

Characteristics of Pressure Confined Concrete under Monotonic Compression

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Abstract

Tests of cylindrical concrete specimens under lateral confining pressure of up to 5,000 psi were conducted for two different axial loading cases: monotonic compression and monotonic tension. The purpose of this experimental investigation is to provide stress-strain characteristics of plain concrete in triaxial stress conditions. Lateral confining pressure levels, loading rates, and strength of concrete specimens are varied as parameters. The loading rates are 34.75×10^{-5} in/in/sec for fast, 6.95×10^{-5} in/in/sec for normal, and 0.579×10^{-5} in/in/sec for slow loading cases. The concrete specimens used in the experiment have compressive strength of 3,500 psi and 6,500 psi, respectively. Findings of this experiment include dependency of the stress-strain behavior of concrete on the above parameters under two different types of loading conditions. The parametric study includes a series of 106 triaxial tests..

keywords : *triaxial stress, confining pressure, loading rates*

1. Introduction

Experimental investigations have shown that the behavior of plain concrete in biaxial and triaxial stress varies considerably from the uniaxial loading case. It is evident in the biaxial loading that the compressive stress in the orthogonal direction tends to inhibit the formation of the microcracks in concrete. Nevertheless, the dilatational behavior in biaxial loading of concrete shows considerable volumetric strain increase at high axial stress levels. Therefore, the consideration of the deformation in the third direction is required, that is, the triaxial effects.

1.1 Literature Survey

A number of experimental studies for concrete under multiaxial states of stress have been reported for the last three decades. Models for high-strength concrete columns confined with lateral reinforcement have been proposed.^(1,2) These models for confined concrete are based on the results

of column test. Confinement models based on triaxial work by fluid pressure (active confinement) do not incorporate these models confined by spirals or ties (passive confinement) in a column. In the case of passive confinement, the confining pressure is not constant as for active confinement. The response under active and passive confinement was found to be similar.⁽³⁾ Strength criteria for the ultimate strength of confined concrete subjected to confining pressure were also proposed.⁽⁴⁻⁶⁾ The response of fiber-reinforced concrete subjected to multiaxial loading condition was studied, by performing uniaxial compression tests, split cylinder tests to acquire tensile properties, and a series of triaxial tests including various loading paths.⁽⁷⁾ Experimental work of the biaxial compressive tests of normal strength concrete⁽⁸⁾ and triaxial tests of normal strength concrete⁽⁹⁻¹¹⁾ were studied.

The behavior of high strength concrete under multiaxial states of stress was also studied. The compressive strength of sample specimen was ranged from 8,600 to 17,000 psi⁽¹²⁾ and from 6,000 to 15,000 psi under confining pressure

ranged from 1,200 to 12,000 psi.⁽¹³⁾

Damage characteristics of concrete under multiaxial states of stress were studied. Recent research results have illustrated that damage in concrete due to microcracking is manifest by volumetric expansion of the material.⁽¹⁴⁻¹⁸⁾ The behavior of plain concrete under triaxial stress states, as well as damage characteristics of concrete under dry and saturated condition was studied.⁽¹⁹⁾ The experimental focus pertains to accumulation of damage and strain history. The effect of stress path on multiaxial compressive strength was studied.⁽²⁰⁾

1.2 Research Significance

In the survey of literature, the behavior of plain concrete in triaxial stress was investigated under normal and high strength of concrete specimens, and low and high confining pressure levels and different stress path, but information about effect of loading rate is still lacking. Accordingly, the purpose of this work is to develop and provide information about the stress-strain characteristics of concrete in triaxial stress states with lateral confinement, loading rates and strength of concrete which is useful and required to predict the behavior of and design concrete structures subjected to confinement. Comprehensive experimental works have been performed in monotonic loadings with compressive and tensile loads.

2. Experimental Program

The short-term triaxial behavior of plain concrete cylinders subjected to lateral confining pressure with two different axial loading cases is investigated. The two axial loading cases are monotonic compression and monotonic tension. The tests were conducted according to the ASTM Standards; lateral stresses provided by fluid pressure were kept at preselected constant levels while axial stress was varied.

2.1 Test Series

Throughout the tests, two types of concrete specimens were used for all two loading cases: lower strength and higher strength concrete with uniaxial strength of 3,500 and 6,500 psi respectively. The test cases studied are described in the following and summarized in Table 1.

2.1.1 Monotonic Compression

Specimens were tested under different lateral confining pressure varied from 0 psi (unconfined, uniaxial) up to 5,000 psi. Three different axial loading rates were used to examine the effect of loading rates on the triaxial behavior of concrete:

fast rate of 1 minute, normal rate of 5 minutes, and slow rate of 60 minutes to reach the uniaxial compressive strength of unconfined concrete specimen by monotonically increasing the axial strain. The corresponding strain rates to the loading rates are 34.75×10^{-5} in/in/sec for fast, 6.95×10^{-5} in/in/sec for normal, and 0.579×10^{-5} in/in/sec for slow loading cases.

Table 1 Test series

Loading type	Confining pressure (psi)	Time to f'_c (T_i) (min)	Number of tests
Compression	0	1	3
		5	3
		60	3
	300	1	3
		5	3
		60	3
	600	1	3
		5	3
		60	3
	1200	1	3
		5	3
		60	3
	2000 ⁺	5	1
	3500 ⁺	1	1
		5	1
60		1	
5000 ⁺	5	1	
Tension	0	5	3
	300	5	3
	600	5	3
	1200	5	3
Total number of tests (53 cases x 2 types of concrete strength)			106

+ Under stress-controlled loadings

2.1.2 Monotonic Tension

The confining pressure of up to 1,200 psi was used. A single loading rate of 5 minutes to f'_c was used. The corresponding strain rate to the loading rates is 6.95×10^{-5} in/in/sec.

2.2 Test Specimens

Vertically cast cylindrical concrete specimens with 3 in. (76 mm) diameter by 7.5 in. (191mm) length (height/diameter = 2.5) were used for the monotonic compression tests. For the tension tests, the length of the specimen was reduced to 7 in. (178mm) by removing a 0.5 in. (13mm) portion of the specimen from the top zones (h/d = 2.3). For vertically cast concrete specimens, the top regions of the specimens typically consist of weaker concrete. This is attributed to segregation during compaction and minor bleeding at the top end surfaces of the specimens. These weak regions are removed to prevent occurrence of premature

tensile failure.

High early strength Portland Type III cement was used. Maximum aggregate size was 0.5 in. The mix ratio of concrete specimens is summarized in Table 2.

Table 2 Mix ratio by weight

Uniaxial strength	Water:Cement:Sand:Gravel
3,500 psi (Type A concrete)	0.74 : 1 : 3.17 : 3.05
6,500 psi (Type B concrete)	0.51 : 1 : 1.33 : 1.76

Specimens were cured in water until about 2 days prior to testing at 7 days. The nominal uniaxial compressive strengths of the unconfined specimen for lower and higher strength concrete were obtained by testing 3 in. diameter by 6 in. (152 mm) long control cylinders. End surfaces of the compression specimens were capped while those of the tension specimens were sliced and ground. Each confined specimen was coated with dual layers of urethane to prevent ingress of confining fluid into the specimen during testing

2.3 Triaxial Test System

Triaxial test systems were developed and used for different axial loading cases and confinement pressure levels. A schematic view of a test system is shown in Fig. 1. In all tests, the test systems were operated under stroke (displacement) control, except for the cases of compression loading with the confining pressure of 2,000 psi and up, in which stress-controlled loading was applied.

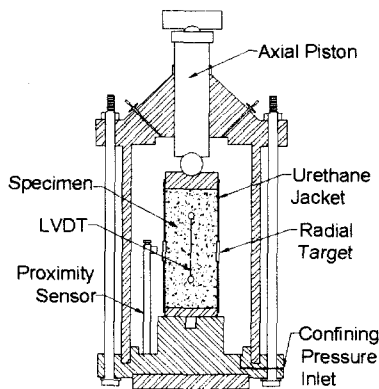


Fig. 1 Triaxial test system

Two pressure-independent linear variable displacement transformers (LVDTs), located diametrically opposite one another, were used to obtain the average axial deformation along the middle half of the specimen. Two pressure-independent proximity sensors, located diametrically opposite each other, were used to obtain the radial deformation at mid-height of the specimen. Aluminum mounting studs

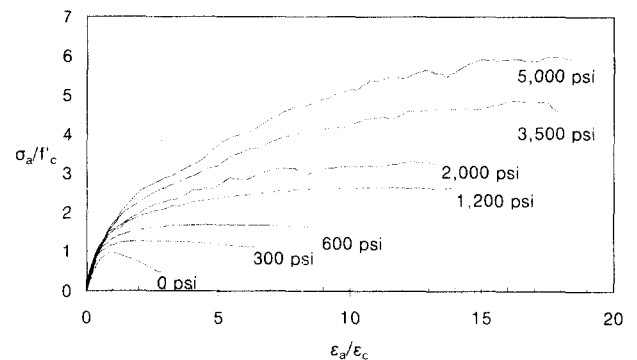
(for LVDTs) and steel targets (for proximity sensors) were glued onto each specimen using the epoxy. The tensile load train consisted of a concrete specimen epoxied at both ends, and attached to a pair of cable assemblies. The cable assemblies ensured that direct tensile loading was applied to the specimen.

3. Test Results and Evaluation

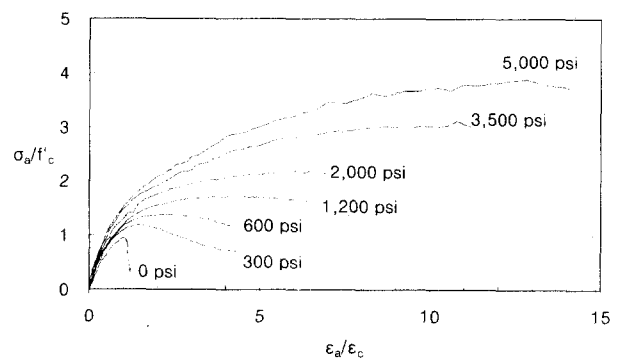
3.1 Monotonic Compression

3.1.1 The Effect of Confining Pressure

The effect of confining pressure on the monotonic stress-strain response is shown in Fig. 2. Increasing confinement resulted in higher ultimate strength and ductility (measured as a percentage of axial strain corresponding to peak stress). The initial tangent moduli also appear to increase with increasing confinements. The flatter post-peak descending branches of the stress-strain responses were observed with higher-confinements.



(a) Type A concrete



(b) Type B concrete

Fig. 2 Effect of confining pressure on the axial stress-strain response

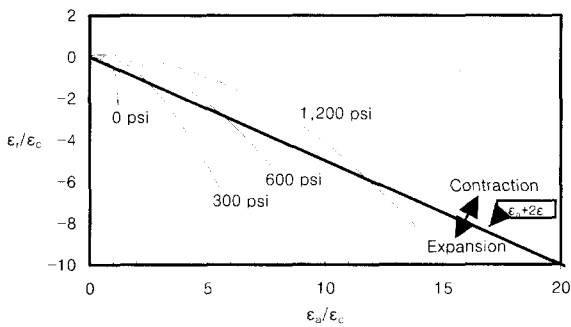
The above observed trends appear to hold both for lower strength (Type A, $f'_c = 3,500$ psi) and higher strength (Type B, $f'_c = 6,500$ psi) concrete investigated.

The response of concrete to mechanical load is characterized by progressive damage of its microstructure, a process

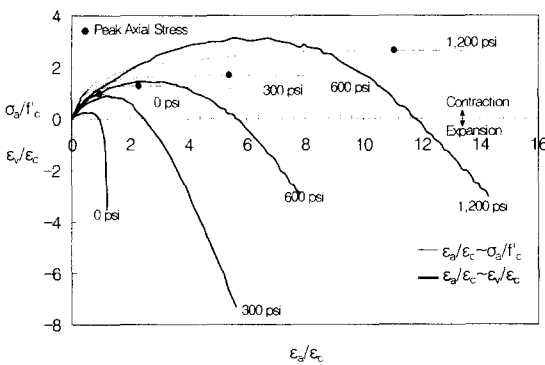
manifest by cracking and volumetric expansion of material mass.¹⁴⁻¹⁹ Therefore, the volumetric transition from contraction to expansion results in the critical characteristic of stress-strain response of concrete under triaxial stress state. Due to axisymmetry, volumetric strain defined by

$$\epsilon_v = \epsilon_a + 2\epsilon_r \quad (1)$$

The sign convention is positive in compression and contraction. In Fig. 3 (a), ϵ'_a/ϵ_c is the coordinate value of an intersection point between $\epsilon_a + 2\epsilon_r = 0$ and each axial-radial strain graph in the ϵ_a/ϵ_c axis. Beyond the value of ϵ'_a/ϵ_c , volumetric strains become expansive. In Fig. 3 (b), the same values of ϵ'_a/ϵ_c are associated with the peak axial stress at a given lateral confining pressure and beyond those values the strength of concrete becomes deteriorated due to the progressive damage of microstructure.¹⁸ Therefore, the higher value of ϵ'_a/ϵ_c results in the higher ultimate strength and the larger contraction area. Increasing confinement resulted in higher value of ϵ'_a/ϵ_c and as a result the ultimate strength becomes higher. This result shows good agreement with previous results.¹⁹ In Fig. 3 (b) higher confining pressure leads to further contraction in the volumetric strain (max. value of ϵ_v)



(a) Axial strain-radial strain response



(b) Axial strain-volumetric strain response

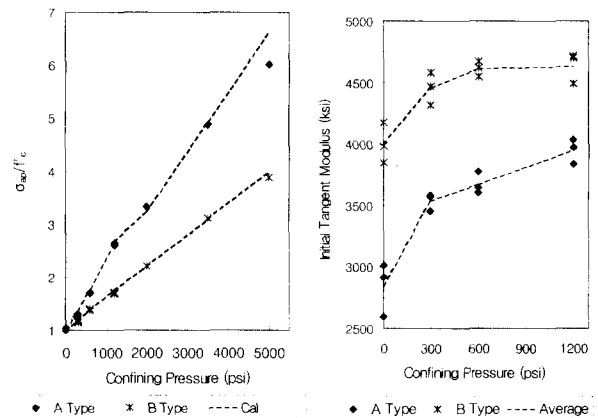
Fig. 3 Effect of confining pressure for type A concrete

A strength criteria for the ultimate strength of confined concrete subjected to confining pressure was proposed⁴ and is defined by

$$\sigma_{ap} = f'_c + 4.1\sigma_{lat} \quad (2)$$

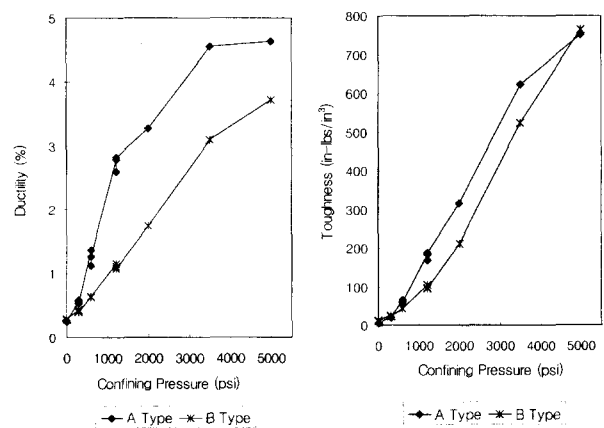
The uniaxial compressive strengths of unconfined concrete specimen (f'_c) used in the experiment vary slightly according to the lateral pressure despite the same strength type.

Fig. 4 (a) shows the effect of confining pressure on the peak axial stress σ_{ap} and the calculated ultimate strength (dotted line) proposed.⁴ Fig. 4 (b) shows the effect of confining pressure on the initial tangent modulus. A relatively higher increase in the toughness (measured as the area under peak load), ductility (measured as the strain at peak load), and ultimate strength were observed in lower strength concrete than in higher strength concrete. This result shows good agreement with previous results.¹³



(a) Peak axial stress

(b) Initial tangent modulus



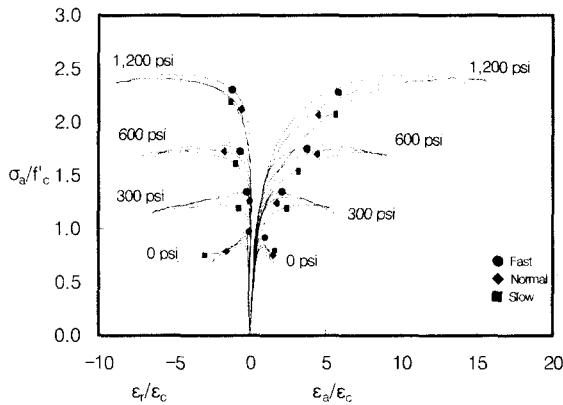
(c) Ductility

(d) Toughness

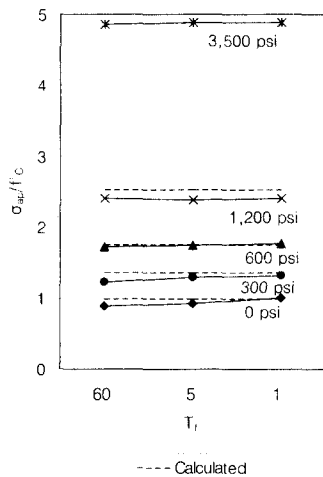
Fig. 4 Effect of confining pressure

3.1.2 The Effect of Strain Rates

The effect of strain rates on the triaxial monotonic stress-strain response was investigated by carrying out compression tests with the strain rates of 34.75×10^{-5} in./in./sec for fast, 6.95×10^{-5} in./in./sec for normal, and 0.579×10^{-5} in./in./sec for slow loadings. Fig. 5 (a) illustrates the effect of strain rates on the stress-strain responses and Fig. 5 (b) shows the effect of strain rates on the peak axial stress σ_{ap} and the calculated ultimate strength (dotted line) proposed.⁽⁴⁾



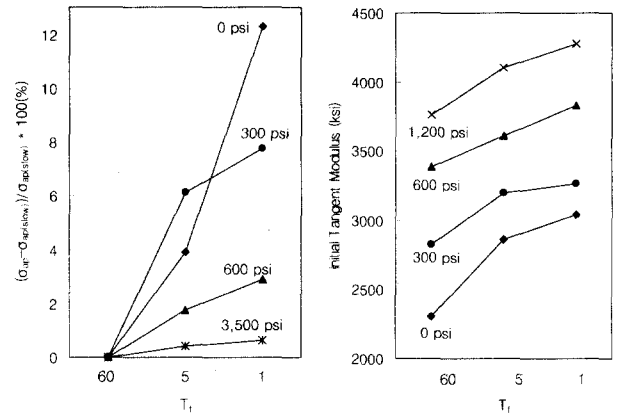
(a) Axial stress-axial strain-radial strain response



(b) Peak axial stress

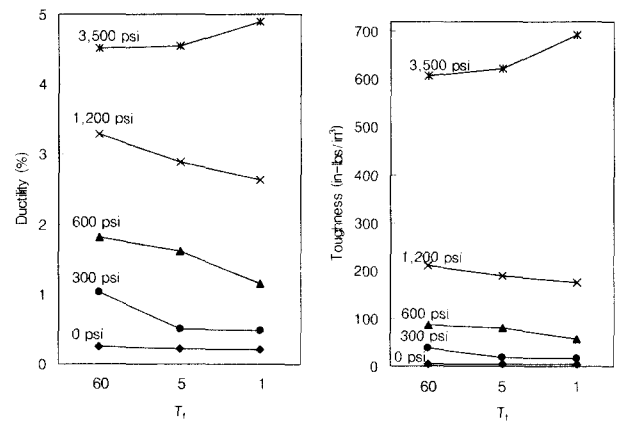
Fig. 5 Effect of strain rates

Higher strain rates generally lead to higher ultimate strength. However, improvement in strength appears to diminish with higher confinement. This result is shown in Fig. 6 (a) which illustrates the percentage of the increment of the peak axial stress. Fig. 6 (b) illustrates that under the same confining pressure level, higher strain rates appeared to produce slight increases in the initial tangent modulus. Effect of strain rates on ductility and toughness is plotted in Fig. 6 (c), (d). When $\sigma_{ur} < 3,500$ psi, higher rates of straining generally yield smaller ductility and toughness. Otherwise, the reverse trend appears to hold.



(a) Peak axial stress

(b) Initial tangent modulus



(c) Ductility

(d) Toughness

Fig. 6 Effect of strain rates for type A concrete

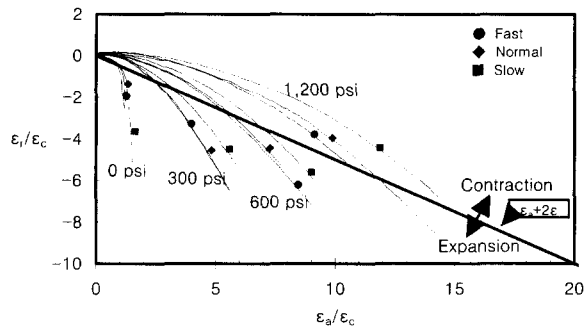
In Fig. 7 (b) and (c), under the same confining pressure further contraction in the volumetric strain (max. value of ϵ_v) is observed with slow rate of straining because in Fig. 7 (a) the slope of fast strain rate graph is steeper. This result is explained by

$$\epsilon_v = \epsilon_a + 2\epsilon_r, \Rightarrow \frac{d\epsilon_v}{d\epsilon_a} = 1 + 2\frac{d\epsilon_r}{d\epsilon_a} \quad (3)$$

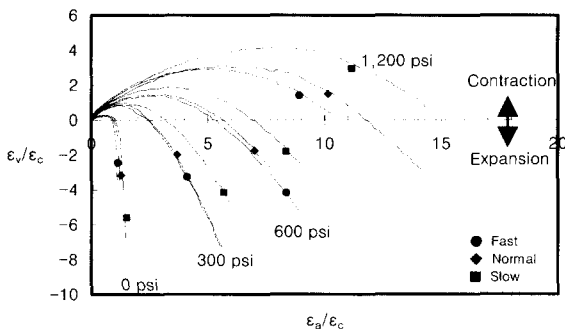
when $\frac{d\epsilon_r}{d\epsilon_a} = -\frac{1}{2}$, ϵ_v gets max. value.

In Fig. 7 (b), under the same confining pressure smaller values of $\epsilon_a^* / \epsilon_c$ are observed as the strain rates become faster. Since it is considered that higher strain rates generally lead to higher ultimate strength, this result conflicts with the recent results that the higher values of $\epsilon_a^* / \epsilon_c$ represent the higher peak axial stress¹⁸ based on the damage characteristics. It is supposed that in higher strain rates, insufficient progress of microcrack results in higher ultimate strength in spite of smaller values of $\epsilon_a^* / \epsilon_c$.

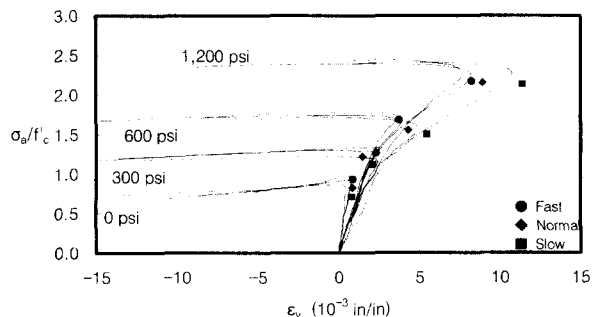
In Fig. 7 (c), Rapid volume expansion of concrete is attributed to the higher rate of increase in the radial strain to axial strain ratio (ϵ_r / ϵ_a) as the axial load is increased to the ultimate and beyond.



(a) Axial strain–radial strain



(b) Axial strain–volumetric strain



(c) Volumetric strain–axial stress

Fig. 7 Effect of strain rates for type A concrete

Fig. 8 and 9 show the strain rates effect on the octahedral stress-strain response of lower strength concrete. It is therefore concluded that the strain rate is an important parameter in the formulation of realistic constitutive models for concrete.

3.1.3 Effect of Concrete Strength

Based on the results reported here, the effect of concrete strength on the mechanical behavior of confined and unconfined concrete in compression is summarized:

- i) The confinement and the strain rates produce similar effects on both lower and higher strength concrete investigated.

- ii) Under the same confining pressure, a relatively higher increase in the toughness (measured as the area under peak load), ductility (measured as the strain at peak load), and ultimate strength were observed in lower strength concrete than in higher strength concrete.
- iii) The amount of compressive strength increment and the effect of strain rates are closely related to the ratio of confining pressure to uniaxial compressive strength (σ_{lat}/f'_c) for the two strength types studied.

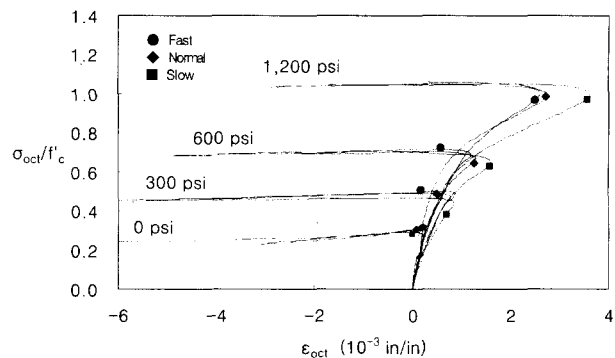


Fig. 8 Effect of strain rates on the octahedral normal stress-strain responses for type A concrete

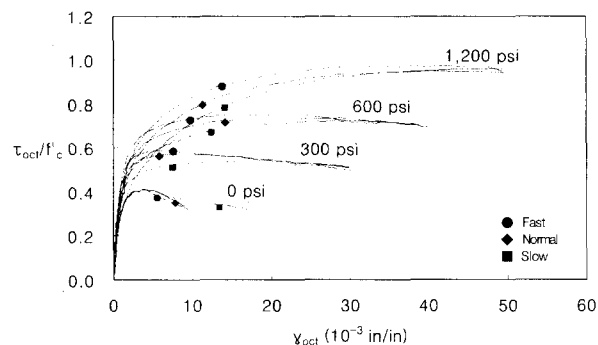


Fig. 9 Effect of strain rates on the octahedral shear stress-strain responses for type A concrete

3.2 Monotonic Tension

For confining pressures up to 1,200 psi, practically no reductions in the ultimate tensile strength of confined concrete are observed. The ultimate tensile strength is obtained at about 0.10 of the respective uniaxial compressive strength. For each of the confined and unconfined tension test conducted in this study, failure of concrete under normal rate of straining is accompanied by a single tensile fracture plane.

It thus appears that for concrete subjected to low confinements, no reduction in the ultimate tensile strength occurs. The various limiting confining pressure levels are not established in this study due to the limited range of confining pressure investigated.

Although not much difference in the ultimate tensile strength is observed, the ductility (axial strain correspond-

ing to peak axial stress) appears to increase with higher confining pressures. However, it is noted that at the zero axial stress level, indirect tensile strains are present due to the Poisson's effect induced by the surrounding confining pressure. If the total tensile strain is adjusted by the corresponding indirect tensile strain in each confined tension test, it appears that both the confined and unconfined concrete fail in tension when a certain strain limit is reached. In other words, the maximum strain failure criterion appears to apply for concrete in tension.⁽²¹⁾

3.3 Variability in Test Results

Various sources have been known to contribute to the variations of results obtained in concrete testing. Especially, exact strain measurements are difficult for concrete under multiaxial stress conditions.⁽¹⁹⁾ In this study, the sources of variability were attributed to material, strain measurements, and adequacy of sealing using urethane jackets. In triaxial testing where fluid confinement is used, the adequacy of the membrane or jacket in protecting the concrete specimens from the surrounding pressurized fluid is an important factor to obtain correct result. Markedly different behaviors can be obtained from unjacketed versus properly jacketed specimens. In this study, normal rate confined compression tests conducted on properly jacketed and unjacketed specimens and it was indicated that reductions in both strength and ductility were obtained from unjacketed specimens. Moreover, the reductions appeared to increase with higher degrees of confinement. Spalling around the mid-sections of the unjacketed specimens tested to failure was also observed.

4. Conclusion

The results obtained from this investigation aiming to explore the influence of a number of significant variables on the triaxial behavior of concrete for the benefit of improved understanding of the stress-strain characteristics can be summarized as follows:

- 1) Behavior of concrete in compression depends on the degree of confinement. Increasing confinements result in higher ultimate strength, ductility, and toughness, as well as flatter descending branches of the stress-strain diagrams. These results are explained by the mechanism that higher pressure restraining radial displacement by confining mechanism results in restraint of volumetric expansion. As a result, the transition value of volumetric strain from contraction to expansion beyond which microcracking deteriorates the strength of concrete under triaxial stress state increases.

- 2) Behavior of concrete in compression also depends on strain rates. Higher strain rates produce higher ultimate strength and stiffnesses, but reduced ductility and toughness. However, the increment of the ultimate strength appears to diminish with higher pressure.
- 3) In the cases of ductility and toughness beyond a certain lateral pressure, higher strain rates appear to produce higher ductility and toughness.
- 4) Under the same confining pressure further contraction in the volumetric strain is observed with slow strain rate
- 5) Under the same confining pressure, although higher strain rate produce higher ultimate strength, smaller values of $\varepsilon^*_a / \varepsilon_c$ is observed.
- 6) Behavior of various strength type concrete in confined compression appears to depend on the ratio of confining pressure to uniaxial compressive strength (σ_{lat} / f'_c). If the above ratio is less than the critical confinement ratio ($\sigma_{lat} / f'_c < \sigma_r / f'_c$), then the behavioral trends of confined concrete follow those observed in unconfined concrete. The critical confinement ratio (σ_r) is estimated at about 0.1 of the ultimate strength.

Acknowledgement

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Notation

- d = diameter of a cylindrical specimen
h = height of a cylindrical specimen
 f'_c = uniaxial compressive strength of unconfined concrete specimen
 T_j = time to reach the f'_c
 f'_t = tensile strength of unconfined concrete specimen
 f'_sp = split strength of unconfined concrete specimen
 ε_c = uniaxial compressive strain of an unconfined concrete specimen corresponding to the peak axial stress f'_c
 ε_a = axial strain of a cylindrical specimen
 ε_r = radial strain of a cylindrical specimen
 ε_v = volumetric strain of a cylindrical specimen
 $\varepsilon^*_a / \varepsilon_c$ = transition coordinate value of volumetric strain from contraction to expansion in the $\varepsilon_a / \varepsilon_c$ axis of $\varepsilon_a / \varepsilon_c$ and $\varepsilon_v / \varepsilon_c$ diagram
 σ_{lat} = lateral confining pressure applied to cylindrical speci-

men, in the radial direction

σ_a = axial stress applied to a cylindrical specimen

σ_{ap} = peak axial stress at a given lateral confining pressure

$\sigma_{ap(slow)}$ = peak axial stress at a given lateral confining pressure when $T_f = 60$ min. (slow strain rate)

σ_{oct} = octahedral normal stress

τ_{oct} = octahedral shear stress

σ_T = critical lateral confining pressure which produces confinement effect

References

1. Collins, M. P., "Stress-Strain Relationships of High-Strength Concrete for Use in Structural Design," Network of Excellence on High-Performance Concrete, Montreal, Apr. 26-28, 1992.
2. Ahmad, S. H., and Shah, S. P., "Stress-Strain Curves of Concrete Confined by Spiral Reinforcement," *ACI Journal*, Proceedings Vol. 79, No. 6, pp. 484-490, Nov.-Dec., 1982.
3. Richart, F. E., Brandtzaeg, A., and Brown, R.L., "Failure of Plain and Spirally Reinforced Concrete in Compression," Bulletin 190, University of Illinois, Engineering Experimental Station, Champaign, Illinois, 1929.
4. Richart, F. E., Brandtzaeg, A., and Brown, R. L., "A Study of the Failure of Concrete under Combined Compressive Stresses," Bulletin 185, University of Illinois, Engineering Experimental Station, Urbana, Illinois, 1928.
5. Setunge, S., Attard, M. M., and Darvall, P. L., "Ultimate Strength of Confined Very High-Strength Concretes," *ACI Structural Journal*, Vol. 90, No. 6, pp. 632-641, 1993.
6. Attard, M.M., and Setunge, S., "Stress-Strain Relationship of Confined and Unconfined Concrete," *ACI Materials Journal*, Vol. 93, No. 5, pp 432-442, Sept.- Oct., 1996.
7. Chern, J. C., Yang, H. J., and Chen, H. W., "Behavior of Steel Fiber Reinforced Concrete in Multiaxial Loading," *ACI Materials Journal*, Vol. 89, No. 1, pp. 32-40, Jan.-Feb., 1992.
8. Kupfer, H., Hilsdorf, H. K., and Rusch, H., "Behavior of Concrete under Biaxial Stresses," *ACI Journal*, Vol. 66, No. 8, pp. 656-666, Aug., 1969.
9. Chinn, J., and Zimmerman, R.M., "Behavior of Plain Concrete under Various High Triaxial Compression Loading Conditions," Technical Report No. WL TR 64-163, Air Force Weapons Laboratory, N. Mex., 146 pp, Aug., 1965.
10. Mills, L.L., and Zimmerman, R.M., "Compressive Strength of Plain Concrete under Multiaxial Loading Conditions," *ACI Journal*, Vol. 67, No. 10, pp. 802-807, Oct., 1970.
11. Launy, P., and Gachon, H., "Strain and Ultimate Strength of Concrete under Triaxial Stress," *Concrete for Nuclear Reactors*, SP-34, American Concrete Institute, Farmington Hills, Mich., pp. 112-121, 1972.
12. Xie, J., Elwi, A. E., and MacGregor, J. G., "Mechanical Properties of Three High-Strength Concretes Containing Silica Fume," *ACI Materials Journal*, Vol. 92, No. 2, pp. 135-145, Mar.-Apr., 1995.
13. Ansari, F., and Li, Q., "High-Strength Concrete Subjected to Triaxial Compression," *ACI Materials Journal*, Vol. 95, No. 6, pp. 747-755, Nov.-Dec., 1998.
14. Mier, V., and M., J. G., "Fracture of Concrete under Complex Stress," *Heron*, Vol. 31, No. 3, pp. 1-90, 1986.
15. Smith, S. S., "On Fundamental Aspects of Concrete Behavior," Structural Research Series No. 87-12, C.E.A.E. Department, University of Colorado, Boulder, 1987.
16. Smith, S. S., Willam, K. J., Gerstle, K. H., and Sture, S., "Concrete over the Top, or: Is There Life after Peak?" *ACI Materials Journal*, Vol. 86, No. 5, pp.491-497, Sept.-Oct., 1989.
17. Imran, I., "Applications of Nonassociated Plasticity in Modeling the Mechanical Response of Concrete," Ph.D. Thesis, Department of Civil Engineering, University of Toronto, 1994.
18. Pantazopoulou, S. J., and Mills, R. H. "Microstructural Aspects of the Mechanical Response of Plain Concrete," *ACI Materials Journal*, Vol. 92, No. 6, pp. 605-616, Nov.-Dec., 1995.
19. Imran, I., and Pantazopoulou, S. J., "Experimental Study of Plain Concrete under Triaxial Stress," *ACI Materials Journal*, Vol. 93, No. 6, pp. 589-601, Nov.-Dec., 1996.
20. Lan, S., and Guo, Z., "Experimental Investigation of Multiaxial Compressive Strength of Concrete under Different Stress Paths," *ACI Materials Journal*, Vol. 94, No. 5, pp. 427-434, Sept.-Oct., 1997.
21. Soon, K. A., "Behavior of Confined Concrete in Monotonic and Cyclic Loadings," PhD Thesis, Department of Civil and Environmental Engineering, MIT, June 1987.