The Onset of Natural Convection and Heat Transfer Correlation in Horizontal Fluid Layer Heated Uniformly from Below

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The critical condition of the onset of buoyancy-driven convective motion of uniformly heated horizontal fluid layer was analysed by the propagation theory which transforms the disturbance quantities similarly. The dimensionless critical time, τ_c, is obtained as a function of the Rayleigh number and the Prandtl number. Based on the stability criteria and the boundary-layer instability model, a new heat transfer correlation which can cover whole range of Rayleigh number was derived. Our theoretical results predict the experimental results quite reasonably.

Key Words : Buoyancy Effect, Stability Analysis, Heat Transfer Correlation, Propagation Theory

Nomenclature

a	: Horizontal wave number [-]
d	: Fluid depth [m]
g	: Gravitational acceleration [m/s^2]
k	: Thermal conductivity [J/mK]
Nu	: Nusselt number ($=q_w/k\Delta T$) [-]
P	: Pressure [Pa]
Pr	: Prandtl number ($=\nu/\alpha$) [-]
Ra	: Rayleigh number ($=g\beta\Delta T d^3/\alpha\nu$) [-]
Ra_q	: Rayleigh number based on the heat flux ($=g\beta q_w d^4/k\alpha\nu$) [-]
q_w	: Wall heat flux [J/m^2]
T	: Temperature [K]
t	: Time [s]
$\vec{U}$	: Velocity vector [m/s]
w	: Dimensionless vertical velocity [-]
X,Y,Z	: Space in Cartesian coordinate [m]

Greeks

α	: Thermal diffusivity [m^2/s]
β	: Thermal expansion coefficient [1/K]
ΔT	: Temperature difference [K]
ζ	: Similarity variable [-]
θ	: Dimensionless temperature [-]
μ	: Viscosity [Pa s]
ν	: Kinematic viscosity [m^2/s]
ρ	: Density [kg/m^3]
τ	: Dimensionless time [-]

Subscripts

0	: Basic quantity
1	: Disturbed quantity

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1. Introduction

When an initially quiescent fluid layer is heated from below with a certain Rayleigh number exceeding critical value, the buoyancy-driven convective motion occurs. This convective motion driven by buoyancy forces has attracted many researcher's attention since Benard's (1901) systematic experiments. It is well known that buoyancy-driven convection plays an important role in many engineering problems, such as chemical vapor deposition, solidification, electroplating and also many other conventional processes involving heat and mass transfer. Most of these processes involve non-linear, developing temperature profiles and therefore it is one of the most important problems to predict when or from where the buoyancy-driven motion sets in.

Morton (1957) is the first to attempt a theoretical analysis of the stability of initially quiescent fluid layer with time-dependent base temperature profiles. Lick (1965) and Currie (1967) also analysed the stability of fluid layers heated from below with time-dependent manner. In their analyses, they froze the base temperature profiles and treated time as a parameter, therefore they neglected the variation of disturbances with time. From these, their analysis called frozen-time model confirms experimental results when base temperature profiles are nearly linear. Foster (1965) set forward an amplification theory to analyze the onset of buoyancy-driven convective motion which is taken when the fastest growing disturbances are amplified by a certain factor of its initial white noise disturbances. However, this method determined the initial disturbances and growth factor arbitrarily to fit experimental data. Wankat and Homsy (1977) introduced the energy method to analyse the convective instability of initially quiescent fluid layer under time -dependent base temperature profiles. The energy method determines a lower bound of onset condition, therefore predicted onset times are lower than experimental ones. Another weak point of this method is that it cant show the dependence of critical conditions on the Prandtl number. Jhaveri and Homsy (1982) analysed the onset time of buoyancy-driven motion under the step and linear change of surface temperature with time by introducing a random forcing function. They identified the onset condition by 1% increases in Nusselt number compared to the conduction state.

Choi et al. (1986) proposed the propagation theory to analyse the buoyancy-driven convection phenomena. In their analysis, they introduce the thermal-boundary layer thickness as a new length scaling factor and transformed disturbance equations similarly under the linear stability theory and the principle of exchange of stabilities. In the propagation theory, the onset conditions are defined as the conditions that the fastest growing disturbances start to grow rapidly. Their predicted results were compared with experimental data of initially quiescent horizontal fluid layers (Kim et al., 1999a), initially quiescent fluid-saturated horizontal porous fluid layers (Yoon and Choi, 1989), laminar forced convection flow (Kim et al., 1999b), and laminar natural convection flow (Chun and Choi, 1991), reasonably well.

Another important problem in buoyancy-driven convection will be the heat transfer characteristics in the thermally fully-developed state. For analysing this problem Howard (1964) proposed the boundary layer instability model that, for very high Rayleigh number case, the heat transfer characteristics have close relationship with stability criteria. Busse (1967) modified Howard's concept by considering the heat transfer resistance of upper boundary. Long (1976), Cheung (1980) and Arpaci (1997) derived backbone equation to predict the heat transfer in horizontal fluid layer. By incorporating their stability criteria into the boundary-layer instability model, Choi and his coworkers have derived new heat transfer correlations for horizontal fluid layer (Lee et al., 1988), fluid saturated porous layer (Yoon and Choi, 1989), plane Couette flow (Choi and Kim, 1994), and plane Poiseuille flow (Kim et al., 1999b). Their resulting heat transfer correlations are in good agreement with a great deal of available experimental data.

In this study we consider the buoyancy effects in horizontal fluid layer heated from below. The onset condition of buoyancy-driven convective motion is analysed and the predicted values are compared with available experimental data. Also, a new heat transfer correlation is proposed and compared with experimental data.

2. Stability Analysis

2.1 Governing equations

The system considered here is a Newtonian fluid with an initial temperature T_i confined by two infinite parallel plates. The fluid layer of depth "d" is heated from below with constant flux q_w. The upper boundary is kept at initial temperature T_i. The schematic diagram of the base system is shown in Fig. 1. For this system the

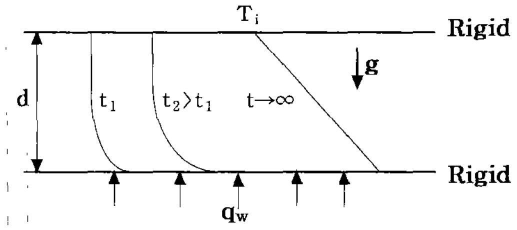

Fig. 1 Schematic diagram of base system

governing equations of flow and temperature fields are expressed by employing the Boussinesq approximation:

$$\nabla\cdot\vec{U}=0 \tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\vec{U}\cdot\nabla\right\}\vec{U}=\frac{1}{\rho_r}\nabla P+\nu\nabla^2\vec{U}+g\beta T\vec{k} \tag{2}$$

$$\left\{\frac{\partial}{\partial t}+\vec{U}\cdot\nabla\right\}T=\alpha\nabla^2 T \tag{3}$$

where $\vec{U}$, T, P, μ, α, g, ρ and β represent velocity vector, temperature, pressure, viscosity, thermal diffusivity, gravitational acceleration, density, and thermal expansion coefficient, respectively. The subscript "r" represents the reference state.

The important parameters to describe the present system are the Prandtl number, Pr, and the Rayleigh number based on the bottom heat flux, Ra_q, defined by

$$Pr=\frac{\nu}{\alpha}\text{ and }Ra_q=\frac{g\beta q_w d^4}{k\alpha\nu} \tag{4}$$

where k and ν denote thermal conductivity and kinematic viscosity, respectively. Ra_q is sometimes called the dimensionless heat flux. In case of slow heating the basic temperature profile is linear and time-independent and its critical condition is independent of Pr and represented by (Sparrow et al., 1964)

$$Ra_{q,c}=1296 \tag{5}$$

For a rapid heating system with large Ra_q, however, the stability problem becomes transient and complicated, and the critical time t_c to mark the onset of buoyancy-driven motion remains unsolved.

During the conduction state the base temperature field can be governed by the following dimensionless forms:

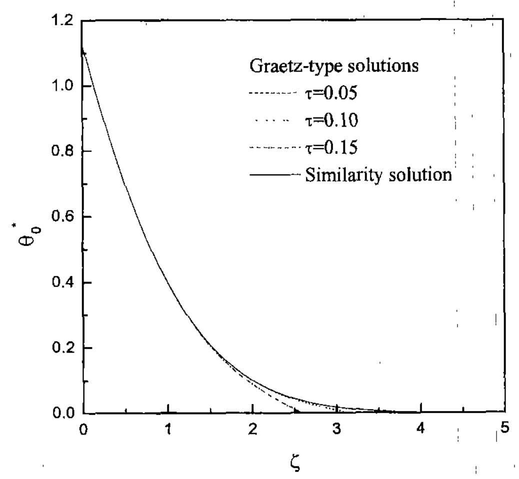

Fig. 2 Base temperature profiles

$$\frac{\partial\theta_0}{\partial\tau}=\frac{\partial^2\theta_0}{\partial z^2} \tag{6}$$

with the following initial and boundary conditions.

$$\theta_0=0\text{ at }\tau=0\text{ and }z=1 \tag{7.a}$$

$$\frac{\partial\theta_0}{\partial z}=-1\text{ at }z=0 \tag{7.b}$$

where $\tau=d^2/(\alpha t)$, $z=Z/d$ and $\theta_0=k(T-T_i)/q_w d$. The subscript "0" denotes the base state. The Graetz-type solution of base temperature field can be obtained by employing conventional separation of variable technique as follows:

$$\theta_0=1-z-2\sum_{n=1}^{\infty}\frac{1}{\mu_n^2}\cos(\mu_n z)\exp(-\mu_n^2\tau) \tag{8}$$

where $\mu_n=(n-1/2)\pi$. For deep-pool systems, the Leveque-type solution can be obtained as follows (Carslaw and Jaeger, 1959):

$$\theta_0=\sqrt{\frac{4\tau}{\pi}}\left\{\exp\left(-\frac{\zeta^2}{4}\right)-\zeta\,\mathrm{erfc}\left(\frac{\zeta}{2}\right)\right\} \tag{9}$$

where $\zeta=z/\sqrt{\tau}$. The above equation is in good agreement with the exact solution (8) in the region of $\tau\leq0.05$, as shown in Fig. 2.

Since we are primarily concerned with the deep-pool case of large Ra_q and small τ, the above Leveque-type solution of Eq. (9) represents the basic temperature profile quite well. Although the above Leveque-type solution represents the base temperature profile, for the mathematical convenience we introduce the dimensionless variable θ_0^*:

$$\theta_0^* = \frac{\theta_0}{\sqrt{\tau}} \tag{10}$$

Then the base temperature field within $\tau \leq 0.05$ can be transformed to

$$\frac{d^2\theta_0^*}{d\zeta^2} + \frac{\zeta}{2}\frac{d\theta_0^*}{d\zeta} - \frac{1}{2}\theta_0^* = 0 \tag{11}$$

with the boundary conditions

$$\frac{d\theta_0^*(0)}{d\zeta} = 0 \text{ and } \theta_0^*(\infty) = 0 \tag{12}$$

The solution of Eq. (11) satisfying Eq. (12) can be obtained by conventional numerical scheme and is the same as the base temperature profile of Eq. (9).

2.2 Stability equations

Under the linear stability theory disturbances caused by the onset of thermal convection can be formulated, in dimensionless form, in terms of the temperature component θ_1 and the vertical velocity component w_1 by transforming Eqs. (1) ~ (3):

$$\left\{\frac{1}{Pr}\frac{\partial}{\partial \tau} - \overline{\nabla}^2\right\}\overline{\nabla}^2 w_1 = \overline{\nabla}_1^2 \theta_1 \tag{13}$$

$$\frac{\partial \theta_1}{\partial \tau} + Ra_q w_1 \frac{\partial \theta_0}{\partial z} = \overline{\nabla}_1^2 \theta_1 \tag{14}$$

where $\overline{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\overline{\nabla}_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Here the velocity component has the scale of α/d and the temperature component has the scale of $\alpha\nu/(g\beta d^3)$. The proper boundary conditions are given by

$$w_1 = \frac{\partial w_1}{\partial z} = \frac{\partial \theta_1}{\partial z} = 0 \text{ at } z=0 \tag{15.a}$$

$$w_1 = \frac{\partial w_1}{\partial z} = \theta_1 = 0 \text{ at } z=1 \tag{15.b}$$

Our goal is to find the critical time τ_c for a given Pr and Ra_q by using Eqs. (13) ~ (15).

Based on the normal mode analysis, the convective motion is assumed to exhibit the horizontal periodicity. Then the perturbed quantities can be expressed as follows:

$$[w_1(\tau, x, y, z), \theta_1(\tau, x, y, z)] = [w_1(\tau, z), \theta_1(\tau, z)]\exp[i(a_x x + a_y y)] \tag{16}$$

where "i" is the imaginary number. The horizontal wave number "a" has the relation of $a = [a_x^2 + a_y^2]^{1/2}$. The propagation theory employed to find the onset time of convective motion (t_c) is based on the assumption that, at the onset of motion, the disturbances are propagated mainly within the thermal boundary layer thickness Δ_T and the following scale analysis by using $\Delta_T(\propto t^{1/2})$ as proper length scale would be valid for perturbed quantities of Eqs. (2) and (3), respectively:

$$\frac{\partial T}{\partial t} \sim W_1\frac{\partial T_0}{\partial Z} \sim \alpha\nabla^2 T_1 \sim \alpha\frac{T_1}{\Delta_T^2} \tag{17}$$

$$g\beta T_1 \sim \nu\nabla^2 W_1 \sim \nu\frac{W_1}{\Delta_T^2},\ W_1 \sim \frac{g\beta T_1 \Delta_T^2}{\nu} \tag{18}$$

From the above equations, the following relation can be obtained:

$$\frac{\partial T_0}{\partial Z} \sim \frac{\alpha\nu}{g\beta\Delta_T^4} = \frac{\Delta T}{\Delta_T}\left(\frac{g\beta\Delta T\Delta_T^3}{\alpha\nu}\right)^{-1} = \frac{\Delta T}{\Delta_T}Ra_{\Delta_T}^{-1} \tag{19}$$

where Ra_{Δ_T} is the Rayleigh number based on the length Δ_T and temperature difference across the boundary layer thickness ΔT. For the present system, the relation between ΔT and q_w has the following form:

$$\Delta T \sim \frac{q_w \Delta_T}{k} \tag{20}$$

Therefore, $Ra_{\Delta_T}\left(\propto \frac{g\beta q_w \Delta_T^4}{k\alpha\nu} = Ra_{q,\Delta_T}\right)$ has the meaning of the Rayleigh number based on the boundary layer thickness Δ_T and the wall heat flux q_w. It is now assumed that for small t the characteristic value of $Ra_{\Delta_T}(\propto Ra_q \tau^2)$ will become a constant since $|\partial T_0/\partial Z| \sim \Delta T/\Delta_T$ in Eq. (19). For the isothermal heating systems, this trend predicted by Patick and Wragg (1975), Foster (1965), and Jhaveri and Homsy (1982) experimentally and theoretically. Based on these evidences, the following relation is obtained from Eq. (18):

$$\left|\frac{w_1}{\theta_1}\right| \sim \delta_T^2 \tag{21}$$

where $\delta_T(\propto\sqrt{\tau})$ is the dimensionless thermal boundary layer thickness.

With the above reasoning the dimensionless amplitude functions of the most dangerous mode are assumed to have the form of

$$[w_1(\tau, z), \theta(\tau, z)] = [\tau w^*(\zeta), \theta^*(\zeta)] \quad (22)$$

where w^* and θ^* are the amplitude functions of disturbances. By using these relations the stability equation is obtained from the Eqs. (13) and (14) as

$$\left\{(D^2-a^{*2})^2+\frac{1}{2Pr}(\zeta D^3-a^{*2}\zeta D+2a^{*2})\right\}w^*=-a^{*2}\theta^* \quad (23)$$

$$\left(D^2+\frac{1}{2}\zeta D-a^{*2}\right)\theta^*=Ra^*w^*D\theta_0^* \quad (24)$$

where $a^*=a\sqrt{\tau}$, $Ra^*=Ra_q\tau^2$ and $D=d/d\zeta$. For the deep-pool case, the boundary conditions, Eq. (15), are transformed as follows;

$$w^*=Dw^*=D\theta^*=0 \text{ at } \zeta=0 \quad (25.a)$$

$$w^*=Dw^*=\theta^*=0 \text{ as } \zeta \to \infty \quad (25.b)$$

It is assumed that a^* and Ra^* are the eigenvalues, and also the onset time of buoyancy-driven convection for a given Ra_q is unique under the principle of exchange of stabilities. The above procedure is the essence of our propagation theory. Our propagation theory relaxed frozen-time model by considering the terms involving $\partial(\cdot)/\partial\tau$ in Eqs. (13) and (14).

2.3 Solution procedure

In order to solve the stability Eqs. (23) ~ (25) the base temperature profile must be obtained from Eqs. (11) and (12). For this purpose the fourth or fifth order Runge-Kutta-Fehlberg method is employed and the stability equations are solved by employing the outward shooting scheme of Chen et al. (1983). In order to integrate these stability equations the proper value of D^2w^*, D^3w^* and θ^* at $\zeta=0$ are assumed for a given Pr and a^*. Since the stability equations and the boundary conditions are all homogeneous, the value of $D^2w^*(0)$ can be assigned arbitrarily and the value of the parameter Ra^* is assumed. This procedure can be understood easily by taking into account of the characteristics of eigenvalue problems. After all the values at $\zeta=0$ are provided, this eigenvalue problem can be proceeded numerically.

Integration is performed from $\zeta=0$ to a fictitious upper boundary with the fourth order Runge-Kutta-Gill method. If the guessed value of Ra^*, $D^3w^*(0)$ and $\theta^*(0)$ are correct, w^*, Dw^* and $D\theta^*$ will vanish at the upper isothermal boundary. To improve the initial guesses the Newton-Raphson iteration is used. When convergence is achieved, the upper boundary is increased by predetermined value and the above procedure is repeated. Since the disturbances decay exponentially outside the thermal boundary layer, the incremental change of Ra^* also decays fast with an increase in upper boundary depth. This behavior enable us to extrapolate the eigenvalue to the infinite depth. The results of the above procedure for $Pr \to \infty$ is shown in Fig. 3. The minimum value of Ra^* on the curve of Ra^* vs. a^* will represent the conditions of the onset of thermal convection.

2.4 Stability analysis results

In the limiting case of infinite or zero Prandtl number, the governing equations are reduced to a simpler form because the inertia or viscous terms are negligible, respectively. For this limiting case, Lee et al. (1988) analysed the stability conditions. They approximated a base temperature profile and distributions by using integral method and WKB approximation. For infinite Prandtl number, the critical condition obtained from Fig. 3 is $Ra^*=20.03$, which is very close to the one predicted by Lee et al. ($Ra^*=20.88$). It shows good agreements between their critical condition

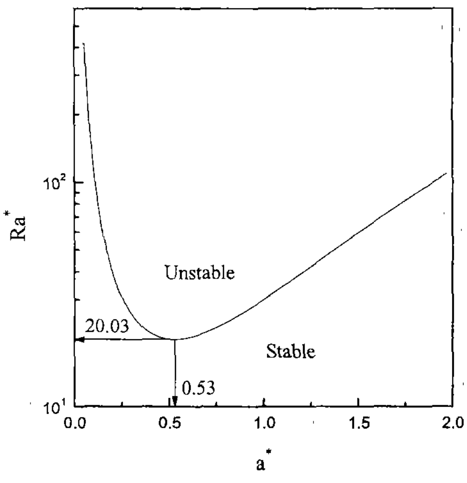

Fig. 3 Neutral stability curve for $Pr \to \infty$

and ours. This means that our numerical scheme is quite favorable to analyse the stability equations.

For finite Prandtl numbers, the critical values of a_c^* and Ra_c^* are summarized in Table 1. Based on these results and Lee et al.'s result for $Pr \to 0$, the critical condition can be represented as:

$$Ra_c^* = 20.03\left[1+\left(\frac{0.43}{Pr}\right)^{2/3}\right]^{3/2} \qquad (26)$$

It seems evident that Ra_c^* increase with decreases in Pr, and the Pr effect on critical conditions is negligible for $Pr \geq 10$. The Pr effect becomes pronounced for $Pr < 1$. This means that the inertia terms make the system more stable. This trend can be shown in Fig. 4 obviously. In order to compare the presented stability criteria with the experimental data, we introduce the experiments of Nielsen and Sabersky (1973) and Chu (1990), where very viscous liquids were used. The Prandtl numbers in their experiments were 45~4700 and 4×10^5, respectively. As mentioned above, the critical conditions are nearly independent of Pr for $Pr \geq 10$, so we adopt the infinite Pr case as the basis of the comparison between theoretical and experimental results. For infinite Pr, the stability criteria can be expressed as follows:

$$\tau_c = 4.48 Ra_q^{-1/2} \text{ and } a_c = 0.25 Ra_q^{1/4} \qquad (27)$$

The above results are compared with the experimental data of Nielsen and Sabersky (1973) and the theoretical results of Kim and Kim (1986) in Fig. 5. As shown in Fig. 5, our τ_c is lower than the experimental data. However, Kim and Kim's results shows fairly good agreement with experimental data. This discrepancy is due to the difference in the definition of critical condition of each study. We define τ_c as the time that infinitesimal disturbances start to grow exponentially, but Kim and Kim (1986) defined as the time when the Nusselt number shows minimum value in the plot of Nu vs. τ. It can be assumed that a certain time is required after the onset of disturbances to amplify the disturbances to affect the Nusselt number. This may explain the difference between our critical time and Kim and Kim's.

Foster (1969) proposed that the onset time of natural convection obtained by using the thermal boundary layer thickness as a length scaling factor should be too short by factor of 4. By accepting Foster's proposal, we suggest that the disturbances set in at τ_c will lead to manifest

Table 1 Numerical values of critical conditions for the various Pr

Pr	0.01	0.1	0.7	1	7	10	100	∞
Ra_c^*	1122.30	158.64	45.90	39.04	23.36	22.41	20.29	20.03
a_c^*	0.73	0.73	0.67	0.66	0.57	0.56	0.52	0.52

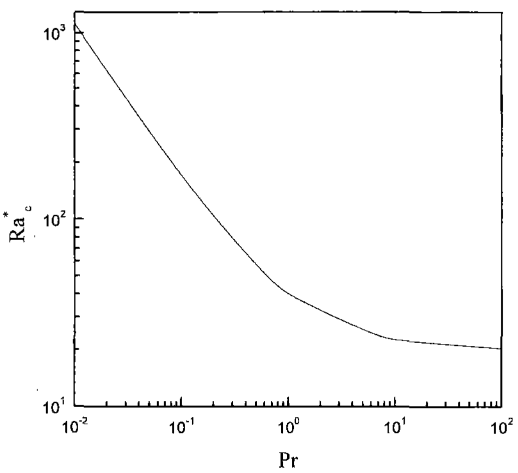

Fig. 4 Effect of Prandtl number on critical condition

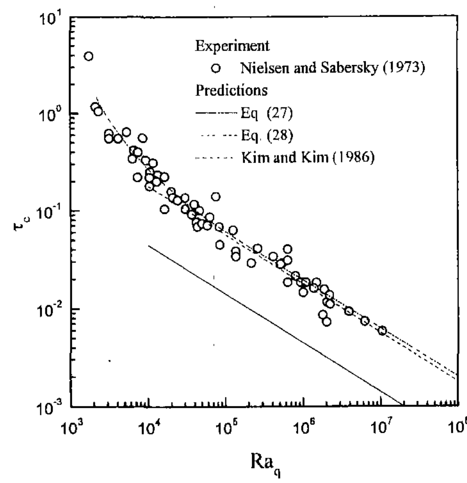

Fig. 5 Comparison of critical conditions with experimental data

convection at $4\tau_c$. Thus, it is assumed that the onset time when the convective motion can be detectable experimentally, τ_0 can be given as follows:

$$\tau_0=17.92Ra_q^{-1/2} \quad (28)$$

The above relation is compared with Nielsen and Sabersky's experimental works in Fig. 5.

Chu (1990) conducted experiments to investigate the relationship between Ra_c and $Ra_{q,c}$, where Ra_c is the critical Rayleigh number based on temperature difference between two plates. He represents his experimental data as Ra_c vs. $Ra_{q,c}$ plot. For convenience of comparison, we reconstruct our stability condition by using the following relation obtained from base temperature profile

$$\frac{k\Delta T}{q_w d}=\frac{2}{\sqrt{\pi}}\sqrt{\tau} \quad (29)$$

as

$$Ra_c=2.39Ra_{q,c}^{3/4} \quad (30)$$

In Fig. 6, Chu's experimental results are compared with the theoretical results of ours and Kim and Kim's (1986).

As shown in Fig. 6, our critical condition represents the incipient motion criteria fairly well, whereas Kim and Kim's results show good agreement with Nu_{min} criteria. It is assumed that incipient motion criteria have nearly same physical meaning of the present τ_c, and Nu_{min} criteria are related with critical condition of Kim and Kim's. From the Chu's visualization results, we can obtain very important information on the growth of disturbances. When the horizontal fluid layer is heated from below, a certain time is required to make the buoyancy-driven convection set in. Once the disturbances set in, they grow continuously and affect the Nusselt number. The Nusselt number follows conduction state to a certain time after onset of disturbances, and deviates from conduction state. And further time is required to show minimum point and undershoot in the Nusselt number.

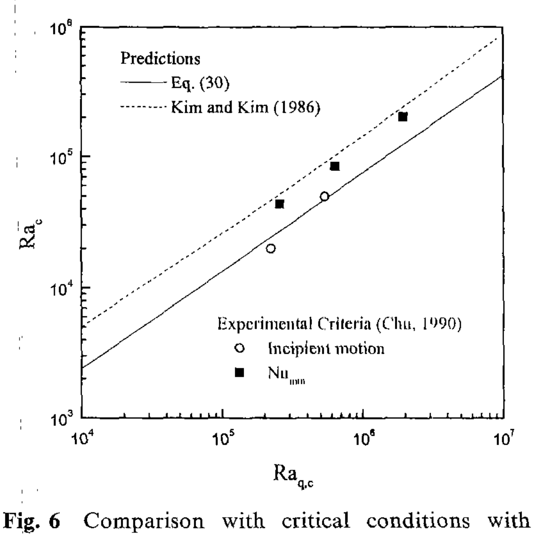

Fig. 6 Comparison with critical conditions with experimental data

Chu's Nu_{min} criteria have good agreements with Nilsen and Sabersky's results. From this, it can be assumed that Nielsen and Sabersky's criteria correspond to Chu's Nu_{min} criteria. From these, it seems evident that the disturbances which set in at the τ_c grow and they will affect the Nusselt number around $4\tau_c$.

3. Heat Transfer

3.1 Turbulent heat transport for large Rayleigh number

The possibility of connecting stability criteria to the heat transport on the turbulent thermal convection was investigated by Howard (1964). He postulated that at a large Rayleigh number the convective instability, in the form of thermal, sets in after a time t*, and the thermal break-up the boundary layer after a time much shorter than t*. It was further assumed that after the break-up of boundary layer the system be restored to quiescent state. According to the Howard's concept, we assume that the onset of thermal can be described by the above stability analysis, and that the turbulent heat transfer would be governed by the narrow boundary layer like a conduction film of thickness δ_* near the heated surface. δ_* is usually called the conduction layer thickness.

Busse (1967) modified the Howard's concept such that the heat transport resistances exist near the upper boundary as well as the lower one. This boundary layer instability model is schematized in Fig. 7. According to this model the Nusselt

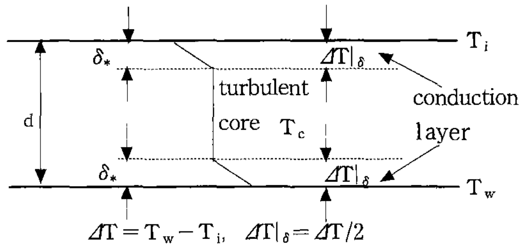

Fig. 7 Turbulent heat transport model

number in the fully-developed turbulent state is expressed as:

$$Nu = \frac{Q_{actual}}{Q_{conduction}} = \frac{1}{2}\frac{d}{\delta_*} \text{ for } Ra \to \infty \quad (31)$$

Long (1976), Cheung (1980), and Arpaci (1997) analysed the buoyancy-driven turbulent heat transport semi-theoretically and showed that the heat transport characteristics for $Ra \to \infty$ would be independent of the fluid-layer depth, like the Howard's and Busse's concept. By slight modification of their model, the following heat transfer correlation for the present system can be obtained:

$$Nu = \frac{ARa_q^{1/4}}{1 - BRa_q^{-1/12}} \quad (32)$$

where A and B are the undetermined constants.

By transforming Eq. (31) the heat transport in the fully-developed turbulent state may be expressed as:

$$Nu = \frac{1}{2}\left(\frac{Ra}{2Ra_\delta}\right)^{1/3} \text{ for } Ra \to \infty \quad (33)$$

where $Ra_\delta = g\beta\Delta T_\delta \delta_*^3/(\alpha\nu)$ is the Rayleigh number based on the conduction thickness δ_* and the temperature difference over the conduction layer thickness ΔT_δ. Following the boundary-layer instability model of Fig. 7, ΔT_δ is the half of the total temperature difference ΔT. By using the relation of $Ra_q = Ra \cdot Nu$, Eq. (33) can be replaced by

$$Nu = \frac{1}{2}\left(\frac{Ra_q}{Ra_\delta}\right)^{1/4} \text{ for } Ra_q \to \infty \quad (34)$$

Following the Howard's concept δ_* may be replaced by $\delta_{T,c}$. $\delta_{T,c}$ is thermal penetration depth at the onset condition of buoyancy-driven convection. From the Eq. (34) and the relations of $Ra^* = Ra_q\tau^2$, $\delta_{T,c} = 3.21\tau_c^{1/2}$, and Eq. (29), $Ra_{\delta_{T,c}}$ can be obtained as:

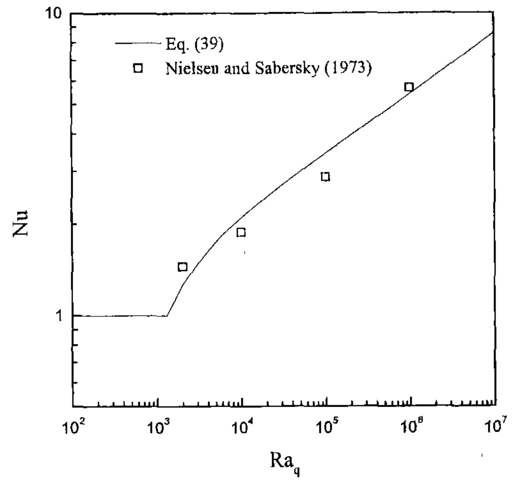

Fig. 8 Comparison with Nu vs. Ra_q for large Prandtl number

$$Ra_{\delta_{T,c}} = 747.3 \quad (35)$$

By combining Eqs. (34) and (35) the resulting heat transfer correlation for the fully-developed turbulent state is expressed as:

$$Nu = 0.0956 Ra_q^{1/4} \text{ for } Ra_q \to \infty \quad (36)$$

3.2 Heat transfer correlation

In order to obtain a heat transfer correlation over whole range of Rayleigh number, we consider the heat transfer near the criticla condition. The finite-amplitude heat transfer characteristics slightly over $Ra_{q,c}(=1296)$ can be obtained by using the shape assumption of Stuart (1964). For the region of $Ra_q \to Ra_{q,c}$, the Nusselt number can be expressed as:

$$\frac{1}{Nu} = 1 - \frac{\Gamma}{Ra_q}(Ra_q - Ra_{q,c}) \text{ for } Ra_q \to Ra_{q,c} \quad (37)$$

The constant Γ is obtained from the distribution of disturbance quantities at $Ra_q = Ra_{q,c}$:

$$\Gamma = \frac{\left(\int_0^1 w_1\theta_1 dz\right)^2}{\int_0^1 (w_1\theta_1)^2 dz} = 0.6313 \quad (38)$$

Assembling the Eqs. (32), (36) and (37), for the whole range of Ra_q, we can derive a new heat transfer correlation of the present system of large Prandtl number fluid as

$$Nu = 1 + \frac{0.0956(Ra_q^{1/4} - 1296^{1/4})}{1 - 1.404Ra_q^{-1/4}} \quad (39)$$

The above prediction agrees fairly well with the experimental results of Nielsen and Sabersky (1973), as shown in Fig. 8. It is noted that $Nu \equiv 1$ for $Ra_q = 1296$. This value corresponds to that of conduction state.

4. Conclusion

The critical condition of the onset of buoyancy-driven motion of uniformly heated horizontal fluid layer has been analyzed by the propagation theory. The predicted critical condition is verified by the comparison with experimental data. It seems evident that the disturbances which set in at τ_c must grow to be detectable experimentally. Incorporating the present stability criteria and the boundary-layer instability model, a new heat transfer correlation is derived. Since our theoretical predictions have close agreement with experimental results, it may be stated that our propagation theory is a powerful tool to examine the buoyancy-driven phenomena in horizontal fluid layers.

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