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〈논 문〉

Dynamic Stability of a Drum Brake Shoe under a Frictional Force

마찰력을 받는 드럼 브레이크-슈의 동적안정성

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ABSTRACT

The paper presents the dynamic stability of a flexible shoe in drum brake systems subjected to a frictional force. The frictional force between the drum and the shoe is assumed as a distributed frictional force, while the shoe is modeled as an elastic beam supported by two translational springs at both ends and elastic foundations. Governing equations of motion are derived by energy expressions, and their numerical results are obtained by employing the finite element method. The critical distributed frictional force and the instability regions are demonstrated by changing the stiffness of two translational springs and elastic foundation parameters. It is also shown that the beam loses its stability by flutter and divergence depending on the stiffness of elastic supports and elastic foundation parameters. Time responses of beams corresponding to their instability types are also demonstrated.

1. INTRODUCTION

The brake is a mechanical device that produces the braking forces using dry friction. The noises generated in the brake system of cars are considered as unstable vibration phenomena due to frictional forces. These noises and vibrations may be referred to as *moan, groan, squeal, judder* and so on, depending on the frequency ranges. These different terminologies for the brake noises imply that there can be many different mechanisms for the noises and vibrations. As to the mechanism of brake noises, the earlier theory suggested that the noise could be caused by negative friction-induced vibration. It is noted that the theory was based upon one-degree of freedom model.(1~4) However the recent theories of brake noises have been based on a multi-degree of freedom model subjected to nonconservative loads induced by

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constant frictional forces between the drum and the shoe.[1~4] The theories state that the drum brake squeal is attributable to dynamic instability of elastic brake systems under a constant nonconservative loads.[1~4]

Leipholz[5] presented both the dynamic stability and the critical distributed friction forces on a beam subjected to the distributed follower forces due to distributed frictional forces for four different boundary conditions such as pin-pin, clamped-pin, clamped- clamped, and clamped-free conditions. He found that the flutter-type instability occurs for the clamped-free case while the divergence-type occurs for the other cases.

The study on the dynamic stability of a beam under distributed follower forces presented above does not intend to apply directly to the drum brake shoe. However, the strip of shoe has been treated in many studies as a beam with various boundary conditions since some experiments on the drum brake system showed that the brake shoe and the associated boundary conditions have a great influence on the brake noise.[6~8]

Recently, Kang and Tan[9] used Galerkin's method to investigate the dynamic stability of the Euler-Bernoulli beam by regarding the frictional force acting on the brake shoe as the pulsating distributed follower force.

As presented above, the dynamic stability of a drum brake shoe has been mostly studied by employing a beam model with conventional boundary conditions subjected to either uniformly or pulsating distributed frictional forces.

In this paper, however, a beam with elastic supports are used since the brake parts such as caliper piston, caliper, and supporting bracket can be regarded as translational springs. While, the lining parts of drum brake systems can be regarded as distributed elastic foundations.

The dynamic stability of the beam under the uniformly distributed frictional forces is studied using the finite element method.

The intended aim of the present paper is to show the possibility of dynamic instability of a brake shoe when it is assumed to be flexible and subjected to a distributed frictional force, while the dynamic instability of drums has been studied well so far[4]

2. ANALYSIS

2.1 Mathematical Model

Fig. 1 depicts a conceptual sketch of a standard drum brake system.[4] Fig. 2 shows an idealized model of a drum-shoe system.

The shoe is now assumed to be elastic and thus flexible, though the real shoes are very stubby and rigid. In reality, there is a lining between the drum and the shoe. The lining is modeled for simplicity into a Winkler-type elastic foundation of

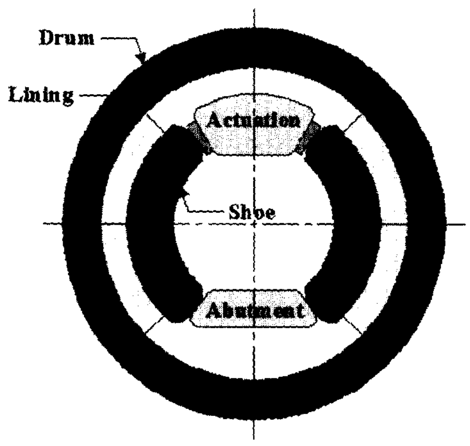

Fig. 1 Conceptual sketch of a drum brake system.

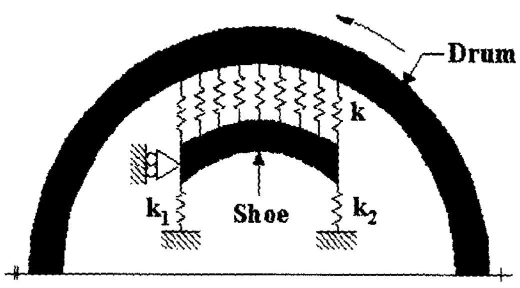

Fig. 2 Idealized drum-shoe system.

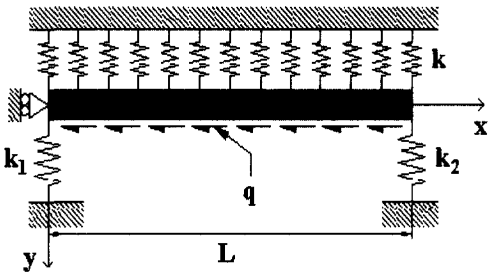

Fig. 3 Simplified beam model of a brake shoe under a distributed frictional force.

the spring constant k. It is assumed that the shoe is supported at both ends elastically by two linear springs of different spring constants k_1 and k_2. The shoe is then assumed to be subjected to a distributed follower force induced by dry friction between the drum and the shoe. Distributed tangential follower force applied to a column was first considered by Pflüger in his book.[10] Fundamental aspects of the effect of distributed follower forces on the stability of columns were complied in a book by Leipholz.[11] However the existence of such kind of follower forces has been an issue of severe criticism so far.[11] Fig. 2 implies one example of the existence of a distributed tangential follower force. Intended aim of the present paper is to demonstrate the possible existence of distributed tangential follower force induced by a dry friction between the drum and the shoe in drum brake systems.

Fig. 3 shows a simplified beam model imaged from the model in Fig. 2. The beam is assumed to be an elastic, uniform, and straight beam of total length L, mass per unit length m, and bending stiffness EI.

2.2 Finite Element Formulation

Extended Hamiton's principle for the nonconservative system under consideration can be written as

$$\delta\int_{t_1}^{t_2}(T+W_c-U-U_s)dt+\int_{t_1}^{t_2}(\delta W_{nc})dt=0 \qquad (1)$$

Energy expressions for the mathematical model in Fig. 3 are written in the following forms :

The kinetic energy of the uniform column :

$$T=\frac{1}{2}\int_0^L m\left(\frac{\partial y}{\partial t}\right)^2dx \qquad (2)$$

The work done by the conservative component of the distributed follower force :

$$W_c=\frac{1}{2}\int_0^L q(L-x)\left(\frac{\partial y}{\partial x}\right)^2dx \qquad (3)$$

The virtual work done by the nonconservative component of the distributed follower force :

$$\delta W_{nc}=-\frac{1}{2}\int_0^L q\left(\frac{\partial y}{\partial x}\right)^2\delta y dx \qquad (4)$$

The potential energy of the beam due to bending :

$$U=\frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2dx \qquad (5)$$

The strain energy stored in the elastic foundation and the two spring supports :

$$U_s=\frac{1}{2}\int_0^L ky^2dx+\frac{1}{2}k_1y^2(0,t)+\frac{1}{2}k_2y^2(L,t) \qquad (6)$$

Substitution of equations (2)~(6) into equation (1), leads to

$$\int_{t_1}^{t_2}\int_0^L[m\left(\frac{\partial y}{\partial t}\right)\delta\left(\frac{\partial y}{\partial t}\right)+q(L-x)\left(\frac{\partial y}{\partial x}\right)\delta\left(\frac{\partial y}{\partial x}\right)$$
$$+EI\left(\frac{\partial^2 y}{\partial x^2}\right)\delta\left(\frac{\partial^2 y}{\partial x^2}\right)-q\left(\frac{\partial y}{\partial x}\right)\delta y-ky\delta y]dxdt$$
$$-\int_{t_1}^{t_2}[(k_1y\delta y)_{x=0}+(k_2y\delta y)_{x=L}]dt=0 \qquad (7)$$

For simplicity the following dimensionless quantities are introduced :

$$\xi=\frac{x}{L},\ \eta=\frac{y}{r},\ \tau=\frac{t}{L^2}\sqrt{\frac{EI}{m}},\ F=\frac{qL^3}{EI},$$
$$K=\frac{kL^4}{EI},\ K_1=\frac{k_1L^3}{EI},\ K_2=\frac{k_2L^3}{EI} \qquad (8)$$

where F is the distributed frictional force parameter. K is the nondimensional elastic foundation parameter. Further, K_1 and K_2 are the nondimensional parameters describing the spring constants of the two elastic supports, respectively.

Then, equation (7) can be rewritten in the dimensionless form.

$$\int_{t_1}^{t_2}\int_0^L [\eta_\tau\delta\eta_\tau + F(1-\xi)\eta_\xi\delta\eta_\xi + F\eta_\xi\delta\eta - \eta_{\xi\xi}\delta\eta_{\xi\xi} - K\eta\delta\eta]$$
$$d\xi d\tau - \int_{t_1}^{t_2}[(K_1\eta\delta\eta)_{\xi=0} + (K_2\eta\delta\eta)_{\xi=1}]d\tau = 0 \qquad (9)$$

In order to obtain a characteristic equation of small motion of the beam, the beam is divided into N equal elements to be compatible with finite element method. Then, equation (9) can be written as

$$\int_{t_1}^{t_2}[\sum_{i=1}^{N}\int_{\frac{1}{N}(i-1)}^{\frac{1}{N}i}[\eta_\tau\delta\eta_\tau + F(1-\xi)\eta_\xi\delta\eta_\xi + F\eta_\xi\delta\eta - \eta_{\xi\xi}\delta\eta_{\xi\xi}$$
$$-K\eta\delta\eta]d\xi - \{[(K_1\eta\delta\eta)_{\xi=0} + (K_2\eta\delta\eta)_{\xi=1}]\}d\tau = 0 \qquad (10)$$

Substitution of local coordinates ($\zeta = N\xi - i + 1$) into equation (10) yields the following discretized equation :

$$\int_{t_1}^{t_2}[\sum_{i=1}^{N}\{\eta_\tau^{(i)}\delta\eta_\tau^{(i)} + FN(N-i+1-\zeta)\eta_\zeta^{(i)}\delta\eta_\zeta^{(i)}$$
$$-FN\eta_\zeta^{(i)}\delta\eta^{(i)} - N^4\eta_{\zeta\zeta}^{(i)}\delta\eta_{\zeta\zeta}^{(i)} - K\eta^{(i)}\delta\eta^{(i)}\}d\zeta$$
$$-\{(K_1\eta^{(1)}\delta\eta^{(1)})_{\zeta=0} + (K_2\eta^{(N)}\delta\eta^{(N)})_{\zeta=1}\}]$$
$$d\tau = 0 \qquad (11)$$

The dimensionless displacement η can be assumed to take the form.

$$\{\eta^{(i)}(\zeta,\tau)\} = \{e^{(i)}(\zeta)\}\cdot\{v^{(i)}(\tau)\} \qquad (12)$$

By substitution of equation (12) into (11), finally the characteristic equation is obtained in the matrix form.

$$[M]\{v\}_{\tau\tau} + [K]\{v\} = 0 \qquad (13)$$

The displacement vector varies with time according to an exponential law.

$$\{v(\tau)\} = \{X\}e^{(\lambda\tau)} \qquad (14)$$

Finally, the global characteristic equation can be obtained in the form.

$$|[M]^{-1}[K] + \lambda^2[E]| = 0 \qquad (15)$$

where, $[E]$ is the unit matrix.

The stability of the system under consideration is determined by the sign of σ, real part of complex eigenvalue, $\lambda = \sigma \pm i\omega(\, i=\sqrt{-1})$. If $\sigma < 0$, the system is stable ; if $\sigma > 0$ and $\omega = 0$, the system is statically unstable, i.e., divergence type instability ; if $\sigma > 0$ and $\omega \neq 0$, the system is dynamically unstable, i.e., flutter type instability ; if $\sigma = 0$, the critical distributed follower force(F_{cr}) arises.

3. NUMERICAL RESULTS

Numerical analyses were performed based on the theoretical development presented above. In order to check the accuracy of numerical results obtained in the study, a comparison was conducted with the results in Reference(11) for the case of simply supported at both ends. The differences between two results are within 0.018 % when 20 elements are used for the present study.

Fig. 4 shows the relation between the critical distributed frictional force and the stiffness of the two equal supports $K_1 = K_2$, for the different values of elastic foundation parameter $K = 0.0$, 10, 100. Flutter type instability can occur in the range $K_1 = K_2 \leq 39.976$.

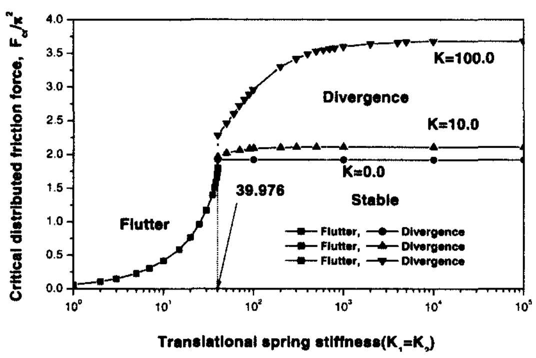

Fig. 4 Critical distributed frictional forces depending on elastic foundations and translational springs.

It is noted that the critical distributed frictional force does not change for the value of K in this range. At the critical point $(K_1 = K_2)_{cr} = 39.976$, the critical distributed frictional force jumps, and the instability type changes from flutter to divergence. Only the divergence type of instability occurs in the range $K_1 = K_2 \geq 39.977$.

When $K = 0.0$, the critical distributed frictional force, $F_{cr}/\pi^2 = 1.921$, is constant for sufficiently large values of $K_1 = K_2$.

The critical flutter value $F_{cr}/\pi^2 = 1.921$ for the beam simply supported at both ends agrees with the earlier results in the Reference(11). When $K = 10.0$, 100.0, the critical distributed frictional force increases with the increasing $K_1 = K_2$.

It is now interesting to observe eigen-frequencies curves to understand how the instability changes from flutter to divergence can take place at the critical stiffness of $K_1 = K_2 = 39.976$.

Fig. 5(a)~(c) show the eigen-frequencies curves of the first and second mode for various values of the support stiffness $K_1 = K_2 = 38$, 39, 40. Fig. 5(a) shows the eigen-frequencies for $K = 0.0$.

It is observed in Fig. 5(a) that the first and second eigen-frequencies coincide each other, and thus the flutter type instability can occurs for $K_1 = K_2 = 38$ and 39. The flutter values of the distributed frictional forces are $F_{cr}/\pi^2 = 1.576$ and 1.654. Divergence occurs as the first eigen-frequency becomes zero, at $F_{cr}/\pi^2 = 1.921$ for $K_1 = K_2 = 40.0$.

It is seen in Fig. 5(b) and (c) that the first and second eigen-frequencies become larger as the elastic embedding is stiffer. It is confirmed that the flutter value of distributed frictional force is constant regardless its elastic foundation, while the eigen-frequencies by themselves depend on the stiffness of the elastic foundation as seen in Fig. 5(a), (b) and (c).

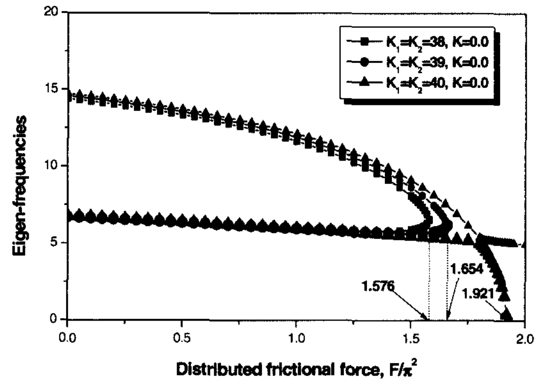

Fig. 5 (a) First and second eigen-frequencies for distributed frictional forces($K = 0.0$).

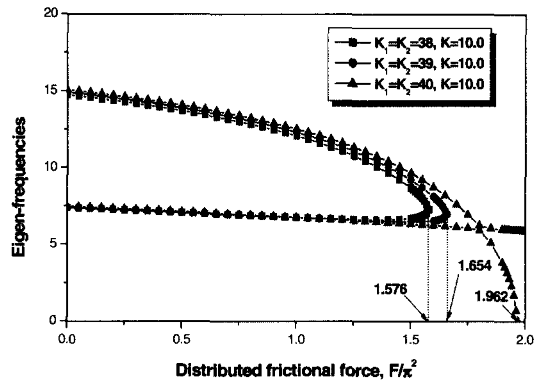

Fig. 5 (b) First and second eigen-frequencies for distributed frictional forces($K = 10.0$).

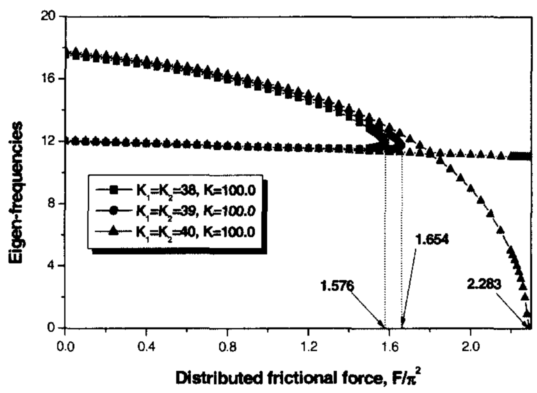

Fig. 5 (c) First and second eigen-frequencies for distributed frictional forces($K = 100.0$).

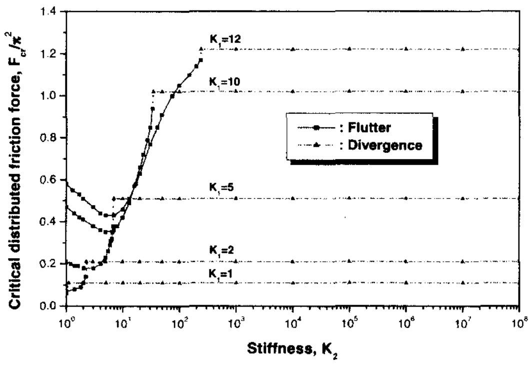

Fig. 6 Instability transitions depending on the spring constants($K_1=1$, 2, 5, 10, 12 and K_2.)

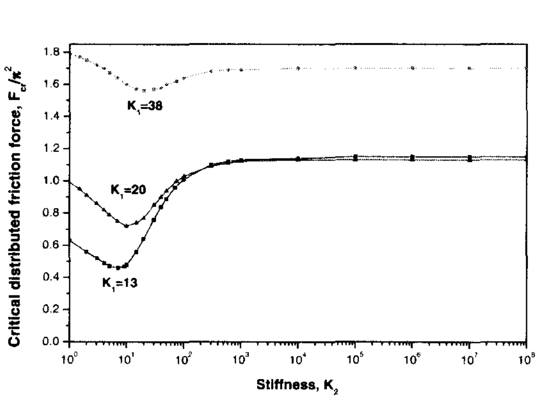

Fig. 7 Flutter instabilities depending on the spring constants ($K_1=13, 20, 38$ and K_2)

Fig. 6 shows both the critical distributed frictional force and the instability type for various right spring constant, K_2, when the left spring constant, $K_1=1$, 2, 5, 10, 12. As shown in the figure, the critical distributed frictional force increases as the spring constant K_1 increases. The transition of the instability type from a flutter to a divergence occurs as K_2 increases for each value of K_1.

Fig. 7 presents the critical distributed frictional forces for various values of K_2, when $K_1=13$, 20, 38. In this case, only the flutter occurs for all values of K_2. And also, the critical distributed frictional force decreases at first and increases later as the spring constant K_2 increases for a fixed value of K_1. Meanwhile, the spring constant $K_2 \geq 10^5$ can be considered as a rigid support since the critical distributed frictional force changes little with K_2 above 10^6.

Fig. 8 presents the critical distributed frictional forces for various values of K_2, when $K_1=39$, 40, 41. The transition from a flutter to a divergence occurs as K_2 increases when $K_1=39$ and 40.

Fig. 9 shows the relation between the critical distributed frictional force and the constant of the left spring support K_1, when $K=0.0$ and $K_2=10.0$, 100.0, 1000.0.

There are three regions of instability, A, B, and C. The critical distributed frictional force for divergence instability increases monotonically as the value of K_1 is small enough and increasing. Then, flutter type instability takes place for the

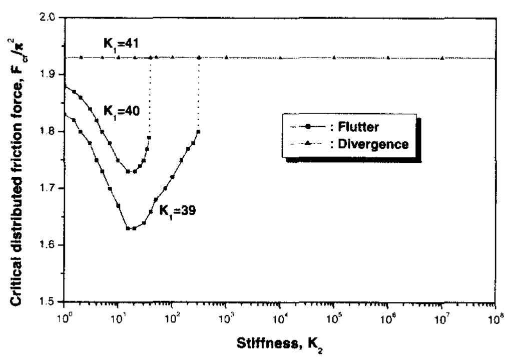

Fig. 8 Instability transitions depending on the spring constants($K_1=39$, 40, 41 and K_2).

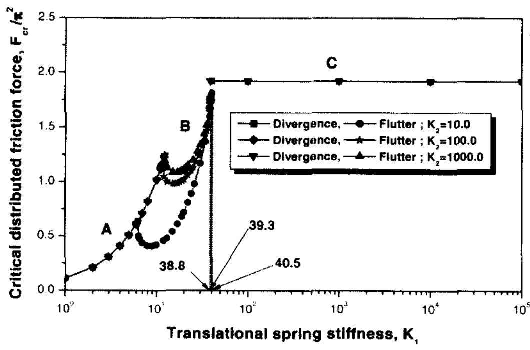

Fig. 9 Effect of the right translational spring on stability of beams, when $K=0.0$.

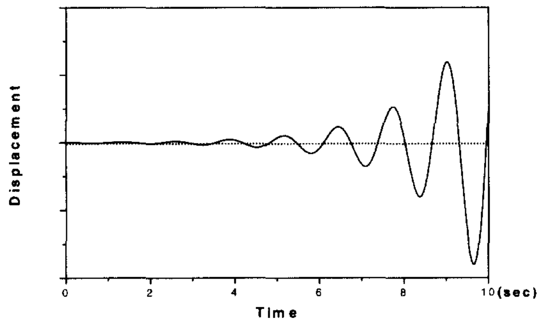

Fig. 10 Time response of the region B in Fig. 9. ($K=0.0, K_1=20.0, K_2=10.0$).

spring constant K_1 of about the order of 10.0. After a sharp climb of flutter value at $K_1=40.5$, divergence occurs again for $K_1>40.5$. The critical distributed frictional force value $F_{cr}/\pi^2=1.921$, remains constant for the considerably large value K_1.

Fig. 10 depicts typical unstable configurations at the B region of instability in Fig. 9. In this figure, we can see the flutter response with increasing time.

4. CONCLUDING REMARKS

The brake shoe is simplified into an elastically supported beam subjected to a distributed frictional force due to a dry friction. The paper has suggested that the instability of shoe can be considered as one of the possible sources for drum brake squeal, if the shoe is weak and thus flexible.

Flutter type instability can occur in the range $K_1=K_2\leq 39.976$ for the different values of K.

When $K=0.0$, the transition of the instability type from flutter to a divergence occurs as K_2 increases for $K_1\leq 12$ and $39\leq K_1\leq 40$. However only the flutter occurs for all values of K_2 when $K=0.0$ and $13\leq K_1\leq 38$. Also, only the divergence occurs for $K=0.0$ and $K_1\geq 41$..

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