Approximate Yield Criterion for Voided Anisotropic Ductile Materials

Youngsuk Kim*
School of Mechanical Engineering, Kyungpook National University, Taegu 702-701, Korea
Sungyeun Won, Dongsoo Kim, Hyunsung Son
Graduate School, Kyungpook National University, Taegu 702-701, Korea

As most fractures of ductile materials in metal forming processes occurred due to the results of evolution of internal damage - void nucleation, growth and coalescence. In this paper, an approximate yield criterion for voided(porous) anisotropic ductile materials is developed. The proposed approximate yield function is based on Gurson's yield function in conjunction with the Hosford's non-quadratic anisotropic yield criterion in order to consider the characteristic of anisotropic properties of matrix material. The associated flow rules are presented and the laws governing void growth with strain are derived. Using the proposed model void growth of an anisotropic sheet under biaxial tensile loading and its effect on sheet metal formability are investigated. The yield surface of voided anisotropic sheet and void growth with strain are predicted and compared with the experimental results.

Key Words : Yield Criterion, Anisotropic Sheet, Damage Evolution

1. Introduction

In sheet metal forming processes, the formability of sheet metals is limited by the occurrence of internal damage evolution which yields eventually a localized necking failures. The formability of sheet metals is often evaluated using a strain analysis based on the concept of forming limit diagrams(FLD) originally introduced by Keeler(1965) and Goodwin(1968). Until now, many research works have focused on the evaluation of theoretical and experimental FLDs as a criterion of formability of sheet metals. Marciniak and Kuczynski(1967) analyzed the causes of necking failure and theoretically predict the forming limits under the assumption of the presence of local inhomogeneities in the original sheet metal. Storen and Rice(1975) developed a localized necking model based on bifurcation theory. Zhao, Sowerby, and Sklad(1996) investigated limit strains using the forming limit stress concept.

In sheet metal forming processes, internal damage characterized mainly by void evolution (nucleation, growth and coalescence) can yield a large local plastic flow and finally results a ductile fracture. Graf(1993) and Parmar(1980) showed that microvoid growth mechanism played a key role in the ductile fracture of plastically deformed solids. Therefore, a damage-based yield function and associated flow rules which are take account of void evolution are desirable for precise prediction of the ductile fracture process.

In order to analyze the plastic flow and fracture of a structural metal, Gurson(1977) carried out an upper bound analysis and proposed an approximate yield criterion for perfectly rigid porous materials where the matrix obeys the von Mises yield criterion are containing two different types of void geometries, that is, a long circular

* Corresponding Author,
E-mail : caekim@knu.ac.kr
http://cae.knu.ac.kr
TEL : +82-53-950-5580; **FAX** : +82-53-956-9914
Professor, School of Mechanical Engineering, Kyungpook National University, Taegu 702-701, Korea. (Manuscript **Received** April 27, 2000; **Revised** August 9, 2001)

cylinder and sphere. Gurson's work successfully applied for the prediction of the effects of void nucleation and growth on the forming limit of voided isotropic matrix materials (Jalinier, 1978). To improve the agreement of Gurson's model with results of micromechanical studies and experimental observations, several theoretical studies have already been performed. Tvergaard (1991) suggested a modified version of the Gurson's yield criterion by comparing the results of shear band instability based on a finite element model for porous materials with those based on a continuum model using the Gurson's yield criterion.

In view of significant effects of plastic anisotropy on the forming limit in sheet forming processes, an anisotropic version for Gurson's yield criterion, which can account for the effect of plastic anisotropic properties of matrix material, is needed to investigate forming limit and to characterize ductile fracture process in sheet forming processes. Considering the plastic anisotropy produced by cold rolling process on yield function, Hill's anisotropy yield function (1950) and others (Hill, 1979; Hosford, 1979; Barlat, 1989) were proposed and often used to deduce the forming limit of various anisotropic sheets . Lee (1998) studied that a theory of continuum damage mechanics for anisotropic solids on the basis of both the strain energy equivalence principle and the equivalent line crack damage modeling.

In the present work, an approximate Gurson's yield function incorporating with Hosford's (1979) non-quadratic anisotropic yield criterion is developed to describe the plastic deformation of voided anisotropic materials. The proposed yield criteria for voided anisotropic materials and its associated flow rules were then used to investigate the yield surface and damage evolution of a normal anisotropic sheet under biaxial tensile loading for various void volume fractions. The predicted results for yield surface and void growth with strain are compared with the experimental ones.

2. Theoretical Analysis

Theories describing voided materials suggest that the yield criterion is a function of the first invariant of the stress tensor, J_1, and the second invariant of the deviatoric stress tensor, J_2'. Gurson (1977) presented a yield function, based on an upper bound solution for spherically symmetric deformations of rigid perfectly plastic materials around a single spherical void. His proposed yield function is given by:

$$g(\sigma_{ij}, \bar{\sigma}_M, f)=\left(\frac{\bar{\sigma}}{\bar{\sigma}_M}\right)^2+2f\cosh\left(\frac{\sigma_{kk}}{2\bar{\sigma}_M}\right)-1-f^2=0 \quad (1)$$

$$\text{or } \bar{\sigma}^2=\bar{\sigma}_M^2(1+f^2)-2f\bar{\sigma}_M^2\cosh\left(\frac{\sigma_{kk}}{2\bar{\sigma}_M}\right) \quad (1a)$$

where $\bar{\sigma}$ and $\bar{\sigma}_M$ are the effective stresses of the voided material and matrix material, respectively, and f is the current void volume fraction. A simplified form of Gurson's yield function is obtained by expanding the hyperbolic cosine in terms of its power series of $(\sigma_{kk}/2\bar{\sigma}_M)$. In many deformation processes, since $(\sigma_{kk}/2\bar{\sigma}_M)$ is less than unity, it is acceptable to neglect higher power terms than 2

$$\bar{\sigma}^2=\bar{\sigma}_M^2(1+f^2)-2f\bar{\sigma}_M^2\left[1+\frac{1}{2}\left(\frac{\sigma_{kk}}{2\bar{\sigma}_M}\right)^2\right] \quad (2)$$

Tvergaard (1987) modified Gurson's model to the following form:

$$\bar{\sigma}^2=\bar{\sigma}_M^2(1+q_1^2f^2)-2fq_1\bar{\sigma}_M^2\left[1+\frac{1}{2}\left(\frac{q_2\sigma_{kk}}{2\bar{\sigma}_M}\right)^2\right] \quad (2a)$$

The fitting parameters q_1 and q_2 have been introduced to get good predictions of the model with numerical analyses for deformations of materials with various voids. It has been found that getting a value in the range of 1.5~2.5 to q_1 and taking q_2 equal to unity gives the good agreement between experimental results and numerical simulation of plastic flow of porous ductile materials for several different powder compacts.

In this work, Gurson's yield function, Eq. (2), can be further modified with Hosford's (1979) anisotropic yield criterion of $\bar{\sigma}$ for normal anisotropy as given by

$$\bar{\sigma}=\left\{\frac{1}{(1+R)}[|\sigma_2-\sigma_3|^a+|\sigma_3-\sigma_1|^a+R|\sigma_1-\sigma_2|^a]\right\}^{\frac{1}{a}} \quad (3)$$

where R is the mean anisotropy parameter which is the averaged anisotropy parameter measured in 0°, 45°, and 90° direction tension tests. Hosford (1979) proposed the yield function of Eq. (3) – based on the Hill-Bishop analysis – for crystallographic textured fcc and bcc metals. Hosford also suggested that the even integer a should be 6 for bcc metals and 8 for fcc metals. Hosford's criterion was better for aluminum and brass compared to the other two Hill's criteria according to Cristian and Hosford (1983). It has been also indicated by Hosford (1988) that the expression (Eq. (3)) describes the yielding of cubic metals better than Hill's (1948) criterion. Our equation (2) in cooperation with Eq. (3) is in agreement with the yield function of Doege (1997) in $a=2$ for an anisotropic voided material. Liao, et al. (1997) investigated the yield surface of anisotropic material and the void growth during strain using Hill's non-quadratic anisotropic yield criterion.

According to the normality rule of the plastic strain vector for the yield surface, the principal plastic strain increments are derived based on the partial differential of the yield function with respect to the stresses, respectively. Thus,

$$d\varepsilon_1^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[-|\sigma_3-\sigma_1|^{a-1}+R|\sigma_1-\sigma_2|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}d\lambda' \quad (4a)$$

$$d\varepsilon_2^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[|\sigma_2-\sigma_3|^{a-1}-R|\sigma_1-\sigma_2|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}d\lambda' \quad (4b)$$

$$d\varepsilon_3^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[-|\sigma_2-\sigma_3|^{a-1}+|\sigma_3-\sigma_1|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}d\lambda' \quad (4c)$$

where $d\lambda$ is a non-negative proportional factor and $d\lambda'=\frac{1}{\bar{\sigma}_M{}^2}d\lambda$. As a result, the volumetric plastic strain, $d\varepsilon_v$, can be obtained by summing

$$d\varepsilon_v=d\varepsilon_1^p+d\varepsilon_2^p+d\varepsilon_3^p=\frac{3f}{2}\sigma_{kk}d\lambda' \quad (5)$$

The porosity of a material is characterized by the void volume fraction, f, which can be defined by

$$f=\frac{V_V}{V_T}=\frac{V_T-V_M}{V_T} \quad (6)$$

where V_T, V_M and V_T designate the void, matrix, and total volume, respectively.

From Eq. (6), $V_M=V_T(1-f)$, which upon differentiation, noting that for an incompressible matrix material, $dV_M=0$, yields:

$$d\varepsilon_v=\frac{dV_T}{V_T}=\frac{dV_V}{V_T}=\frac{df}{1-f} \quad (7)$$

Substituting Eq. (7) into Eqs. (4) gives the plastic strain increments as follows

$$d\varepsilon_1^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[-|\sigma_3-\sigma_1|^{a-1}+R|\sigma_1-\sigma_2|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}\frac{2}{3\sigma_{kk}}\frac{df}{f(1-f)} \quad (8a)$$

$$d\varepsilon_2^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[|\sigma_2-\sigma_3|^{a-1}-R|\sigma_1-\sigma_2|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}\frac{2}{3\sigma_{kk}}\frac{df}{f(1-f)} \quad (8b)$$

$$d\varepsilon_3^p=\left\{2\bar{\sigma}^{(2-a)}\frac{1}{(1+R)}[-|\sigma_2-\sigma_3|^{a-1}+|\sigma_3-\sigma_1|^{a-1}]+\frac{f}{2}\sigma_{kk}\right\}\frac{2}{3\sigma_{kk}}\frac{df}{f(1-f)} \quad (8c)$$

The yield condition, Eq. (1), and flow rule in Eqs. (8) yield the relations between stress and plastic strain as long as the current volume fraction f is known.

Here we consider void growth of sheet metal under the condition of plane stress, where $\sigma_3=0$ and $\sigma_2=\alpha\sigma_1$; thus, from Eq. (8a)

$$\frac{d\varepsilon_1^p}{df}=\left\{\frac{1}{(1+R)}[\alpha^a+1+R(1-\alpha)^a]\right\}^{(2/a-1)} \times\frac{1}{(1+R)}[1+R(1-\alpha)^{a-1}]\frac{4}{3(1+\alpha)}\frac{1}{f(1-f)}+\frac{1}{3(1-f)} \quad (9)$$

This can be integrated for an initial void volume f_o to give

$$\varepsilon_1^p=\left\{\frac{1}{(1+R)}[\alpha^a+1+R(1-\alpha)^a]\right\}^{(2/a-1)} \times\frac{1}{(1+R)}[1+R(1-\alpha)^{a-1}]\frac{4}{3(1+\alpha)} \ln\frac{f(f_o-1)}{f_o(f-1)}+\frac{1}{3}\ln\frac{f_o-1}{f-1} \quad (10)$$

Assuming isotropic hardening, the effective stress of the matrix material for Eq. (1) can be obtained as

$$\bar{\sigma}_M=\frac{1}{(1-f)}\left\{\left[\frac{1}{1+R}(|\sigma_2-\sigma_3|^a+|\sigma_3-\sigma_1|^a+R|\sigma_1-\sigma_2|^a)\right]^{2/a}+\left(\frac{3}{2}\sigma_m\sqrt{f}\right)^2\right\}^{1/2} \quad (11)$$

If the effective plastic strain increment of matrix material is denoted by $d\bar{\varepsilon}_M^p$, from the equivalent plastic work, in that the plastic work done by the unit volume of a porous material is identical to that by the matrix material with the volume $(1-f)$,

$$dw^p=\sigma_{ij}d\varepsilon_{ij}^p=(1-f)\,\bar{\sigma}_M d\bar{\varepsilon}_M^p \quad (12)$$

By substituting Eqs. (4) and (11) into Eq. (12), the rearrangement gives

$$d\bar{\varepsilon}_M^p=\left\{\left[\frac{1}{(1+R)}(|\sigma_2-\sigma_3|^a+|\sigma_3-\sigma_1|^a+R|\sigma_1-\sigma_2|^a)\right]^{2/a}+\left(\frac{3}{2}\sigma_m\sqrt{f}\right)^2\right\}^{1/2}\frac{4}{3f\sigma_{kk}}d\varepsilon_v \quad (13)$$

Using $\rho=\frac{d\varepsilon_2}{d\varepsilon_1}$ and $\alpha=\frac{\sigma_2}{\sigma_1}$, the above provides the following relation

$$d\bar{\varepsilon}_M^p=\frac{\sigma_1}{\bar{\sigma}}(1+\alpha\rho)\,d\varepsilon_1 \quad (14)$$

3. Results and Discussion

Figures 1(a) and (b) represent the yield surfaces in the normalized principal stress plane as represented by Eq. (11) for a void volume fraction with a given value of $R=1.87$. This is then compared with the measured yield surface from the biaxial tensile testing of a cruciform specimen by Kuwabara(1998). The shrinkages of the yield surfaces were self-evident as the void volume fraction was increased. This shows that the strain softening of the material occurred through damage caused by deformation.

Figure 1(b) shows the vertex effect on the yield surface characterized by the yield surface shape considered for a crystallographic slip of an anisotropic yield surface of a void material using Hosford's form. When comparing $a=2.0$ in Fig. 1 (a) with $a=6.0$ in Fig. 1(b), the shrinkage of the yield surface in the case of $a=6.0$ was lower than that of $a=2.0$ with a variation in the void volume fraction. The result of the yield surface in the case

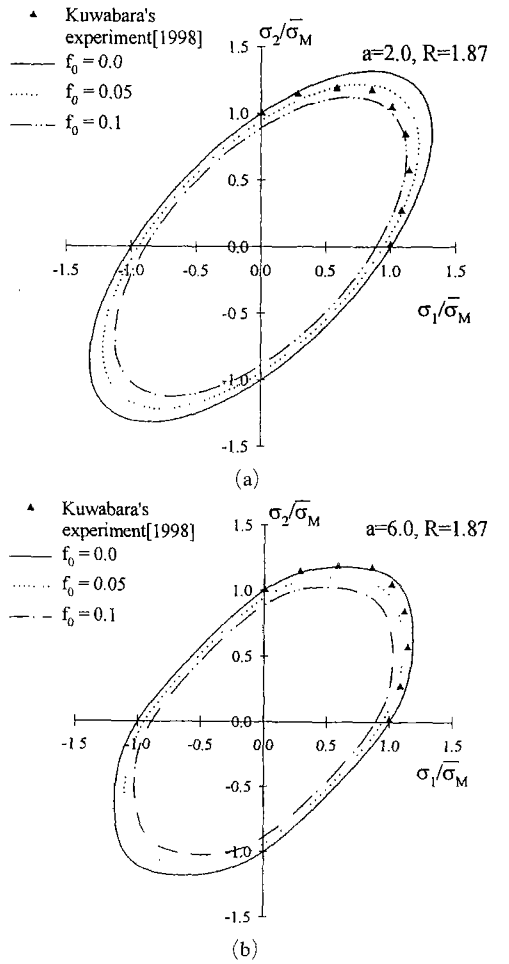

Fig. 1 Yield loci for (a) $a=2.0$ and (b) $a=6.0$ with voided anisotropic yield function

of $a=2.0$ using Hill's form did not agree well with that obtained experimentally by Kuwabara, yet that of $a=6.0$ using Hosford's form agreed well with the experimental result.

Figures 2 shows the yield surfaces for a varying R value at $a=2.0$ and Fig. 3 corresponds to non-quadratic power $a=6.0$ with given values of the initial void volume fraction $f_o=0.05$, respectively. For $a=2.0$, the yield surface shape was clear with a varying R value, as shown in Fig. 2. However, the yield surface shape of $a=6.0$ was not clearer than that of $a=2.0$. The distinct difference in the yield surface shape implies a reduction in the influence of the anisotropy parameter R with an

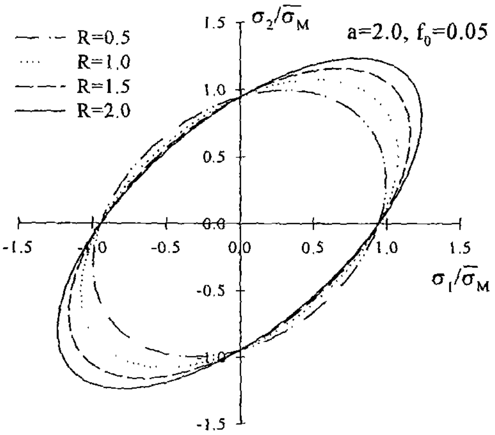

Fig. 2 Effect of plastic anisotropy parameter on the yield loci for $a=2.0$ and $f_o=0.05$

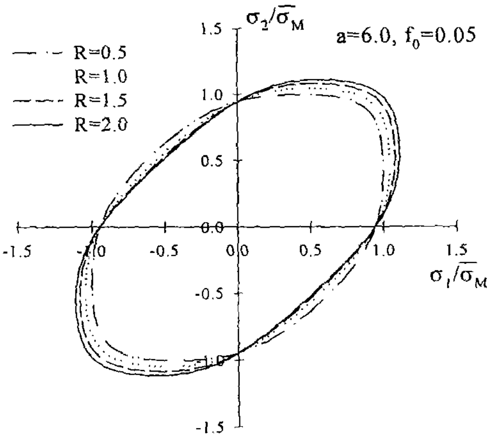

Fig. 3 Effect of plastic anisotropy parameter on the yield loci for $a=6.0$ and $f_o=0.05$

increased power of a.

The variation in the void growth rate relative to the current void volume fraction f for different stress ratios and power of a, as illustrated in Fig. 4, shows variation of void growth rate relative to void volume fraction for the three modes of equibiaxial tension (BT), plane strain tension (PS) and uniaxial tension (UT). The void growth rate increased when the power of a was reduced. It should be noted that the loading pattern had a significant effect on the void growth, as confirmed experimentally by Jalinier (1978) and Luo et al. (1992)

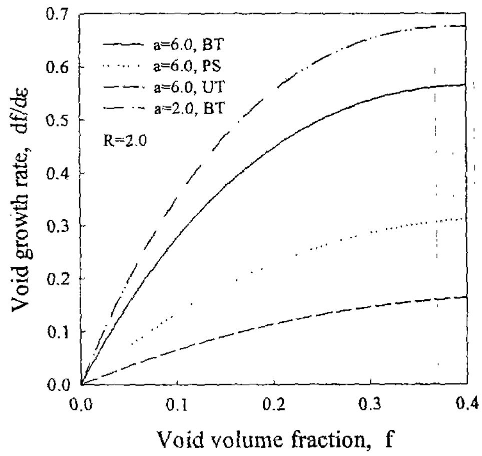

Fig. 4 Variation of void growth rate with void volume fraction for $a=2.0$, 6.0 and $R=2.0$

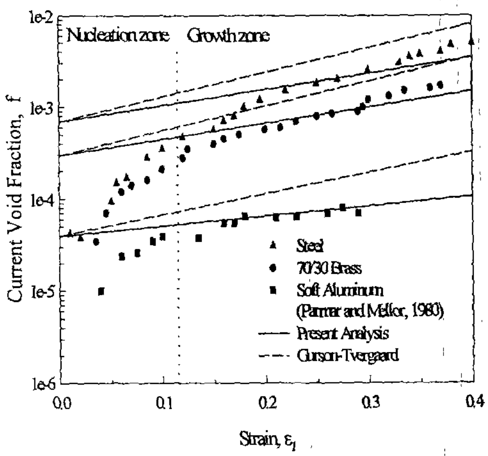

Fig. 5 Comparison between experimental and calculated void growth under equibiaxial tension for different materials

The results shown in Fig. 5 are compared with the measured void growth under equibiaxial tension for three alloys tested by Parmar and Mellor (1980). As comparison, Gurson-Tvergaard's model of Eq. (2a) for isotropic matrix material is used. The results of the proposed approximate Gurson's model with Hosford's anisotropic yield criterion predict well experiments than those of Gurson-Tvegaard's model with fitting parameters $q_1=2.3$ and $q_2=0.9$. The initial value of void volume fraction employed in the analysis is taken as that value resulting from extrapolating the

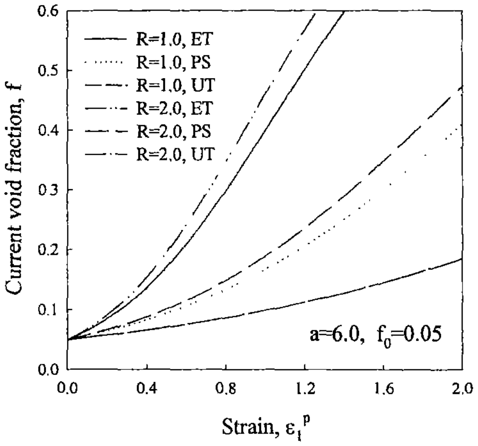

Fig. 6 Growth of void volume fraction with tensile strain for different stress states for $R=1.0$ and $R=2.0$

curve representing the growth zone to the zero strain (vertical) axis. The predicted void growth with strain, for steel ($R=1.68$), 70/30 brass ($R=1.68$) and soft aluminum ($R=0.64$) assumes values of $f_o=7\times10^{-4}$, $f_o=3\times10^{-4}$ and $f_o=4\times10^{-5}$, respectively.

Figure 6 represents the void volume fraction f with the plastic strain ε_1^p for the three modes o f loading pattern with an initial void volume fraction $f_o=0.05$ for $R=1.0$ and $R=2.0$. The result of the uniaxial tension for $R=2.0$ is overlapped with that of the case $R=1.0$

There was a higher increase in the void volume fraction with equibiaxial tension than with the other loading state modes. Also, the void volume fraction increased as the anisotropy parameter increased. However, for the case of uniaxial tension the variation in the void volume fraction did not appear to be influenced by the anisotropy parameter.

4. Conclusion

In this work, yield criterion of voided anisotropic material and associated flow rule are studied. Theoretical formulation for normal anisotropic materials under plane stress state - more suited to sheet metal deformation - is given. The yield surface relative to the void volume fraction and the damage evolution of an anisotropic sheet under biaxial stress state were both investigated. The results of the approximate anisotropic yield function for voided materials produced an effective representation of the yielding condition and plastic deformation behavior of an anisotropic sheet. There is no need to use the Gurson-Tvergaard modified models which should get fitting parameters for obtaining good predictions to the experimental results. These introduced parameters in the Gurson-Tvergaard modified models are only fitting parameters where they could not be related to the materials or even the initial porosity level. Accordingly, the proposed approximate yield function in conjunction with Hosford's anisotropic yield criterion for voided anisotropic materials can be used to evaluate the forming limit and damage of voided sheet metal better than other models do.

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References

Barlat, F. and Lian, J., 1989, "Plastic Behavior and Stretchability of Sheet Metals, Part I : A Yield Function for Orthotropic Sheets Under Plane Stress Conditions," *Int. J. of Plasticity*, Vol. 5, pp. 51~66.

Cristian, V., Hosford, F., 1983, "Yield Loci of Anisotropic Sheet Metals," *Int. J. Mech. Sci.*, Vol. 25(12), pp. 889~915.

Doege, E., Bagaviev, A. and Dohrmann, H., 1997, "Formability Analysis Based on the Anisotropically Extended Gurson Model," *Advanced Methods in Materials Processing Defects*, Ed. M. Predeleanu and P. Gilormini, pp. 281~288.

Goodwin, G. M., 1968 "Application of Strain Analysis to Sheet Metal Forming Problems in the Press Shop," *SAE Paper*, No. 680093.

Graf, A. and Hosford, W. F., 1993, "Effect of Changing Strain Paths on Forming Limit Diagrams of Al 2008-T4," *Meta. Trans.*, Vol. 24A,

p. 2503.

Gurson, A. L., 1977, "Continuum Theory of Ductile Rupture by Void Nucleation and Growth : Part I - Yield Criteria and Flow Rules for Porous Ductile Materials," *J. Eng. Mater. Tech. ASME*, Vol. 99, pp. 2～15.

Hill, R., 1948, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals," *Proc. R. Soc. London*, pp. 281～297.

Hill, R., 1950, *The Mathematical Theory of Plasticity*, Oxford University Press, London.

Hill, R., 1979, "Theoretical Plasticity of Textured Aggregates," *Math. Proc. Camb. Phil. Soc.*, Vol. 85, pp. 179～191.

Hosford, W. F., 1979, "On Yield Loci of Anisotropic Cubic Metals," *Proc. 7th North Am. Metalworking Conf.*, SME, Dearborn, MI, pp. 191～196.

Hosford, W.F., 1988, "Limitations of Non-Quadratic Anisotropic Yield Criteria and Their Use in Analysis of Sheet Forming, In: *Proc. of the 15th IDDRG Congr. A.S.M.*, Dearborn, MI, pp. 163～170.

Jalinier, J., 1978, Endommagement, Instabilite Plastique et Courbes Limites de Formage. Docteur-Ingenier Thesis, L'institut National Polytechnique de Lorraine, France.

Keeler, S. P., 1965 "Determination of Forming Limits in Automotive Stampings," *SAE paper*, No. 650535.

Kuwabara, T., IKeda, S. and Kuroda, K., 1998, "Measurement and Analysis of Differential Work Hardening in Cold-Rolled Steel Sheet Under Biaxial Tension," *J. Mater. Proc. Tech.*, Vol. 80, pp. 517～523.

Lee, U., 1998, "A Continuum Mechanics for Damaged Anisotropic Solids," *KSME Int. J.*, Vol. 12, No. 3, pp. 405～413.

Liao, K.C., Pan, J. and Tang, S.C., 1997, "Approximate Yield Criteria for Anisotropic Porous Ductile Sheet Metals," *Mechanics of Materials*, Vol. 26, pp. 213～226.

Luo, Z.J., Ji, W.H., Guo, N.C., Xu, X.Y., Xu, Q.S., Zhang, Y.Y., 1992, "A Ductile-Damage Model and Its Application to Metal-Forming Processes," *J. Mat. Proc. Tech.* Vol. 30, pp. 31～43.

Marciniak, Z. and Kuczynski, K., 1967, "Limit Strains in the Process of Stretch-Forming Sheet Metal," *Int. J. Mech. Sci. Vol.* 9, pp. 609～620.

Parmar, A. and Mellor, P.B., 1980, "Growth of Voids in Biaxial Stress Fields," *Int. J. Mech. Sci.*, Vol. 22, pp. 133～150.

Storen, S. and Rice, J., 1975, "Localized Necking in Thin Sheets," *J. Mech. Phys. Solids*, Vol. 23, pp. 421～441.

Tvergaard, V., 1987, "Effect of Yield Surfaces Curvatures and Void Nucleation on Plastic Flow Localization," *J. Mech. Phys. Solids*, Vol. 35(1), pp. 43～60.

Tvergaard, V., 1991, "Mechanical Modelling of Ductile Fracture," *Mechanica*, Vol. 26, pp. 11～16.

Zhao, L., Sowerby, R. and Sklad, M.P., 1996, "A Theoretical and Experimental Investigation of Limit Strains in Sheet Metal Forming," *Int. J. Mech. Sci.*, Vol. 38, pp. 1307～1317.