{N}(1/T_i), \qquad \text{where } \bar{T} = \sum_{i=1}^{N} T_i/N \text{ and } 0 < r \leq 1. \tag{21}$$

Note that r takes the value of one when the pattern is balanced, but it takes smaller values as the pattern gets more severely unbalanced. Denoting by r_i as the measure of unbalancedness for pattern P_i and r as the vector of r_i's, then $r = (0.918, 0.841, 0.813, 0.754, 0.519, 0.490)$. Note that the degree of unbalancedness increases as the subscript of P gets large.

For each experiment, 1000 replications are performed and in each experiment we calculated the rejection numbers of the following test statistics : For H_0^a and H_0^b, the two-sided BP_a and BP_b test, the one-sided LM_a and LM_b test, its standardized version (SLM_a and SLM_b) test and corresponding LR_a and LR_b test. For H_0^c and H_0^d, the conditional LM tests (LM_c and LM_d), and the LR tests(LR_c and LR_d).

3.2. Results

Table 1 gives the number of rejections for the various test for testing H_0^a : $\sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 = 0$) and $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$) for all considered patterns. Similar tables for testing $H_0^b : \sigma_\lambda^2 = 0$ (assuming $\sigma_\mu^2 = 0$) and H_0^d : $\sigma_\lambda^2 = 0$ (assuming $\sigma_\mu^2 > 0$) are obtained, but they are not produced here to save space(These results are available upon request from the authors). We first consider a results for $H_0^a : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 = 0$). BP_a, LM_a, SLM_a and LR_a give the result of testing $H_0^a : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 = 0$) for all considered patterns. When H_0^a is true which is in the top block of Table 1, all the tests perform badly since they ignore the time effect, i.e, $\sigma_\lambda^2 > 0$. In fact, the two-sided BP test badly overrejects the null hypothesis H_0^a, while the one-sided LM_a, SLM_a and the LR_a test badly underestimate the nominal size. This is coincide with the result of balanced two-way error component model given by Baltagi, Chang and Li (1992). They explain that the poor performance of two-sided BP test is caused by the large negative value of A. Also, they show that when the true model has only time-specific effects and one is testing $\sigma_\mu^2 = 0$(ignoring the fact that σ_λ^2), then $plimA$ may tend to ∞ as both N and T tend to ∞. Our results for the unbalanced two-way error component model confirm the results of balanced two-way error component model. When σ_μ^2 is large($\gamma_1 \geq 0.2$), all the tests perform well in rejecting the null hypothesis, but the power is slightly decreases as γ_2 increases.

Next, we consider the results of conditional tests. LM_c and LR_c give the result of testing $H_0^c : \sigma_\mu^2 = 0$ (assuming $\sigma_\lambda^2 > 0$) for all considered patterns. From the Table 1, the estimated size of the LM_c test is not significantly different from the nominal size while the LR_c test slightly underestimate the nominal size for all patterns. For 1000 replications, counts between 37 and 63 are not significantly different from 50 at the 0.05 level. The result of underestimation of the LR test is in sharp contrast to the Table 3 of Baltagi, Chang and Li (1992) where the estimated size of the LR test for the balanced two-way error component model is not significantly different from the nominal size. The power of all the tests increases as γ_1 increases. In fact, $\gamma_1 \geq 0.2$, all the tests have high power rejecting the null in 95 % to 97 % of the cases for all patterns.

Also, even if $\gamma_2 = 0$, i.e., the true model is one-way, all the tests in H_0^c perform well compared with the corresponding tests employed in H_0^a. Hence, overspecifying the model, i.e., assuming the model is two-way ($\sigma_\lambda^2 > 0$) when it is one-way($\sigma_\lambda^2 = 0$), does not hurt the power of tests. This confirms similar results by Baltagi, Chang and Li (1992) for the balanced two-way error component model.

The result in this subsection emphasis that one should not ignore the possibility of σ_λ^2 when testing $\sigma_\mu^2 = 0$. In other words, if there is any sign of time effect(i.e., $\sigma_\lambda^2 > 0$) when testing $\sigma_\mu^2 = 0$, one would expect that the test statistic based on H_0^c to be preferable to one test statistic based on H_0^a. In fact, our results suggest that it may be better to overspecify the model rather than underspecify it in testing the variance components.

4. CONCLUSION

This paper deals with the testing problem of one variance component for the unbalanced two-way error component model. We derive the one-sided LM and standardized LM test statistics for the assumption of the other variance component is zero, and conditional LM test statistics for the other effects are given. Using the Monte Carlo experiments, we obtain the following conclusions: (1) the one directional LM tests and LR tests that assume the other variance component is zero have low power when this other variance component is large for H_0^a and H_0^b. (2) the computationally more demanding LR tests slightly underestimate the nominal size and have the low powers relative to conditional LM test statistics for H_0^c and H_0^d.

Table 1. Rejection number of tests for testing $H_0 : \sigma_\mu^2 = 0$

		P_1						P_2					
γ_1	γ_2	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c
0.00	0.00	49	36	49	34	36	36	44	36	56	30	39	32
0.00	0.05	36	22	42	14	34	26	44	26	40	21	39	33
0.00	0.10	46	17	26	12	33	33	42	13	22	13	37	35
0.00	0.20	72	24	31	15	63	41	60	18	27	9	54	29
0.00	0.40	176	1	5	6	47	46	191	1	2	3	42	42
0.00	0.60	375	1	2	4	41	44	388	1	1	2	46	39
0.00	0.80	676	0	0	3	51	47	674	1	1	3	49	40
0.05	0.00	212	285	358	245	294	253	193	264	344	224	272	234
0.05	0.05	192	247	301	218	312	259	183	252	315	217	304	266
0.05	0.10	165	221	274	192	328	275	182	240	302	210	342	306
0.05	0.20	131	176	223	149	404	349	122	167	215	143	375	330
0.05	0.40	117	124	152	115	550	498	113	126	162	120	543	505
0.05	0.60	145	105	133	108	783	748	147	96	122	95	783	744
0.05	0.80	232	73	96	84	989	985	231	67	92	76	989	988
0.10	0.00	533	615	670	594	622	600	533	619	674	607	627	613
0.10	0.05	517	604	684	563	663	618	496	591	664	552	669	623
0.10	0.10	484	579	638	580	708	689	494	584	650	545	695	678
0.10	0.20	442	544	597	493	773	754	448	527	596	491	753	754
0.10	0.40	346	412	451	397	906	878	353	417	480	390	909	897
0.10	0.60	307	365	417	373	993	990	300	357	418	387	992	985
0.10	0.80	313	316	345	322	1000	1000	326	324	362	331	1000	1000
0.20	0.00	917	951	965	933	953	937	930	953	965	948	955	948
0.20	0.05	921	952	967	940	968	960	926	940	961	935	962	952
0.20	0.10	929	947	961	925	981	958	936	957	973	936	988	975
0.20	0.20	907	934	954	911	992	987	903	941	952	919	990	991
0.20	0.40	834	864	891	887	998	998	829	859	882	873	996	999
0.20	0.60	796	842	876	824	1000	1000	783	825	860	813	1000	1000
0.40	0.00	1000	1000	1000	999	1000	999	1000	1000	1000	1000	1000	1000
0.40	0.05	999	1000	1000	1000	1000	1000	999	1000	1000	999	1000	1000
0.40	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.20	999	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.40	997	997	997	995	1000	1000	996	997	998	994	1000	1000
0.60	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.20	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

* BP_a, LM_a, SLM_a, LR_a : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 = 0$*)*

* LM_c, LR_c : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 > 0$*)*

Table 1. Rejection number of tests for testing $H_0 : \sigma_\mu^2 = 0$(Continue)

		P_3						P_4					
γ_1	γ_2	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c
0.00	0.00	32	30	53	24	35	28	31	42	56	22	43	21
0.00	0.05	47	37	52	31	47	41	37	30	44	24	36	24
0.00	0.10	48	24	35	14	44	31	24	16	33	24	35	40
0.00	0.20	49	15	24	19	65	46	50	24	32	16	66	37
0.00	0.40	145	6	10	1	46	36	103	5	10	4	40	37
0.00	0.60	304	0	0	6	41	40	254	0	1	5	43	34
0.00	0.80	559	0	0	9	47	37	487	0	2	6	53	34
0.05	0.00	223	282	336	247	290	254	219	281	342	248	290	253
0.05	0.05	185	258	328	224	309	274	200	281	346	233	329	272
0.05	0.10	175	244	292	204	340	298	170	228	286	192	321	281
0.05	0.20	145	206	251	182	402	360	144	195	255	174	408	359
0.05	0.40	107	140	176	115	561	508	125	155	183	138	558	507
0.05	0.60	94	77	112	80	757	719	106	95	117	88	755	713
0.05	0.80	163	88	109	92	980	977	163	102	130	99	984	981
0.10	0.00	573	660	727	603	669	609	565	643	705	573	649	580
0.10	0.05	510	602	650	552	651	608	526	619	675	570	667	630
0.10	0.10	552	633	694	548	737	657	532	627	683	517	726	654
0.10	0.20	456	548	613	473	776	718	461	556	620	501	789	741
0.10	0.40	361	430	506	447	898	882	377	466	528	434	920	885
0.10	0.60	371	430	477	384	986	978	386	430	474	398	989	989
0.10	0.80	328	364	405	358	1000	1000	350	390	430	382	1000	1000
0.20	0.00	925	956	969	953	956	956	925	951	968	953	954	952
0.20	0.05	928	957	976	944	974	964	926	949	965	941	964	961
0.20	0.10	933	954	967	930	977	972	925	959	970	943	982	972
0.20	0.20	910	939	954	925	982	984	916	940	957	931	989	986
0.20	0.40	845	887	913	880	998	999	863	898	918	904	1000	998
0.20	0.60	811	852	880	841	1000	1000	812	851	879	856	1000	1000
0.40	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.10	1000	1000	1000	999	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.20	999	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.40	996	998	998	997	1000	1000	998	999	999	996	1000	1000
0.60	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.20	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

* BP_a, LM_a, SLM_a, LR_a : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 = 0$*)*

* LM_c, LR_c : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 > 0$*)*

Table 1. Rejection number of tests for testing $H_0 : \sigma_\mu^2 = 0$(Continue)

		P_5						P_6					
γ_1	γ_2	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c	BP_a	LM_a	SLM_a	LR_a	LM_c	LR_c
0.00	0.00	49	57	69	34	58	35	39	41	59	30	41	30
0.00	0.05	37	41	56	29	51	37	35	33	47	25	45	38
0.00	0.10	49	38	54	39	52	42	33	22	31	23	37	39
0.00	0.20	45	24	38	17	64	42	44	22	30	15	47	39
0.00	0.40	69	17	24	10	52	32	84	8	13	9	52	33
0.00	0.60	117	6	11	10	53	46	160	2	2	7	56	44
0.00	0.80	227	3	3	3	47	47	328	0	1	5	63	35
0.05	0.00	214	295	337	256	297	258	221	305	374	253	316	262
0.05	0.05	218	291	346	251	335	282	214	279	332	242	320	278
0.05	0.10	190	261	314	219	333	285	207	274	326	238	355	305
0.05	0.20	184	252	291	209	405	360	145	209	259	182	389	339
0.05	0.40	141	182	244	154	508	463	122	168	213	163	534	487
0.05	0.60	135	173	215	140	771	729	101	124	150	111	768	730
0.05	0.80	149	166	192	140	986	979	115	96	122	91	985	982
0.10	0.00	553	654	700	590	662	598	537	605	658	601	612	606
0.10	0.05	556	635	685	591	683	648	547	639	695	588	681	629
0.10	0.10	525	627	680	567	695	646	528	620	677	584	715	676
0.10	0.20	497	578	631	536	739	702	489	562	625	528	767	730
0.10	0.40	424	488	531	510	881	879	447	526	594	492	915	895
0.10	0.60	433	502	565	490	981	975	364	433	476	443	990	986
0.10	0.80	394	469	516	430	1000	1000	337	405	458	389	1000	1000
0.20	0.00	937	957	965	954	957	955	956	973	982	947	976	949
0.20	0.05	929	964	974	953	973	964	928	954	965	948	967	961
0.20	0.10	923	950	964	943	970	960	930	949	961	951	970	970
0.20	0.20	924	951	958	938	988	983	905	937	952	935	991	982
0.20	0.40	892	922	939	911	1000	999	882	913	926	905	999	997
0.20	0.60	868	912	930	900	1000	1000	846	885	901	876	1000	1000
0.40	0.00	999	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.20	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.40	0.40	1000	1000	1000	998	1000	1000	997	997	998	999	1000	1000
0.60	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.60	0.20	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.00	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.05	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
0.80	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

* BP_a, LM_a, SLM_a, LR_a : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 = 0$*)*

* LM_c, LR_c : *Test statistics for testing* $H_0 : \sigma_\mu^2 = 0$*(assuming* $\sigma_\lambda^2 > 0$*)*

APPENDIX

Let us consider the LM test for $\sigma_\mu^2 = 0$ given the existence of ranom time effects . The null hypothesis for this model is $H_0^c : \sigma_\mu^2 = 0$ (given $\sigma_\lambda^2 > 0$) vs $H_1^c : \sigma_\mu^2 \neq 0$ (given $\sigma_\lambda^2 > 0$). The disturbance covariance matrix $E(uu')$ can be written as

$$\Omega = \sigma_\mu^2 \Delta_1 \Delta_1' + \sigma_\lambda^2 \Delta_2 \Delta_2' + \sigma_\nu^2 I_n, \tag{A.1}$$

where $n = \sum N_t$. Note that under the null hypothesis of $H_0^c : \sigma_\mu^2 = 0$,

$$\Omega = \sigma_\lambda^2 diag(J_{N_t}) + \sigma_\nu^2 I_n. \tag{A.2}$$

Therefore, under the null hypothesis Ω^{-1} becomes, see Baltagi, Chang and Li (1994)

$$\Omega^{-1} = diag\Big(\frac{1}{N_t\sigma_\lambda^2 + \sigma_\nu^2}\bar{J}_{N_t}\Big) + diag\Big(\frac{1}{\sigma_\nu^2}E_{N_t}\Big) = \frac{1}{\sigma_\nu^2}I_n - diag\Big(a_t \bar{J}_{N_t}\Big) \tag{A.3}$$

where $\bar{J}_{N_t} = J_{N_t}/N_t$ and $a_t = 1/\sigma_\nu^2 - 1/(N_t\sigma_\lambda^2 + \sigma_\nu^2)$. Using the formula of Hemmerle and Hartly (1973), we obtain

$$\begin{aligned}
\frac{\partial L}{\partial \sigma_\nu^2} &= D(\widehat{\sigma}_\nu^2) = -\frac{1}{2}tr\Big[diag\Big(\frac{1}{N_t\sigma_\lambda^2 + \sigma_\nu^2}\bar{J}_{N_t}\Big) + diag\Big(\frac{1}{\sigma_\nu^2}E_{N_t}\Big)\Big] \\
&\quad + \frac{1}{2}u'\Big\{diag\Big(\frac{1}{(N_t\sigma_\lambda^2 + \sigma_\nu^2)^2}\bar{J}_{N_t}\Big) + diag\Big(\frac{1}{\sigma_\nu^4}E_{N_t}\Big)\Big\}u \\
&= -\frac{1}{2}\Big[\sum \frac{1}{N_t\sigma_\lambda^2 + \sigma_\nu^2} + \sum \frac{N_t - 1}{\sigma_\nu^2}\Big] + \frac{1}{2}u'\Big\{diag(\Big(\frac{1}{(N_t\sigma_\lambda^2 + \sigma_\nu^2)^2}\bar{J}_{N_t}\Big) \\
&\quad + diag\Big(\frac{1}{\sigma_\nu^4}E_{N_t}\Big)\Big\}u = 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial \sigma_\lambda^2} &= D(\widehat{\sigma}_\lambda^2) = -\frac{1}{2}tr\Big[diag\Big(\frac{N_t}{N_t\sigma_\lambda^2 + \sigma_\nu^2}\bar{J}_{N_t}\Big)\Big] + \frac{1}{2}u'\Big\{diag\Big(\frac{N_t}{(N_t\sigma_\lambda^2 + \sigma_\nu^2)^2}\bar{J}_{N_t}\Big)\Big\}u \\
&= -\frac{1}{2}\Big[\sum \frac{N_t}{N_t\sigma_\lambda^2 + \sigma_\nu^2}\Big] + \frac{1}{2}u'\Big\{diag(\Big(\frac{N_t}{(N_t\sigma_\lambda^2 + \sigma_\nu^2)^2}\bar{J}_{N_t}\Big)\Big\}u = 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial \sigma_\mu^2} &= D(\widehat{\sigma}_\mu^2) = -\frac{1}{2}tr\Big[\Big\{\frac{1}{\sigma_\nu^2}I_n - diag\Big(a_t\bar{J}_{N_t}\Big)\Big\}\Delta_1\Delta_1'\Big] + \frac{1}{2}u'\Big\{\Omega^{-1}\Delta_1\Delta_1'\Omega^{-1}\Big\}u \\
&= -\frac{1}{2}\Big[\frac{n-T}{\sigma_\nu^2} + \sum_{t=1}^{T}\frac{1}{N_t\sigma_\lambda^2 + \sigma_\nu^2}\Big] + \frac{1}{2}u'\Big\{\Omega^{-1}\Delta_1\Delta_1'\Omega^{-1}\Big\}u,
\end{aligned} \tag{A.4}$$

where the fourth equation follows from the fact that $D_tD_t' = I_{N_t}, t = 1, \cdots, T$. Also, using the the formula of Harville (1977), we obtain

$$
\begin{aligned}
E\Big[-\frac{\partial^2 L}{\partial(\sigma_\nu^2)^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[\Big\{diag\Big(\frac{1}{N_t\sigma_\lambda^2+\sigma_\nu^2}\bar{J}_{N_t}\Big) + diag\Big(\frac{1}{\sigma_\nu^2}E_{N_t}\Big)\Big\}^2\Big] \\
&= \frac{1}{2}\Big[\sum \frac{1}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2} + \sum \frac{N_t-1}{\sigma_\nu^4}\Big], \\
E\Big[-\frac{\partial^2 L}{\partial(\sigma_\lambda^2)^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[\Big\{diag\Big(\frac{N_t}{N_t\sigma_\lambda^2+\sigma_\nu^2}\bar{J}_{N_t}\Big)\Big\}^2\Big] = \frac{1}{2}\sum \frac{N_t^2}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}, \\
E\Big[-\frac{\partial^2 L}{\partial\sigma_\nu^2\partial\sigma_\lambda^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[diag\Big(\frac{N_t}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}\bar{J}_{N_t}\Big)\Big] = \frac{1}{2}\sum \frac{N_t}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}, \\
E\Big[-\frac{\partial^2 L}{\partial\sigma_\nu^2\partial\sigma_\mu^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[\Big\{diag\Big(\frac{1}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}\bar{J}_{N_t}\Big) + diag\Big(\frac{1}{\sigma_\nu^4}E_{N_t}\Big)\Big\}\Delta_1\Delta_1'\Big] \\
&= \frac{1}{2}\Big[\frac{n-T}{\sigma_\nu^4} + \sum_{t=1}^{T}\frac{1}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}\Big], \\
E\Big[-\frac{\partial^2 L}{\partial\sigma_\lambda^2\partial\sigma_\mu^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[diag\Big(\frac{N_t}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}\bar{J}_{N_t}\Big)\Delta_1\Delta_1'\Big] = \sum_{t=1}^{T}\frac{N_t}{(N_t\sigma_\lambda^2+\sigma_\nu^2)^2}, \\
E\Big[-\frac{\partial^2 L}{\partial(\sigma_\mu^2)^2}\Big]_{H_0} &= \frac{1}{2}tr\Big[\Big\{\frac{1}{\sigma_\nu^2}\Delta_1\Delta_1' - diag\Big(a_t\bar{J}_{N_t}\Big)\Big\}^2 \\
&\qquad + diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1' diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1'\Big] \\
&= \frac{1}{2}\Big[\sum_{i=1}^{N}\frac{T_i^2}{\sigma_\nu^4} - 2\sum_{s=1}^{T}\sum_{t=1}^{T}\frac{a_tC_{ts}}{N_t\sigma_\nu^2} + \sum_{s=1}^{T}\sum_{t=1}^{T}\frac{a_ta_s}{N_tN_s}C_{ts}^2\Big], \qquad (A.5)
\end{aligned}
$$

where C_{ts} is the number of observations that simultaneously observed at time t and s. The element of information matrix with respect to σ_μ^2 given by last term of equation (A.5) is obtained as follows :

The first term of $E\Big[-\frac{\partial^2 L}{\partial(\sigma_\mu^2)^2}\Big]_{H_0}$ is obtained by

$$
tr\Big[\Delta_1\Delta_1'\Delta_1\Delta_1'\Big] = tr\Big[\Big\{\Delta_1'\Delta_1\Big\}^2\Big] = \sum_{i=1}^{N}T_i^2. \qquad (A.6)
$$

Next, the second term of $E\Big[-\frac{\partial^2 L}{\partial(\sigma_\mu^2)^2}\Big]_{H_0}$ is obtained by

$$diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1' = \begin{pmatrix} a_1\bar{J}_{N_1}D_1D_1' & a_1\bar{J}_{N_1}D_1D_2' & \cdots & a_1\bar{J}_{N_1}D_1D_T' \\ a_2\bar{J}_{N_2}D_2D_1' & a_2\bar{J}_{N_2}D_2D_2' & \cdots & a_2\bar{J}_{N_2}D_2D_T' \\ \vdots & \vdots & \vdots & \vdots \\ a_T\bar{J}_{N_T}D_TD_1' & a_T\bar{J}_{N_T}D_TD_2' & \cdots & a_T\bar{J}_{N_T}D_TD_T' \end{pmatrix}.$$

Therefore, we obtain

$$\begin{aligned} tr\Big[\Delta_1\Delta_1' diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1'\Big] &= tr\Big[\sum_{s=1}^{T}\sum_{t=1}^{T} a_t D_s D_t' \bar{J}_{N_t} D_t D_s'\Big] \\ &= tr\Big[\sum_{s=1}^{T}\sum_{t=1}^{T} a_t D_{st}\bar{J}_{N_t} D_{st}'\Big], \qquad (A.7) \end{aligned}$$

where $D_{st} = D_s D_t'$ is the $N_s \times N_t$ matrix such that the (i, j) th element is equal to 1 when the same individuals show i th position in time s and j th position in time t, and zero elsewhere. Therefore,

$$tr\Big(D_{st}\bar{J}_{N_t}D_{st}'\Big) = \frac{1}{N_t}C_{ts}, \qquad (A.8)$$

and equation (A.7) becomes

$$tr\Big[\Delta_1\Delta_1' diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1'\Big] = \sum_{s=1}^{T}\sum_{t=1}^{T}\frac{a_tC_{ts}}{N_t}. \qquad (A.9)$$

Finally, using the similar derivation of equation (A.7), (A.8) and (A.9), we obtain

$$\begin{aligned} tr\Big[diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1' diag\Big(a_t\bar{J}_{N_t}\Big)\Delta_1\Delta_1'\Big] &= \sum_{s=1}^{T}\sum_{t=1}^{T} a_s a_t tr\Big[\bar{J}_{N_s}D_{st}\bar{J}_{N_t}D_{st}'\Big] \\ &= \sum_{s=1}^{T}\sum_{t=1}^{T} a_s a_t \frac{C_{ts}^2}{N_tN_s}. \qquad (A.10) \end{aligned}$$

Using (A.6), (A.9) and (A.10), the element of information matrix with respect to σ_μ^2 is obtained. Therefore, the information matrix, when evaluated under the null hypothesis ($\sigma_\mu^2 = 0$) is

$$\widehat{J} = \frac{1}{2}\begin{bmatrix} \sum \frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} + \sum \frac{N_t-1}{\widehat{\sigma}_\nu^4} & \frac{n-T}{\widehat{\sigma}_\nu^4} + \sum_{t=1}^T \frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum \frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \\ \frac{n-T}{\widehat{\sigma}_\nu^4} + \sum_{t=1}^T \frac{1}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \widehat{J}_{\mu\mu} & \sum_{t=1}^T \frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \\ \sum \frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum_{t=1}^T \frac{N_t}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} & \sum_{t=1}^T \frac{N_t^2}{(N_t\widehat{\sigma}_\lambda^2+\widehat{\sigma}_\nu^2)^2} \end{bmatrix}, \tag{A.11}$$

where $\widehat{J}_{\mu\mu} = \sum_{i=1}^N \frac{T_i^2}{\widehat{\sigma}_\nu^4} - 2\sum_{s=1}^T\sum_{t=1}^T \frac{\widehat{a}_t C_{ts}}{N_t\widehat{\sigma}_\nu^2} + \sum_{s=1}^T\sum_{t=1}^T \frac{\widehat{a}_t\widehat{a}_s}{N_tN_s} C_{ts}^2$. Thus the resulting LM test statistic is

$$LM = \widehat{D}'\widehat{J}^{-1}\widehat{D} = \frac{\widehat{J}^{\mu\mu}}{det(\widehat{J})} D(\widehat{\sigma}_\mu^2)^2. \tag{A.12}$$

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