Jour. of the KGS Vol. 17, No. 3. June 2001, pp 95~102

Analysis of Group Pile-Cap Interaction by Load Transfer Approach

하중전이법에 의한 말뚝-캡 상호작용 해석

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요 지

본 연구에서는 말뚝-지반, 말뚝-캡 상호작용을 고려한 군말뚝 거동을 해석하는 효율적인 알고리즘을 제안하였다. 단독말뚝의 말뚝-지반 상호작용은 하중전이법을 사용하여 비선형적인 특성을 갖는 지반스프링(p-y, t-z, q-z 곡선등)을 이용하여 모델링하였으며 Beam-column 방법을 이용하여 말뚝-지반 시스템을 수치적으로 모델링하였다. 좀 더 실제적인 군말뚝의 해석을 위해서 지반-말뚝-지반 상호작용에 의한 그룹효과와 더불어 군말뚝내에서 말뚝의 위치 및 말뚝과 말뚝캡의 결합조건에 따른 말뚝-캡 상호작용에 의한 효과를 고려하였다. 본 연구에서는 말뚝-캡의 상호작용에 중점을 두었으며 Beam-column 방법을 이용하여 이를 해석할 수 있는 타당한 수치해석적 방법을 개발하고자 하였다. 개발된 알고리즘을 수치해석 예제를 통해 비교, 검증 하였다.

Abstract

A computationally efficient algorithm to analyze a group pile behavior is proposed by consideration of both Soil-pile and pile-cap interactions. Using load transfer method the nonlinear characteristics of the soil-pile interaction for a single pile is modeled by piecewise linear soil springs (p-y, t-z, and q-z curves). Beam-column method, one of the most practical approaches, is used for numerical modeling of the soil-pile system. In addition to the group effect resulting from the pile-soil-pile interaction, for a more realistic analysis it is essential to consider the effect that comes from the pile-cap interaction including geometric configuration of the piles in a group and connectivity conditions between piles and the cap. This paper mainly focuses on the subject of the pile-cap interaction and the development of a rational numerical procedure of its incorporation with the beam-column method. The algorithm is further elaborated with the help of numerical examples.

Keywords : Beam-column method, Load transfer, Pile-cap interaction, Pile connectivity, Soil-pile interaction

1. Introduction

Since 1970's, rapid economic and population growth in Korea has led to a need for the efficient development of the land. As a result, the construction of infrastructure facilities in the soft ground (hazard zone) was inevitable and has considerably increased. The pile foundation, which consists of a number of piles in a group, has been most generally used in such cases since the geological soil profiles in most regions of Korea are composed

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of surface soil, weathered soil, and base rocks, and the depth (or distance) from the surface to base rock is relatively short. Pile foundations can be classified as an important infrastructure to support various superstructures such as bridges, buildings, lifeline structures, and offshore structure, etc.

Two major resources of group effect of the pile foundation are pile-soil-pile interaction and pile-cap interaction. The pile-soil-pile interaction is closely related to the mechanical behavior, which is known as "mechanical effects", caused by strain superposition in the soil mass and alteration of failure zones (O'Neill, 1983). Much work has been done in this subject. Consequently, several theoretical and numerical approaches have been vigorously developed and implemented to consider these effects for the design and analysis of group piles used as pile foundations, including the finite element method (Ottaviani, 1975; Randolph, 1981; Yeagian and Wright, 1973), the boundary element method (Butterfield and Banerjee, 1971, Butterfield and Douglas, 1981), the elastic solutions based on Mindlin's solution (Poulos et al., 1968 and 1979; Focht and Koch 1973), modified unit load transfer method (Stevens et al., 1979), etc. On the other hand, the pile-cap interaction mainly influenced by the geometric arrangement of piles and the condition of pile head fixity has not received the attention it deserves. Recently, McVay et al. (1996) reported the significance of the head fixity for both battered and plumb 3x3 group piles in loose sand. Consequently, consideration of the pile-cap interaction is inevitable in the design and analysis of the group pile foundation.

This paper focuses on the subject of the evaluation of stiffness characteristics and analysis of pile groups by taking consideration of the pile-cap interaction including the configuration of the piles and head fixity between the piles and the pile cap, nonlinearity of soil, inhomogeneous layering characteristics of soil with respect to depth, and coupling of axial and lateral loading condition. To accomplish the goal, a computationally efficient and accurate numerical procedure is proposed. It uses the beam-column method (Haliburton 1968; Matlock et al., 1981) based on Winkler foundation model to model the behavior of a single pile subjected to axial load, lateral load, or both. The complex pheno- menon of the pile-soil interaction is modeled by discrete nonlinear soil spring supports in the form of p-y curve for lateral support springs, t-z curve for side friction springs, and q-z curve for an end bearing spring due to its computational efficiency. The pile-cap interaction is incorporated into the procedure using the equivalent stiffness matrix method proposed by Lam and Martin (1986). To investigate the accuracy, efficiency, and applicability of the proposed procedure, its results are compared with results obtained from lab test and field case studies.

2. Analysis of Single Pile Behavior

As explained before, various methods are available for the analysis of a single pile. For practical design and analysis, the method to be used is required not only to consider the inhomogeneous layering and the nonlinear nature of soil behavior but also to be computationally efficient. One such a method among the alternatives is the beam-column method (Matlock et al., 1981). It has been being widely used in engineering practices for past two decades. In this method, a pile member is described as a series of beam column elements with discrete spring to represent the soil support condition as shown in Fig. 1.

To obtain the governing differential equation for the axially loaded pile (Seed and Reese, 1957), an axially loaded element shown in Fig. 1(a) is considered. By taking force equilibrium with respect to vertical direction with constitutive relations of the pile in compression

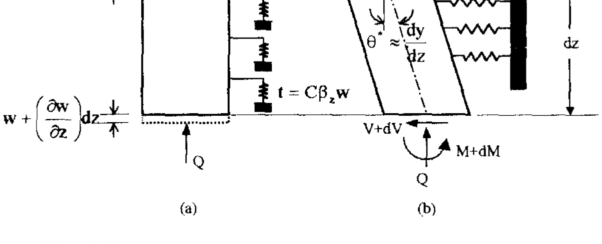


Fig. 1 Force acting on beam-column element of a pile (a)Axial element (b) Beam-column element

($Q = EA\,dw/dz$), the first governing differential equation for the axially loaded pile can be expressed as:

$$EA\frac{d^2w}{dz^2} - C\beta_z w = 0 \tag{1}$$

where E is modulus of the pile, A is cross sectional area of the pile, w is vertical deflection of the pile at point z, β_z is stiffness/circumference for the axial reaction represented by the secant modulus of the soil-response ($t-z$ or $q-z$ or both), which depends on the depth z and pile movement w, and C is circumference of the pile at point z. The load transfer is expressed as a function of the pile movement w, as follows:

If the pile is subdivided in increments of length h, as shown in Fig. 2, the governing differential equation for the axially loaded pile, Eq. (1), can be written in a form of finite difference, for arbitrary node i, as:

$$\frac{Q_{i+1}}{(EA)_{i+1}} - \frac{Q_{i-1}}{(EA)_{i-1}} = 2h\frac{C}{(EA)_i}\beta_i w_i \tag{2}$$

For the laterally loaded pile, a beam-column element (Hetenyi, 1946) shown in Fig. 1(b), in which lateral loads are applied as well as axial load, is considered next. Using horizontal equilibrium with constitutive relations of the pile in bending ($M = EId^2y/dz^2$), the corresponding governing differential equation can be obtained as:

$$EI\frac{d^4y}{dz^4} + Q\frac{d^2y}{dz^2} + q - E_s y = 0 \tag{3}$$

where I is moment of inertial of the pile, y is lateral deflection of the pile at a point z, Q is axial load on the pile, q is distributed load along the length of the pile and E_S is stiffness for the lateral soil reaction represented by the secant modulus of the soil-response ($p-y$) curve.

The finite (central) difference approximation of the second and fourth derivatives of y is expressed, respectively, as:

$$\frac{d^2y}{dz^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \tag{4}$$

$$\frac{d^4y}{dz^4} = \frac{y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}}{h^4} \tag{5}$$

Similarly, if the pile is subdivided in increments of length h, as shown in Fig. 2, the governing differential equation for the beam-column, Eq. (3), can be written using Eqs. (4) and (5), in difference form for arbitrary node i as:

$$\begin{aligned} &y_{i-2}(EI)_{i-1} + y_{i-1}[-2(EI)_{i-1} - 2(EI)_i + Qh^2] \\ &+ y_i[(EI)_{i-1} + 4(EI)_i + (EI)_{i+1} - 2Qh^2 + E_s h^4] \\ &+ y_{i+1}[-2(EI)_i - 2(EI)_{i+1} + Qh^2] + y_{i+2}(EI)_{i+1} + qh^4 = 0 \end{aligned} \tag{6}$$

Since the pile is discretized into n elements with $n+1$ nodes, as shown in Fig. 2, the primary unknowns, i.e., the $n+1$ vertical deflections w_i and the $n+1$ lateral displacement y_i, can be solved for the $n+1$ equations after introducing the boundary conditions. As a result, the secondary unknowns such as the bending moment M, the shear V, the soil reaction p, and the axial load, etc. are subsequently obtained.

3. Pile Head Stiffness of A Single Pile

The stiffness of a pile group in consideration of pile-cap interaction mostly depends on the stiffness characteristics of the individual pile in a group and the configuration such as pile head fixity, number of piles,

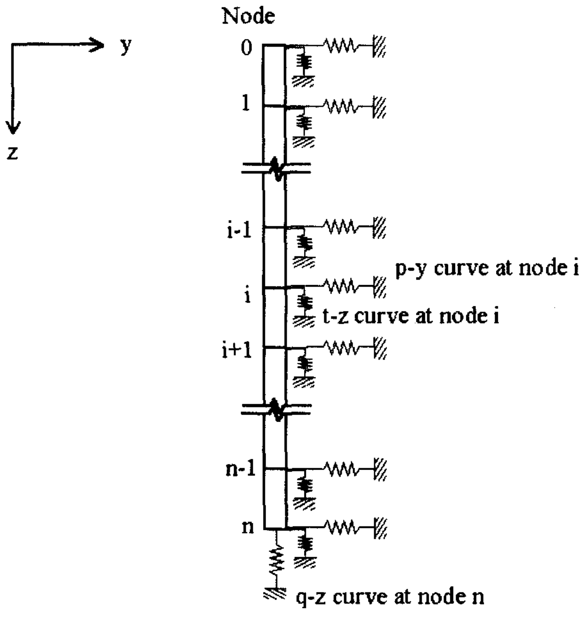


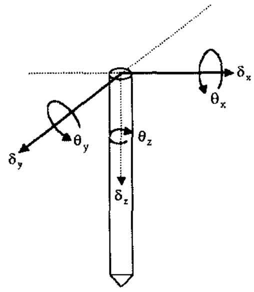
Fig. 2 Beam-column discretization

pile head coordinates, loading position, etc. Therefore, at this stage it is necessary to investigate the pile head stiffness of a single pile. Since this subject is extensively studied and available in the literature (Lam and Martin, 1986), only some of its essential features are briefly discussed below. When a single pile is considered to be rigidly connected to a pile cap, the full pile-head stiffness matrix with consideration of translation, rotation, and cross-coupling components should be used in the formulation of the stiffness of a pile group, as shown in Fig. 3.

When a pile-cap connection, on the other hand, is pinned, the condensed pile-head stiffness matrix with consideration of only translation as shown in Fig. 4 needs to be used.

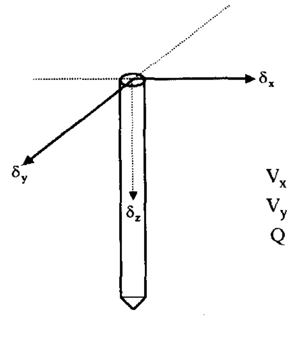
k_{ij} in Figs. 3 and 4 can be obtained from the single pile analysis with necessary boundary condition. For example, if lateral force V is applied at the top of a single pile with boundary condition that $\theta_x = \delta_z = 0$ at the pile head, k_{11} and k_{51} are simultaneously obtained as follows:

$$k_{11} = \frac{V}{\delta_x} \tag{7}$$



$$\begin{array}{c|cccccc} & \delta_x & \delta_y & \delta_z & \theta_x & \theta_y & \theta_z \\ V_x & k_{11} & 0 & 0 & 0 & -k_{15} & 0 \\ V_y & 0 & k_{22} & 0 & k_{24} & 0 & 0 \\ Q & 0 & 0 & k_{33} & 0 & 0 & 0 \\ M_x & 0 & k_{42} & 0 & k_{44} & 0 & 0 \\ M_y & -k_{51} & 0 & 0 & 0 & k_{55} & 0 \\ M_z & 0 & 0 & 0 & 0 & 0 & k_{66} \end{array}$$

Fig. 3 Pile head stiffness matrix for a rigid pile-cap connection



$$\begin{array}{c|ccc} & \delta_x & \delta_y & \delta_z \\ V_x & k'_{11} & 0 & 0 \\ V_y & 0 & k'_{22} & 0 \\ Q & 0 & 0 & k'_{33} \end{array} \qquad \begin{aligned} k'_{11} &= k_{11} - \frac{k_{15}k_{51}}{k_{55}} \\ k'_{22} &= k_{22} - \frac{k_{24}k_{42}}{k_{44}} \\ k'_{33} &= k_{33} \end{aligned}$$

Fig. 4 Pile head stiffness matrix for a free pile-cap connection

and

$$k_{51} = \frac{-M}{\delta_X} \tag{8}$$

where δ_X and M are the computed displacement and the reaction moment at the pile head, respectively.

4. Stiffness of a Pile Group

The stiffness of a pile group in consideration of pile-cap interaction can be developed by integration of the reaction forces at each pile for a prescribed unit displacement for each degree of freedom at the point of loading at the pile cap. Lam and Martin (1986) suggested such a procedure that estimates the full stiffness matrix of the pile group at the pile cap using a linear representation of the stiffness matrix of a single pile, as briefly discussed below, and it is adopted in this paper.

Utilizing a rigid pile cap assumption, the displacements at each pile head can be evaluated after a unit displacement with respect to a specific degree of freedom at the point of loading is applied. The reaction forces at each pile head thus can be obtained by multiplying the pile-head displacements and the pile-head stiffness matrix depending on the pile-cap connection, i.e.,

$$\{F_i\} = [K]\{\delta_i\} \tag{9}$$

where i represents i^{th} degree of freedom.

Next, calculate the statically equivalent set of forces at the point where loading is applied to each set of pile-head forces. Finally, the resultant forces just obtained for all the piles for a group pile are accumulated. This force vector, which actually represents stiffness vector, is a specific column of the pile group stiffness matrix with respect to the degree of freedom under consideration. This entire process needs to be repeated for all the degrees of freedom for the pile cap and the full stiffness matrix of the pile cap is thus obtained.

5. Estimation of Individual Pile Head Forces and Moments

For a complete group pile analysis, it is necessary to estimate each individual pile head forces and moments distributed from the total pile group load so that the response of each individual pile in the group pile can be examined with consideration of pile-cap interaction. The estimation of individual pile head forces and moments can be successfully achieved in reverse order of the procedure to obtain the stiffness of a pile group, discussed in the previous section. The displacements of each individual pile head can be obtained using the displacement of the pile group at the loading point. Thus, the pile head loads in terms of forces and moments can be calculated by multiplying this displacement vector of each pile head by the pile head stiffness matrix, one of either matrix defined in Fig. 3 or 4 according to the pile head-pile cap connectivity condition.

6. An Unified Group Pile Analysis Algorithm

In the proposed algorithm, the single pile head stiffnesses in a group pile are initially obtained by assuming that all piles in the group pile are subjected to the equally distributed load, which is the total applied load divided by the number of piles in the group pile. However, the load at an individual pile head, distributed from the total pile group load, is expected not to be the same as those of other piles since the geometric location of the pile in

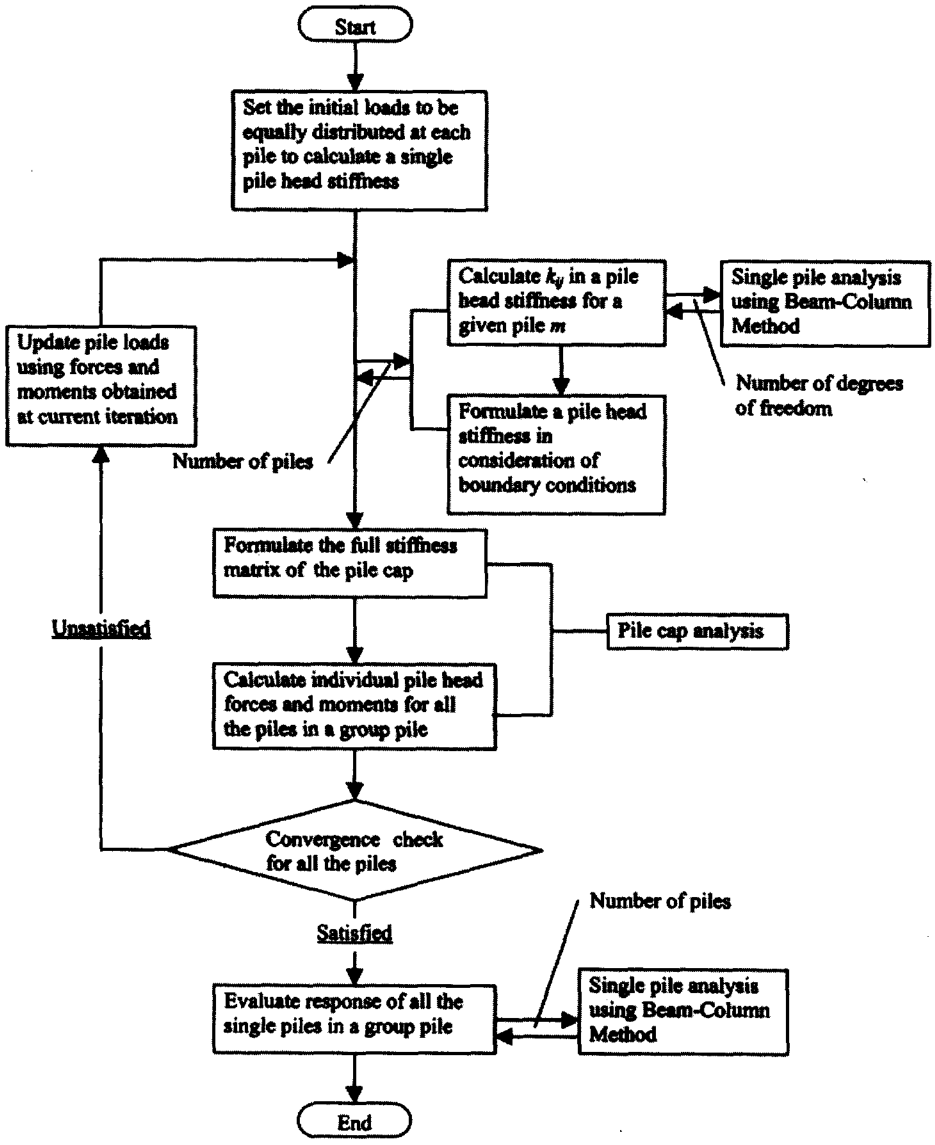


Fig. 5 Flowchart of the proposed algorithm (YG-group program)

the group is different from each other. Furthermore, due to the way to define the stiffness of a single pile in this study, the magnitude of the applied load component may largely control the single pile head stiffness. Therefore, each single pile head stiffness for all the piles needs to be updated for the loading status under consideration. Consequently, in this study the iteration procedure is used to update the individual pile head stiffness of all the piles in a group pile until the difference between the applied load and the distributed load at each individual pile head is within the convergence criteria.

The main steps of the proposed algorithm are as follows:

1. Initially, it is assumed that total pile group loads are equally distributed at each individual pile in the magnitude of total loads divided by the number of piles in a group.
2. Apply only one distributed load component, which is related to the degree of freedom under consideration, along with the proper boundary conditions and obtain the corresponding stiffness components in a pile head stiffness matrix. This process should be conducted for all degrees of freedom.
3. Formulate the pile head stiffness of single piles in a group pile using the stiffness components obtained in step 2 with consideration of the pile-cap connectivity condition. Carry out this procedure as many times as the number of piles, excluding the piles that have the identical loading conditions to the one considered already.
4. Using all the individual pile head stiffnesses, formulate the full stiffness matrix of the pile cap according to the procedure described in previous section, 'STIFFNESS OF A PILE GROUP'.
5. Calculate individual pile head forces and moments for all the piles in the group pile.
6. Compare the individual pile head forces and moments calculated in step 5 with the applied distributed load components used in step 2 for all the individual piles if the difference between them meets the user-specified closure tolerance level.
7. If the convergence criterion in step 6 is not satisfied, use the pile head forces and moments obtained in step 5 as the new load components to be applied for the estimation of a new individual pile head stiffness matrix for each pile and repeat this iteration process (step 2 ~ step 7). If it is satisfied, go to next step.
8. Using the final individual pile head forces and moments, evaluate responses of all the single piles in a group.

The flowchart that shows all the steps described above is also given in Fig. 5. Based on the proposed algorithm, a new computer program YS-GROUP has been developed to analyze the group pile behavior with consideration of both soil-pile and pile-cap interactions. The case studies provided later in the examples are conducted using the program and its capabilities are also discussed.

7. Comprison with Other Case Histories

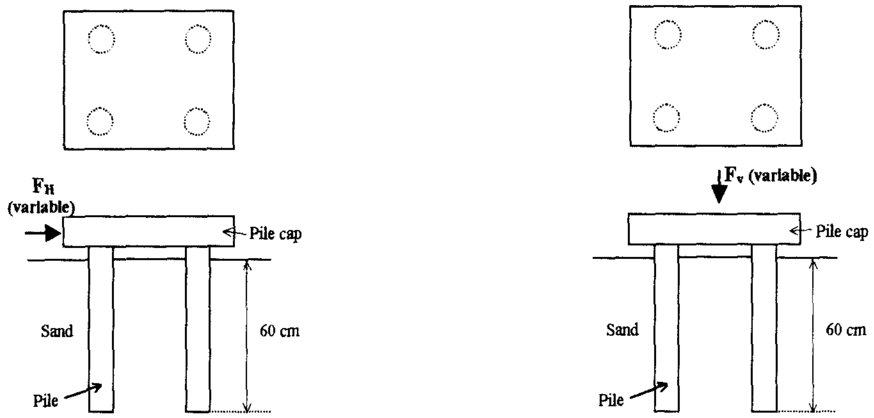
The validity of the proposed algorithm was tested by comparing the results from the present approach with some of the measured results in detail in the following section.

Chung and Jeong (2001)

Small scale model tests were carried out to study the behavior of pile groups subjected by axial and lateral loadings on sand soil. The soil conditions and the model pile data are summarized in Table 1. Fig. 6 shows an idealization of the subsurface profile and pile embedments for test piles. Fig. 7 and Fig. 8 show the predicted

Table 1. Material properties of the test soil and pile

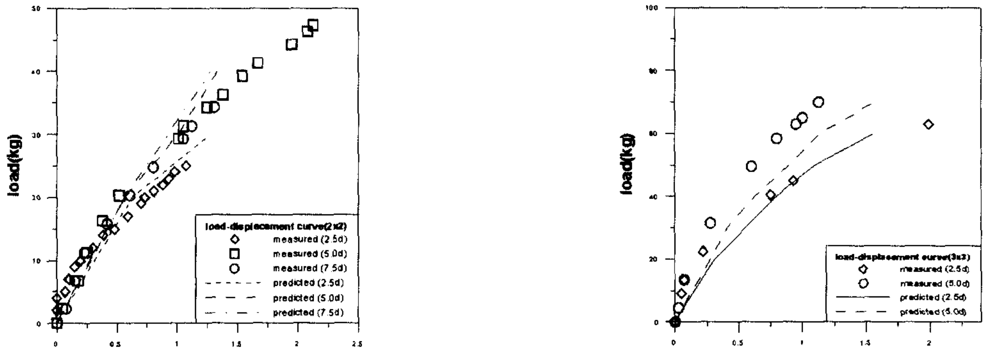
Soil (sand)	Unit weight, γ_t(kN/m^3)	15.3
	Relative density, Dr(%)	73
	Cohesion, c(kN/m^2)	N/a
	Internal friction angle, Φ(°)	37
Pile	Material	PVC pipe
	Length(mm)	630
	Diameter(mm)	22
	Thickness(mm)	2.5
	Elastic Modulus, E(kN/m^2)	3.84×10^4
	Flexural Rigidity EI(kN · m^2)	28264.89



(a) Lateral loading

(b) Axial loading

Fig. 6 Test pile group configurations (Chung and Jeong, 2001)



(a) 2 × 2 Pile groups

(b) 3 × 3 Pile groups

Fig. 7 Lateral load-displacement curves at pile head

and observed lateral load-settlement curves (Fig.7) and axial load-settlement curves (Fig.8), respectively. The analysis of pile groups was performed for fixed head condition and spacing-to-diameter ratios varying from 2.5 to 7.5. The present method relatively well predicts the general trend of the measured lateral loads and axial loads if the measured deflections are small (say less than 15 mm).

Stevens and Holloway (1979)

Comprehensive measurments were carried out by Stevens and Holloway (1979). A full scale test piles were constructed on the site of Mississipi River, Alton, Illinoi. The lateral load capacities of the 8-pile group were determined under combined axial and lateral loads. The configurations of the groups are presented in Fig. 9. The soil conditions and the pile properties are summarized in Table 2. Fig. 10 shows the predicted and measured lateral load-displacement curves. A reasonably good agreement between the present solution and measured ones was obtained.

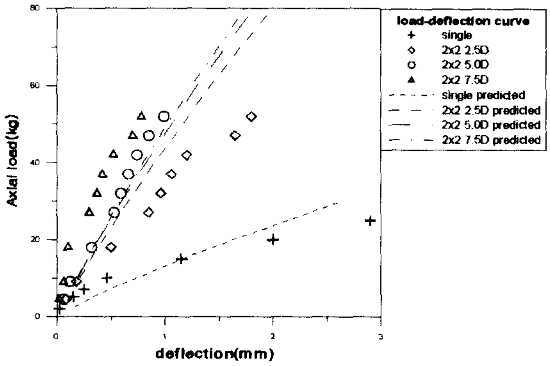


Fig. 8 Axial load-displacement curves at pile head
(2 × 2 pile groups)

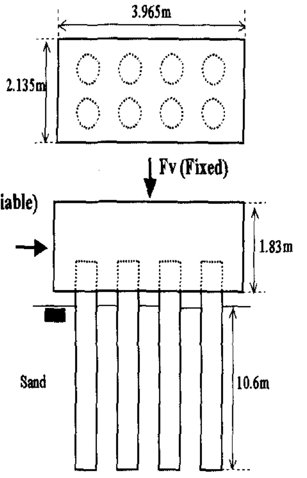


Fig. 9 Test pile group configuration (Stevens and Holloway, 1979)

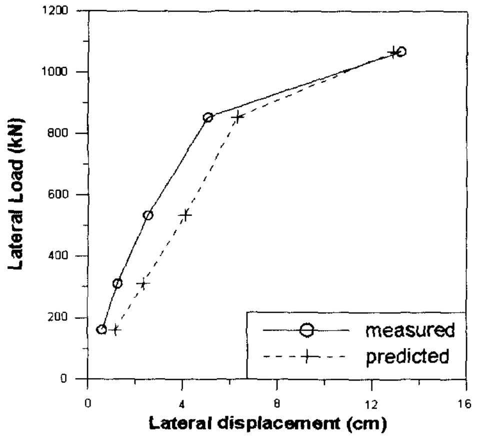


Fig. 10 Load-displacement curves at pile head (2 × 4 pile groups)

8. Conclusions

The main objective of the analysis described herein was to investigate the load distribution and deformation of pile groups using an analytical study and experimental tests. From the findings of this study, the following conclusions are drawn :

1. The load-settlement of pile group can be calculated by load transfer approach which is modeled by piecewise linear soil springs (p-y, t-z, and q-z cerves), without recourse to 3D finite-element analysis.
2. The beam-column method gives an idea of the vertical and horizontal load distribution in the pile groups. The predictions are close to the measurements if the measured deflections are small (less than 15 mm).

Table 2. Material properties of the iest soil and pile

Soil	Unit weight, γ_t(kN/m^3)	17.7
	Cohesion, c(kN/m^2)	N/a
	Internal friction angle, Φ(°)	39
Pile	Material	Concrete
	Length(m)	10.5
	Diameter(cm)	30
	Elastic Modulus, E(kN/m^2)	7.14×10^6
	Axial load (kN)	2135

Acknowledgements

This paper was supported by grant No.1999-1-311-02-3 from the interdisciplinary Research program of the KOSEF.

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