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An Approximated Reasoning with Compensation

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Abstract

In this paper, a fuzzy hyperresolution principle called CFHR, Compensatory Fuzzy Hyperresolution, with positive compensation facility is proposed. Usually hyperresolution has several terms of condition parts. These terms have to be connected by the and connective. If the min/max operator to be used the and operation, there is some dependency problem of the min/max operator. So, we propose a compensatory operator EGM and applied it to the CFHR. We show the CFHR does more meaningful reasoning than existing method. We also prove the completeness of CFHR.

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1. Introduction

The resolution principle proposed by Robinson[1] is a fundamental technique for mechanical reasoning or question answering system. The resolution principle is an inference rule based on binary logic that deduces null clause from some unsatisfiable clause set. But there is imprecise facts - about forty years old - or queries and also will be happened unpredictable situation. For example, let us suppose that there are following (rule 1) and (fact 1).

(Rule 1) If the color of a tomato is red, then the tomato is ripen.
(Fact 1) Tomato A is red
(Result 1) The tomato A is ripen.

We can get the (result 1) "The tomato A is ripen" to infer above (rule 1) and (fact 1). But we can not make an inference using the binary resolution if we have "tomato B is deep red" or "tomato C is a light red". To make an inference these facts, binary logic based system have to keep additional rules as follows.

(Rule 2) If the color of a tomato is deep red, then the tomato is ripen very well.
(Rule 3) If the color of a tomato is light red, then the tomato is not ripen sufficiently.

The binary resolution based on string matching needs every kinds of rules expecting for joining the inference. But it is impossible to represent all of the rules for the whole facts. It is possible to represent truth value [0, 1] about some expression in fuzzy resolution. So "the tomato is ripen very well (Rule2)" or "tomato is not ripen sufficiently (Rule 3)" are can be derived.

Many researchers have studied reasoning about fuzzy logic[2, 3]. Lee[4] discussed the resolution principle in fuzzy logic. He introduced the "half-truth" concepts and showed that if every clause in a set of clauses is something more than a "half-truth" then we are guaranteed that all the logical consequences obtained by repeatedly applying the resolution principle will have the truth-value. Shen[5] introduced the concepts of fuzzy contradictory, contradictory degree, fuzzy resolvent and confidence of resolvent. He extended binary resolution to the fuzzy resolution and proved its completeness.

Shen's work is very interesting. But he did not consider dependency problem to calculate a contradictory degree[6]. For example, consider one man P wants to marry a girl Q1 ($\mu = 0.60$), and the man also like to marry another girl Q2 ($\mu = 0.55$). While the girl Q1 hopes to marry P with truth value $\mu = 0.60$ and the Q2 hopes to marry P with truth value $\mu = 0.90$. It can be expressed as follows :

$$\min(P(0.60), Q1(0.60)) = 0.60.$$
$$\min(P(0.55), Q2(0.90)) = 0.55.$$

We can see the possibility for P to marry the Q1 is 0.60 and the possibility for P to marry the Q2 is 0.55. Though the Q2 want to marry P eagerly (0.90) but the result turned out 0.55 less than P and Q1 (0.60). It caused by dependency problem of min/max operator.

Hyperresolution usually has several terms of condition parts[7]. These term have to be connected by the and connective. If the min/max operator to be used the and operation, there is some dependency problem of the min/max operator of fuzzy set theory.

In this paper, we propose an inference rule called Compensatory *Fuzzy Hyperresolution (CFHR)* which has a compensation operator *Extended Generalized Mean (EGM)*. Theoretical background and effectiveness

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of the CFHR are shown. Finally the completeness of the CFHR are proven.

2. Theoretical Background

2.1 Fuzzy resolution and hyperresolution

Fuzzy logic is defined as {[0, 1], ∧, ∨, ¬}. [0, 1] means real number of truth value between 0 and 1. (and), (or), (not) are defined as follows[5, 7, 8, 9, 11, 12].

P ∧ Q = EGMAND(P, Q).
P ∨ Q = EGMOR(P, Q).
¬P = 1 - P (where P, Q ∈ [0, 1]).

Here, EGM_{AND} and EGM_{OR} means that *compensatory and* and *compensatory or*, respectively. Each compensatory operator includes *min* and *max* operators.

Definition 1 Variable Pi (i = 1, 2, ..., n) or its negation ¬Pi is said to be literal, Pi and ¬Pi are said to be complements of each other. A clause C is disjunction of literals.

Definition 2 Given interpretation I, the truth value of a clause C is $T_I(C) \in [0, 1]$.

Definition 3 For a set of S of clauses, the truth value of S under a given interpretation I as $T_I(S)$, where if S = {C_1, C_2, ... , C_n}, is EGM_{AND} of each truth value of C_1, ... , C_n.

$$T_I(S) = EGM_{AND}(T_I(C_1 \wedge C_2 \wedge \ldots \wedge C_n)) = (T_I(C_1), \ldots, T_I(C_n)).$$

Definition 4 Let a pair of complement literal P and ¬P. P ∧¬P is said to be a contradiction.

Definition 5 Let contradictory degree of P ∧¬P be cd(P) = $EGM_{OR}(T_I(P), T_I(\neg P)) - EGM_{AND}(T_I(P), T_I(\neg P))$.

There is some kinds of resolutions[9] The *hyperresolution* of them resembles a If-Then rule of expert systems or reasoning systems. Fuzzy concepts applies to the hyperresolution. The definition and concept of the hyperresolution are proposed by Robinson[3]. In the hyperresolution, the positive clauses A_1, A_2, ... , A_m and negative or mixed clause B generate a positive clause. Any intermediate results are not generated.

Definition 6 The inference rule hyperresolution considers simultaneously a clause B that contains at least one negative literal and a set of clauses Ai each of which contains only positive literals, and yields a clause C containing only positive literals. The clause B is termed the nucleus and the clauses A_i the satellites for the application of hyperresolution. The clause B is termed a hyperresolvent.

¬P ∨ ¬Q ∨ R ∨ S	: (It means that "if P and Q the R or S")
P ∨ T	: A_1
Q ∨ W	: A_2
T ∨ W ∨ R ∨ S	: C

In the above example, we can see that all the negative literals of B must be clashed with the positive literals of A_1 and A_2

2.2 EGM compensation operator

Let us consider Dykhoff[10]'s generalized mean operator (GM) and show some problem for using it compensation operator. Dykhoff's GM was defined as follows.

Definiton 7 The GM has number of operands, A_1, A_2, ... , A_n, ($A_i \in [0, 1]$), is defined as follows[10].

$$GM(A_1, A_2, \ldots, A_n; p, w_1, w_2, \ldots, w_n) = (\sum_{i=0}^{n} w_i \cdot A_i^{p})^{1/}$$

Here, p is compensation parameter which can have all the range of real number (p ∈ R). wi는 is weighting factor, it has to get more than 0. Sum of the all the weighting factor is 1.

$$\sum_{i=1}^{n} w_i = 1$$

Dykhoff' GM has several merit but it also has a problem. When compensation and or compensation or calculate the truth values, if any operand (A_i) is 0 and $\sum_{i=1}^{n} w_i \cdot A_i^{p}$is less than 1, then the result value of compensation is not between 0 and 1.

Theorem 1

$A_i = 0$ and ($\sum_{i=1}^{n} w_i \cdot A_i^{p}$) < 1, ($\sum_{i=1}^{n} w_i \cdot A_i^{p}$)1/p ∉ [0, 1] 이다.

proof If $A_i = 0$ and ($\sum_{i=1}^{n} w_i \cdot A_i^{p}$) < 1 then Dykhoff's GM is more than 1. It is violated that all the truth value is between 0 and 1 ($A_i \in [0, 1]$).

To satisfy the property $[0, 1]^n \rightarrow [0, 1]$, we propose a new compensatory mean operator, so called Extended Generalized Mean operator (EGM).

Theorem 2 There are the number of operands A_1, A_2, ... , A_n ($A_i \in [0, 1]$) which each operand can have weighting factor wi. The following EGM always is in [0, 1] regardless of Ai or ($\sum_{i=1}^{n} w_i \cdot A_i^{p}$).

$$EGM(A_1, \ldots, A_n; p, w_1, \ldots, w_n) = (\sum_{i=0}^{m} w_i \cdot A_i^{p})^{1/p} (\prod_{i=m+1}^{n} A_i)$$

proof If $A_i = 0$ and ($\sum_{i=1}^{n} w_i \cdot A_i^{p}$) < 1 then EGM is between 0 and 1 ($A_i \in [0, 1]$).

Here, p is compensation parameter which can have all the range of real number. wi is more than 0 and sum of all the wi is 1.

EGM has eight property of Dykhoff'GM and also satisfies basic property of connectives (EGM ∈ [0, 1]). To adapt compensation parameter p, we get the *and* *compensation operator* EGM_{AND} and *or compensation operator* EGM_{OR}.

EGM_{AND} : $-\infty < p \leq 1$.
EGM_{OR} : $1 \leq p < +\infty$.

Property of EGM is as follows.

(1) If p = 1 then it is arithmetic mean
(2) If $p \to 0$ then it is geometric mean.
(3) If $p \to -1$ then it is harmonic mean.
(4) If $p \to -\infty$ then it is min.
(5) If $p \to +\infty$ then it is max.
(6) If $p > 1$ then it can be used OR(including max) operator.
(7) If $p < 1$ then it can be used AND(including min) operator.
(8) If $-\infty < p < q < +\infty$ then it is $EGM(p) = EGM(A_1, A_2, \ldots, A_n ; p, w_1, w_2, \ldots, w_n) \leq EGM(A_1, A_2, \ldots, A_n ; p, w_1, w_2, \ldots, w_n) = EGM(q)$.

3. Compensatory Fuzzy Hyperresolution

CFHR concepts and its execution are shown in this section. We can see the negative dependency of min operator to comparing CFHR with EGM and existing fuzzy resolution with min operator. First of all, we define CFHR.

Definition 8 The inference rule *Compensatory Fuzzy Hyperresolution* considers simultaneously a clause B that contains at least one negative literal and a set of clauses Ai each of which contains only positive literals, and yields a clause C containing only positive literals with confidence of fuzzy resolvent δ successfully. The clause B is termed the nucleus and the clauses Ai the satellites for the application of hyperresolution. The clause B is termed a compensatory fuzzy hyperresolvent.

$\neg P \vee \neg Q \vee R \vee S$: (It means that "if P and Q the R or S")
P(truth value = 0.55) $\vee$ T : A_1
Q(truth value = 0.90) $\vee$ W : A_2

$(T \vee W \vee R \vee S)_{\delta \; EGMAND(\delta(P),\ \delta(Q))}$: C

Example 1 Consider one man P wants to marry a girl Q1 with $\mu = 0.60$, and the girl Q1 also like to marry the man P ($\mu = 0.6$)(case 1). While the man P want marry another girl Q2 with $\mu = 0.59$ and the girl Q2 hope to marry P with truth value $\mu = 0.99$ (case 2). Make an inference with CFHR and find which couple has more possibility to marry. Apply min/max and EGM operator to the calculation for contradictory degree.

Solution [ ] clause can be derived form satisfiable set S = {marry(P, Q1), marry(P, Q2), marry(Q1, P), marry(Q2, P), $\neg$marry(x, y) $\vee$ $\neg$marry(x, y) $\vee$ marriage(x, y), marriage(P, Q1), marriage(P, Q2)}

(case 1 : min operator)
δ = min((0.60 - 0.40), (0.60 - 0.40))
= min(0.20, 0.20) = 0.20
(case 1 : EGM_{AND} operator, here p = 1)
δ = EGMAND((0.60 - 0.40), (0.60 - 0.40))
= EGMAND(0.20, 0.20) = 0.20
= (0.5 × 0.201 + 0.5 × 0.201)1 = 0.20
(case 2 : min operator)
δ = min((0.59 - 0.41), (0.99 - 0.01))
= min(0.18, 0.98) = 0.18
(case 2 : EGMAND operator, here p = 1.5)
δ = EGMAND((0.59 - 0.41), (0.99 - 0.01))
= EGMAND(0.18, 0.98)
= (0.5 × 0.181.5 + 0.5 × 0.981.5)1/1.5 = 0.65

The min operator yields case 1 (0.20) is more possible to marry than case 2 (0.18). But EGM_{AND} operator yields couple of case 2 (0.65) have higher possibility than case 1 (0.20). Considering the truth value of P, Q1 and Q2, we can see the P and Q2 couple's possibility is more high than the couple P and Q1.

Example 2 Show a [ ] clause is deducing from S = {$\neg$A $\vee$ $\neg$B $\vee$ C, A, B, $\neg$C} using the min/max operator and EGM operator respectively. Assume, in case 1: T(A) = 0.60, T(B) = 0.70, T(C) = 0.90. In case 2: T(A) = 0.60, T(B) = 0.60, T(C) = 0.60.

Solution [ ] clause can be derived form satisfiable set S = {$\neg$A $\vee$ $\neg$B $\vee$ C, A, B, $\neg$C}.

(case 1 : min operator)
δ = min((0.60 - 0.40), (0.70 - 0.30), (0.90 - 0.10))
= 0.20
(case 1 : EGM_{AND} operator, here p = 1.5)
δ = EGM_{AND}((0.60 - 0.40), (0.70-0.30), (0.90-0.10))
= EGM_{AND}(0.20, 0.40, 0.80)
$= (0.33 \times 0.20^{1.5} + 0.33 \times 0.40^{1.5} + 0.33 \times 0.80^{1.5})^{1/1.5}$
= 0.50
(case 2 : min operator)
δ = min((0.60 - 0.40), (0.60 - 0.40), (0.60 - 0.40))
= 0.20
(case 2 : EGM_{AND} operator, here p = 1)
δ = EGMAND((0.60 - 0.40), (0.60-0.40), (0.60 -0.40))
= 0.20

CFHR show that case 1 get higher possibility than case 2. But min operator can not discriminate case 1 and case 2 though varying truth value B and C.

4. Completeness of Compensatory Fuzzy Hyperresolution

Binary logic or classical logic has a number of inference rule such as binary resolution, hyperresolution, demodulation, paramodulation, and subsumption strategy[9]. A new inference rule have to be proven the completeness.

The first resolution was published and proven by Robinson in 1965[1]. He also proposed the hyperresolution which is more powerful inference rule in the same year[7]. Seven years later in 1972, Lee[4] introduced fuzzy logic to the resolution principle. Ding[5, 8] proposed and proved fuzzy resolution principle. The CFHR solved the dependency problem of min/max operator to make compensation for biased result of min or max value.

Completeness of CFHR is described in the following. We have some assumption to simplify and proof procedures.

Lemma 4.1 *Let C_1 and C_2 be two clauses. Let $R(C_1, C_2)$ denotes any resolvent of C_1 and C_2. Let $EGMMAX[T(C_1), T(C_2)] = b$ and $EGM_{MIN}[T(C_1), T(C_2)] = \beta > 0.5$. Then $b \le T(R(C_1, C_2)) \le b$ ($\beta \le b$).*

Proof :
Let $C_1 = P \vee L_1$, $C_2 = P \vee L_2$, we can represent $R(C_1, C_2) = L_1 \vee L_2$.

$$T(C_1) = EGM_{MAX}[T(P), T(L1)] = \alpha \quad (1)$$
$$T(C_2) = EGM_{MAX}[T(P), T(L2)] = b \quad (2)$$

From (1) and (2), we get $T(L_1) \le \alpha$ and $T(L_2) \le b$.

Case (a) : We suppose $T(L_1) = \alpha$.
$T(R(C_1, C_2)) = T(L_1 \vee L_2)$
$= EGM_{MAX}[T(L_1), T(L_2)] = EGMMAX[\beta, T(L_2)]$
Thus, $b \le T(R(C_1, C_2)) \le b$.

Case (b) : We assume $T(L_1) < \beta$.
From equation (1) and $T(P) = \beta$, $\beta > 0.5$.
Therefore $T(\neg P) = 1 - (P) < 0.5 < \beta$.
And from equation (2), $T(L_2) = b \ge \beta$.
$T(R(C_1, C_2)) = T(L_1 \vee L_2)$
$= EGM_{MAX}[T(L_1), T(L_2)] = b$.
Therefore, we get the results from both case $\beta \le T(R(C_1, C_2)) \le b$.

Lemma 4.2 *Let S be a set of clauses $C_1, C_2, ..., C_m$. Let $EGM_{MAX}[T(C_1), T(C_2), ..., T(C_m)] = b$ and $EGM_{MIN}[T(C_1), T(C_2), ..., T(C_m)] = \beta$. Let Cn denoted by any clause in the set $R^n(S)$. Thus $b \le T(C_1) \le b$ (for all $n \ge 0$).*

Proof :
Proof by Lemma 4.3 and definition 2.9 in pp 117 of [1].

Theorem 4.1 (Completeness of CFHR)
A set S of fuzzy clauses is unsatisfiable if and only if there is a compensation fuzzy hyper-deduction of empty clause [ ] with its confidence of fuzzy resolvent $\delta \neq 0$ from S.

Proof :
(a) (→)
Assume that S is unsatisfiable. Then it is unsatisfiable in binary logic by Lemma 4.1. Thus empty clause [ ] must be deduced from Lemma 4.2 and S. $\delta \neq 0$ or $T([\ ]_{\delta}) \le 0.5$ is meaningless, then $\delta = 0$.

(b) (←)
Suppose there is a deduction of [ ] with $\delta \neq 0$. And assume that S is satisfiable. Then we get $T(S) \ge 0.5$ and get $T([\ ]) < 0.5$. But this is impossible. Therefore we can represent $T(C^n)$ as $0.5 < T([\ ])$. It is not accepted because of $0.5 > T([\ ])$. Thus S must be unsatisfiable.

5. Conclusion

We proposed an intelligent fuzzy resolution for rule-based systems or intelligent systems. Some existing resolution principles with fuzzy concepts produce non-commonsense resolvents because they use the min/max operation of fuzzy set theory to compute the confidence of resolvents for executing resolution. But proposed *CFHR* with positive compensation facility did more meaningful reasoning than existing method. We showed the effectiveness of *CFHR* and completeness.

We expect that fuzzy knowledge-based systems with *CFHR* can be applied to several fields such as uncertain knowledge-based environments, imprecise query processing and intelligent information integration. Further research should be concentrated on such topics as finding suitable model for applying real world application and selection strategies for proper clause.

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