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Tooth modification of helical gears for minimization of vibration and noise

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ABSTRACT

Vibration and noise of gears is due to the transmission error and the vibration exciting force caused by the periodically alternating tooth stiffness. Transmission error is the rotation delay between driving and driven gear caused by manufacturing error, alignment error in assembly and so on. Tooth stiffness changes with the proceeding mesh of teeth. The purpose of this study is to develop how to calculate simultaneously the optimum amounts of tooth profile modification, end relief and crowning by minimizing the vibration exciting force of helical gears. We estimate the vibration exciting force by the meshing analysis of gears. Formulated constraints of this problem consist of contact ratio and strengths of gear teeth such as tooth bending strength, surface durability, and scoring. ADS(Automated Design Synthesis) is used as an optimization tool. We also investigate the relation between the aspect ratio and the optimum values of tooth modification. The proposed method can calculate the optimum amount of tooth modification automatically and is expected to be practically useful to resolve the problem of vibration of helical gears.

Key Words : Helical gear, Tooth Modification, Tooth Profile Modification, End Relief, Crowning, Vibration Exciting Force

1. Introduction

The meshing vibration of gear occurs due to not only the meshing impact and the change of mesh stiffness of gear tooth but also the manufacturing error and the assembly error of gear train(1). In general, tooth modification methods are used in order to reduce the meshing vibration and noise of gear train. Kinds of such methods are, for example, tooth profile modification towards involute curve, crowning towards face width, end relief and so on. Several Researches(2)~(6) have been made focusing on reducing vibration of gears utilizing these tooth modification methods, but none of those could determine simultaneously the optimum amounts of tooth profile modification as well as those of tooth width modification for helical gears commonly used in power transmission devices. Therefore, in this research, the design methods of tooth modification amounts for helical gears are developed to reduce meshing vibration and noise of gears by not only reducing the meshing impact of gears but also improving the attributes of gear pair contact applying the optimization techniques.

2. Method of tooth modification

2.1 Tooth profile modification

Tooth profile modification is the method by which tooth profile is subject to change from theoretical involute curve by means of reducing tiny amounts at tooth tip or root fillet of tooth. Such kinds are tip relief and full profile modification. In this research, linear tooth profile modification is applied only to the case where tip relief is chosen as tooth profile modification. Fig. 1 shows 2-dimensional geometry of half-size tooth having tip relief amount divided using the centerline of tooth as basis. In fig. 1, s means the tooth tip thickness, s_o the length of tip relief and h_o means the amount of tip relief. The amount of tip relief is calculated by transforming into the amount of tooth error and this error $e(h_i)$ is determined by equation (1) using the coordinate h_i in radial direction from tooth tip. The amount of tip relief $e(h_i)$ is then entered into the meshing analysis program as input parameter.

$$e(h_i) = \begin{cases} \dfrac{h_o - h_i}{h_o} \cdot s_o, & h_i < h_o \\ 0, & h_i \geq h_o \end{cases} \quad (1)$$

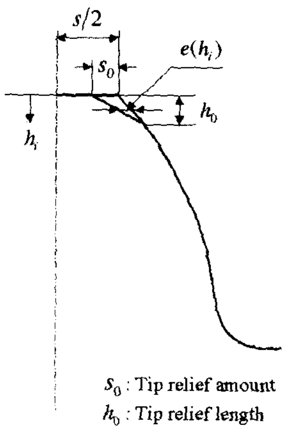

Fig. 1 Tip relief

2.2 Crowning and end relief

Fig. 2 and fig. 3 show symmetrical crowning and skew-symmetrical crowning respectively. The radius of curvature of symmetrical crowning ρ_{sym} is calculated using equation (2) and the amount of modification $e(b_j)$ is calculated by equation (3). These calculated values are then used as input parameters of the meshing analysis program. c_c is the amount of crowning and b is the face width.

$$\rho_{sym} = \frac{4c_c^2 + b^2}{8c_c} \quad (2)$$

$$e(b_j) = \rho_{sym} - \sqrt{\rho_{sym}^2 - b_j^2}\ , -\frac{b}{2} \leq b_j \leq \frac{b}{2} \quad (3)$$

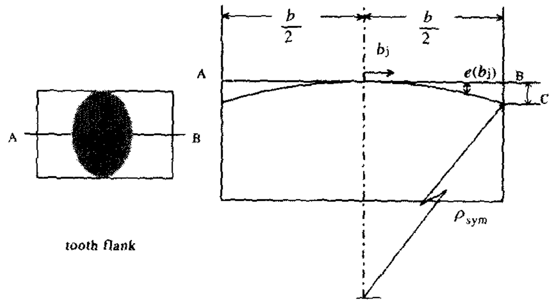

Fig. 2 Symmetrical crowning

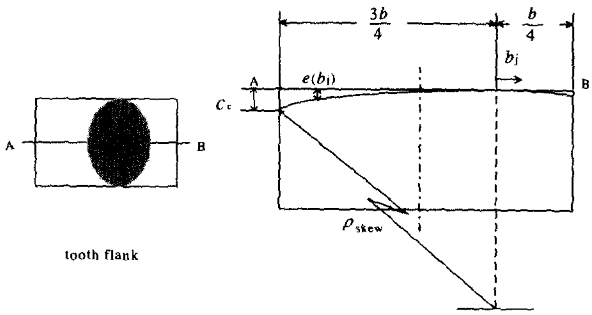

Fig. 3 Skew-symmetrical crowning

The radius of curvature of skew-symmetrical crowning ρ_{skew} and the amount of face width modification $e(b_j)$ are calculated by equation (4) and (5) respectively. In this research, skew-symmetrical crowning having 3 to 1 aspect ratio is used.

$$\rho_{skew} = \frac{4c_c^2 + (1.5b)^2}{8c_c} \quad (4)$$

$$e(b_j) = \rho_{skew} - \sqrt{\rho_{skew}^2 - b_j^2}\ , -\frac{3b}{4} \leq b_j \leq \frac{b}{4} \quad (5)$$

The amount of face width modification by means of linear tip relief(shown in fig. 4) $e(b_j)$ is calculated through equation (6).

$$e(b_j) = \begin{cases} 0 & , -\dfrac{b_{red}}{2} \leq b_j \leq \dfrac{b_{red}}{2} \\ c_e \cdot \dfrac{|b_j| - \dfrac{b_{red}}{2}}{b_e} & , b_j < -\dfrac{b_{red}}{2}\ ,\ b_j > \dfrac{b_{red}}{2} \end{cases} \quad (6)$$

In equation (6), c_e is the amount of end relief, b_e is the length of end relief, and b_{red} is the width of end relief. Besides, the amount of face width modification by quadratic end relief $e(b_j)$ is calculated by equation (7).

$$e(b_j) = \begin{cases} 0 & , -\frac{b_{red}}{2} \le b_j \le \frac{b_{red}}{2} \\ c_e \cdot \dfrac{\left(|b_j| - \frac{b_{red}}{2}\right)^2}{b_e^2} & , b_j < -\frac{b_{red}}{2}, b_j > \frac{b_{red}}{2} \end{cases} \tag{7}$$

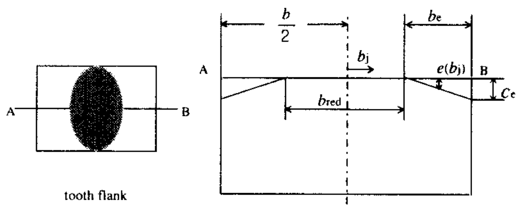

Fig. 4 End relief

3. Optimization method of the amount of tooth modification

Actually, only the variation amount of transmission error affects vibration of gears, so reducing the amplitude of this variation amount enables to reduce vibration. In addition to that, by means of reducing the effective value of vibrational exciting force through the proportional relation between transmission error and vibrational exciting force of gears, vibration level of gear falls(1). This research uses the optimization method with open source program ADS (Automated Design Synthesis)(7) in order to determine the optimum amount of tooth modification reducing vibration of gear pairs and choose vibrational exciting force as an objective function. Constraints are root fillet strength, surface durability, scoring resistance and contact ratio of gears. Design variables are, for driving gear and driven gear, the amount and length of tooth profile modification for profile modification, the amount of crowning for crowning, and the amount and length of end relief for end relief modification.

3.1 Objective function

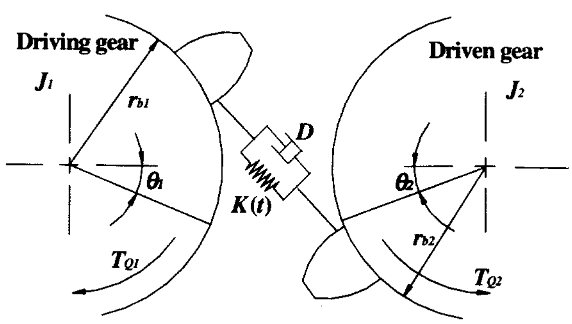

Fig. 5 Vibrational modeling of a pair of helical gear

The motion of helical gear pair transmitting torque T_{Qi} as fig. 5 is formulated into equation (8) as the form of the 1st order differential equation(1).

$$M\ddot{\Delta} + D\dot{\Delta} + \sum k_i \Delta = W + \sum k_i e_i \tag{8}$$

In equation (8), M is converted mass of gear body in line of action and $M = M_1 M_2 / (M_1 + M_2)$, $M_1 = J_1 / r_{b1}^2$, $M_2 = J_2 / r_{b2}^2$. J is mass moment of inertia, r_b is base circle radius, D is damping coefficient, W is transverse normal load, e_i is the composite error of gear pair i , k_i is the spring stiffness of gear pair i, $\sum$ is the summation for simultaneously meshed gear pair, and Δ is the transformed value of rotation delay of driven gear with respect to driving gear in line of action and that $\Delta = \Delta_1 - \Delta_2$, $\Delta_1 = r_{b1} \cdot \theta_1$, $\Delta_2 = r_{b2} \cdot \theta_2$. By referring the average value of $\sum k_i$ with respect to time t as K , making the terms Δ_s , λ, ω_e^2 , τ, $\kappa(\tau)$, $\varphi(\tau)$ dimensionless as $\Delta_s = W/K$, $\lambda = \Delta / \Delta_s$, $\omega_e^2 = K/M$, $\tau = t\omega_e$, $\kappa(\tau) = \sum k_i / K$, $\varphi(\tau) = \sum k_i e_i / W$, and replacing λ^*, φ_0 with $\lambda^* = \lambda - 1 - \varphi_0'$, $\varphi_0 = 1 + \varphi_0'$, equation (8) can be expressed as equation (9).

$$\ddot{\lambda}^* + 2\zeta\sqrt{\kappa(\tau)}\,\dot{\lambda}^* + \kappa(\tau)\lambda^* = \varphi^* - \kappa^*(1 + \varphi_0') \tag{9}$$

The right side of equation (9) goes to the periodic function whose average value is 0, and average value of $\kappa(\tau)$ is 1. So, the solution of equation (9) can be approximated into that of equation (10)(1).

$$\ddot{\lambda}^* + 2\zeta\dot{\lambda}^* + \lambda^* = \varphi^* - \kappa^*(1+\varphi_0') \quad (10)$$

Equation (10) is the dimensionless form, so transforming the right side of equation (10) into the magnitude of real vibrational exciting force of gears makes the synthesized exciting force per unit face width E_v equal to the equation (11).

$$E_v = (W/b)(K_0/K)\{\varphi^* - \kappa^*(1+\varphi_0')\} \quad (11)$$

In equation (11), b is face width, K_0 and K are the average values of composite spring stiffness of tooth in transverse plane for ideal gear pair having no errors and for gear pair having errors, respectively.

The magnitude of Fourier coefficients for each order of vibrational exciting force A_m is equal to equation (12) and both vibration and noise level of gears can be evaluated by the effective value $E_{v,eff}$ for A_m as equation (13).

$$A_m = \sqrt{(\eta_m - \varphi_0\alpha_m)^2 + (\gamma_m - \varphi_0\beta_m)^2} \quad (12)$$

$$E_{v,eff} = \sqrt{\sum_{m=1}^{\infty} \frac{A_m^2}{2}} \quad (13)$$

3.2 Constraints

Inappropriate tooth modification may have severe harmful influences on both the strength conditions and the contact ratio. Therefore, strength conditions like bending strength, surface durability and scoring resistance, and contact ratio must be considered as constraints in optimization procedure.

Root fillet stress on the point Y_i at the tooth root fillet $\sigma_{Bi}(Y_i)$ is calculated as the equation (14)(8).

$$\sigma_{Bi}(Y_i) = \int_{b/\cos\beta_b} \left(\frac{\sigma^*}{\sigma_{Nbl}}\right)_{\xi_i} \cdot \frac{6}{S_{nF}^2} \beta_i(Y_i, \xi_i)\cos\psi_i(\xi_i) p_i(\xi_i) d\xi_i \quad (14)$$

In equation (14), β_b is base helix angle, σ^* is actual tooth root stress when the unit load is applied at point ξ_i of equivalent spur gear, σ_{Nbl} is nominal bending stress generated on tooth root due to the same unit load, S_{nF} is tooth thickness at critical section, $\beta_i(Y_i, \xi_i)$ is the bending moment at point Y_i of tooth root when the unit load is applied on point ξ_i of tooth face, $\psi_i(\xi_i)$ is the angle between line normal to tooth face and elastic surface of tooth at point ξ_i, and $p_i(\xi_i)$ is the magnitude of distributed load at point ξ_i.

Using the Herzian contact stress formula for two contact surfaces, contact stress at point x_i is calculated using equation (15)(9),(10).

$$\sigma_H(x_i) = \frac{2p_i(x_i)}{\pi a(x_i)} \quad (15)$$

In equation (15), $p_i(x_i)$ is distributed load per unit face width and $a(x_i)$ is half the Hertz contact width.

In order to estimate scoring resistance, maximum value of flash temperature distribution T_f is used. Maximum value of flash temperature distribution occurring at point x_i is calculated using equation (16)(8),(9),(11).

$$T_f(x_i) = \frac{1.11\mu p_i(x_i)|\sqrt{V_1(x_i)} - \sqrt{V_2(x_i)}|}{\sqrt{\lambda\rho_s c}\sqrt{2a(x_i)}} \quad (16)$$

In equation (16), $V_1(x_i)$ and $V_2(x_i)$ are the tangential velocities at point x_i of driving gear and driven gear, respectively, λ is the heat transfer rate between tooth faces, ρ_s is the density of the material of gear, c is the specific heat of the material of gear, and μ is the friction coefficient.

3.3 Formulation of optimization procedure

Optimization procedure in order to determine the amount of tooth modification minimizing vibration exciting force of gears satisfying both the strength constraints and the contact ratio constraints is formulated as follows.

Minimize $F(x) = E_{v,eff} = \sqrt{\sum_{m=1}^{\infty} \frac{A_m^2}{2}}$

subject to

$$G_1(x) = \max[\sigma_{Bi}(Y_i)] / \sigma_{B\lim} - 1 \le 0$$
$$G_2(x) = \max[\sigma_H(x_i)] / \sigma_{H\lim} - 1 \le 0$$
$$G_3(x) = T_f / T_{f\lim} - 1 \le 0$$
$$G_4(x) = 1 - \varepsilon_a / \varepsilon_{a,lower} \le 0$$
$$G_5(x) = \varepsilon_a / \varepsilon_{a,upper} - 1 \le 0$$
$$x_j^l \le x_j \le x_j^u \qquad j = 1,8 \le 0$$

In the formulation above, ε_a is the transverse contact ratio which ranges between 1.2 and 2.5(2).

4. Determination of the optimum amount of tooth modification

To investigate the effect of face width on determination of the amount of tooth modification among several parameters, aspect ratio is used. Aspect ratio m_a is the ratio of face width to pitch diameter of pinion and is expressed as equation (17).

$$m_a = \left(\frac{b \cdot \cos\beta_0}{m_n \cdot z_1} \right) \tag{17}$$

In equation (17), b is face width, β_0 is helix angle, m_n is normal module, and z_1 is the number of pinion teeth. As varying the value of aspect ratio from 0.25 to 2.0 by changing face width, the optimum amount of tooth modification is obtained that the relation between aspect ratio and the amount of tooth modification is examined.

Table 1 Specification of helical gears

Specification	Pinion	Wheel
Normal module [mm]	2	
Number of teeth	23	156
Normal pressure angle [deg]	20	
Helix angle [deg]	25	
Input torque [N · m]	63.1	
Driving speed [rpm]	1134	
Material	SCM415	

Table 1 shows the dimension of helical gears used in this research. Table 2 shows the optimization result in case when both linear tip relief and linear end relief are applied. Vibrational exciting force without tooth modification decreases as aspect ratio increases except that it increases to some extent when aspect ratio 2.0. This is due to the rise of vibrational exciting force from increased transmission error as the transmitted load per unit face width becomes too small. Vibrational exciting force through optimization of tooth modification decreases remarkably than without modification. In case of aspect ratio 0.25, the effect of tip relief appears relatively greater than that of end relief, since face width is considerably smaller than pitch diameter. When aspect ratio is 0.5, the effect of tip relief diminishes more than aspect ratio 0.25. In case of aspect ratio 1.0, the effect of end relief increases remarkably, and when aspect ratio becomes 1.5, tip relief hardly affect vibrational exciting force, thus end relief mainly makes tooth modification. When aspect ratio is 2.0, the effect of tip relief is showing itself again due to the effect of rotation delay by transmission error.

Fig. 6 shows real contact area on the plane of action whose width is face width and whose height is length of contact for the optimization result of table 2.

Table 2 Optimum for linear tip relief and linear end relief

Aspect ratio			0.25	0.5	1.0	1.5	2.0
Pinion	s_o	[μm]	15.0	10.3	7.9	0.5	15.3
	h_o	[m_n]	0.586	0.476	0.246	0.010	0.508
	c_e	[μm]	0.9	0.5	9.6	5.2	5.1
	b_e	[b]	0.050	0.020	0.065	0.106	0.308
Gear	s_o	[μm]	12.9	10.3	8.3	0.5	15.0
	h_o	[m_n]	0.527	0.430	0.194	0.010	0.507
	c_e	[μm]	0.9	0.5	9.6	5.2	5.1
	b_e	[b]	0.050	0.020	0.065	0.106	0.308
E_v [N/mm]	no modification		5.33	1.78	0.22	0.07	0.11
	modification		0.22	0.06	0.06	0.03	0.02

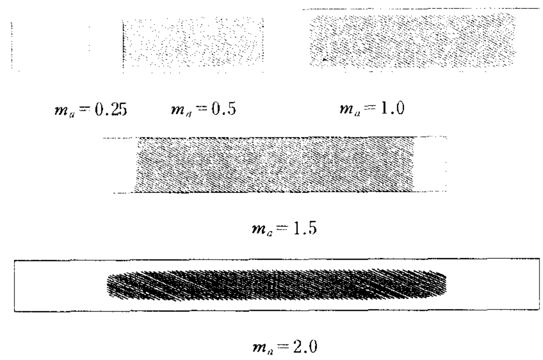

Fig. 6 Real contact area on the plane of action

Table 3 Optimum for linear tip relief and quadratic end relief

Aspect ratio			0.25	0.5	1.0	1.5	2.0
Pinion	s_o	[μm]	14.9	9.6	5.1	0.5	20.1
	h_o	[m_n]	0.546	0.466	0.503	0.010	0.300
	c_e	[μm]	0.9	0.5	15.0	15.0	5.0
	b_e	[b]	0.050	0.020	0.100	0.100	0.100
Gear	s_o	[μm]	13.0	9.6	4.9	0.5	20.0
	h_o	[m_n]	0.545	0.468	0.507	0.010	0.299
	c_e	[μm]	0.9	0.5	15.0	15.0	5.0
	b_e	[b]	0.050	0.020	0.100	0.100	0.100
E_v	no modification		5.33	1.78	0.22	0.07	0.11
[N/㎜]	modification		0.38	0.11	0.04	0.03	0.01

Table 4 Optimum for linear tip relief and symmetrical crowning

Aspect ratio			0.25	0.5	1.0	1.5	2.0
Pinion	s_o	[μm]	15.6	9.5	0.1	5.1	5.4
	h_o	[m_n]	0.569	0.439	0.122	0.296	0.310
	c_c	[μm]	0.9	0.5	1.9	1.0	1.0
Gear	s_o	[μm]	14.4	9.7	1.1	5.1	5.4
	h_o	[m_n]	0.605	0.444	0.012	0.280	0.341
	c_c	[μm]	0.9	0.5	1.9	1.0	1.0
E_v	no modification		5.33	1.78	0.22	0.07	0.11
[N/㎜]	modification		0.58	0.14	0.06	0.02	0.01

Table 3 shows the optimization result in case of both linear tip relief and quadratic tip relief applied, which has similar tendency of the case of linear end relief applied on the whole.

Table 4 shows the optimization result for both linear tip relief and symmetrical crowning applied. In case of crowning, vibrational exciting force with respect to the optimization of tooth modification also decreases considerably than in case of no modification like that end relief modification is applied. And the fact that optimum amount of crowning is less than that of end relief means small amount of crowning takes full effect of tooth modification since crowning makes the whole face modified.

Table 5 Optimum for linear tip relief and skew-symmetrical crowning

Aspect ratio			0.25	0.5	1.0	1.5	2.0
Pinion	s_o	[μm]	14.9	9.9	5.2	4.4	15.0
	h_o	[m_n]	0.523	0.520	0.533	0.474	0.298
	c_c	[μm]	0.5	3.2	2.1	1.9	0.5
Gear	s_o	[μm]	12.8	8.4	4.8	4.2	15.0
	h_o	[m_n]	0.570	0.416	0.471	0.410	0.302
	c_c	[μm]	0.5	3.2	2.1	1.9	0.5
E_v	no modification		5.33	1.78	0.22	0.07	0.11
[N/㎜]	modification		0.71	0.09	0.06	0.02	0.01

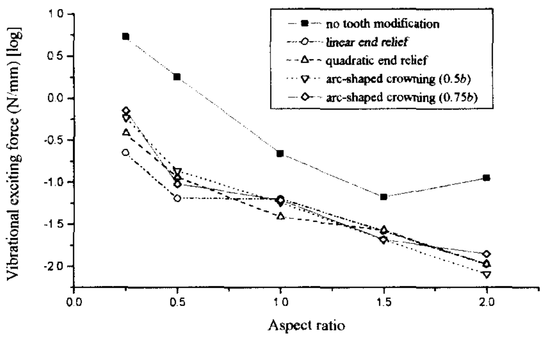

Fig. 7 Vibrational exciting force due to change of aspect ratio

Table 5 shows the optimization result for both linear tip relief and skew-symmetrical crowning applied. Skew-symmetrical crowning is pretty much similar to symmetrical tooth modification method, but it differs from symmetrical crowning in case of aspect ratio 0.5 such that it has larger value than symmetrical crowning.

Fig. 7 shows the comparison of each result of four tooth modification methods described. In fig. 7, it shows that for aspect ratio 0.25 and 0.5, linear end relief minimizes vibrational exciting force, and for aspect ratio 1.0, quadratic end relief does. For aspect ratio 1.5, crowning tends to reduce vibrational exciting force rather than end relief. In case of aspect ratio 2.0, it shows that symmetrical crowning which is less affected by load has likely to minimize vibrational exciting force of gears.

5. Conclusions

As the result of determining the amount of tooth modification using optimization method in order to minimize vibration and noise of gears, the following is summarized as conclusions.

(1) According to the proposed method in this research, the optimum amount of tooth modification minimizing vibration of gears with respect to several kinds of load distribution in tooth face can be determined.

(2) In case of both aspect ratio being 0.25 and 0.5, design by end relief minimizes vibrational exciting force. When aspect ratio is 1.0, design by quadratic relief minimizes vibrational exciting force. Finally, in cases when aspect ratio is 1.5 and 2.0, where face width is relatively larger rather than pitch diameter, design by arc-shaped crowning minimizes vibrational exciting force.

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