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Constitutive Modeling of Confined Concrete under Concentric Loading

Cha-Don Lee* Ki-Bong Choi** Jun-Sil Cha***

* Professor, Department of Architectural Engineering, Chungang University, Korea.
** Professor, Department of Architectural Engineering, Kyungwon University, Korea.
*** Graduate Student, Department of Architectural Engineering, Chungang University, Korea

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Abstract

The inelastic behavior of a reinforced concrete columns is influenced by a number of factors : 1) level of axial load, 2) tie spacing, 3) volumetric ratio of lateral steel, 4) concrete strength, 5) distribution of longitudinal steel, 6) strength of lateral steel, 7) cover thickness, 8) configuration of lateral steel, 9) strain gradient, 10) strain rate, 11) the effectively confined concrete core area, and 12) amount of longitudinal steel.

A new constitutive model of a confined concrete is suggested in order to investigate the nonlinear behavior of the reinforced concrete columns under concentric loading. The developed constitutive model for the confined concrete takes into account the effects of effectively confined area as well as the horizontal and longitudinal distributions of the confining pressures. None of the existing models incorporated these two main effects at the same time. A total of different six constitutive models for the behavior of the confined concrete under concentric compression were compared with the sixty-one test results reported by different researchers. The superiority of the developed model in its accuracy is demonstrated by evaluating the error function, which compares the weighted averages for the sum of squared relative differences in peak compressive strength and corresponding strain, stress at strain equal to 0.015, and total area under stress-strain curve up to strain equal to 0.015.

***Keywords** : inelastic behavior, confined concrete, constitutive model, ductility, strength, confining pressure, effectively confined area, error function*

1. Introduction

Seismic design of framed structures is mostly based on the ductility approach. The safety of the structures during a major earthquake depends on the ability to deform plastically while maintaining the maximum load-carrying capacity. In seismic design, the reinforced concrete columns need to be well confined by the lateral reinforcement to provide sufficient deformation capability against large displacement occurring in the inelastic rage. Confinement requirements of various codes are based on maintaining the axial strength of a column after cover concrete is spalled off.

The inelastic behavior of columns is influenced by a number of factors such as : 1) level of axial load, 2) tie spacing, 3) volumetric ratio of lateral steel, 4) concrete strength, 5) distribution of longitudinal steel, 6) strength of lateral steel, 7) cover thickness, 8) configuration of lateral steel, 9) strain gradient, 10) strain rate, 11) the effectively confined concrete core area, and 12) amount of longitudinal steel. Some of these important parameters are ignored in the code equations and thus, in certain circumstances code provisions maybe unsafe, while in others, they may be unnecessarily conservative.

Semi-empirical and pure empirical equation for the complete stress-strain curves for the confined concrete under axial compression and/or flexural compression have been developed on the basis of experimental observations [8-19]. These models mainly differ in their derivations from assumptions made for effectively confined area and distributions of lateral pressure by confinement effect. Currently available models, however, do not consider these two influencing factors simultaneously, but separately.

A new model which takes into account the effects of effectively confined concrete area as well as distribution of confining pressure are developed in this study for the confined concrete under concentric loading. Main characteristic parameters are identified by minimizing the error function which measures the differences between model predictions and test results. The detailed descriptions are given in the following sections.

2. Modeling of the Stress-Strain Relationship of Confined Concrete

Beneficial inelastic ductile behavior of a column is a direct outcome of the constitutive characteristics of the confined concrete in a column. Analytical simulation of this effect is only possible if a model for the confined concrete is able to represent the degree of confinement accurately in terms of constituent material properties, quantity, and column geometry.

As mentioned before, it is necessary that a reasonable constitutive model for the confined concrete be expressed by both the distribution of confining pressure and the effectively confined concrete area. None of the existing models incorporated these two main effects at the same time.

The parabolic ascending branch and bilinear descending branches are used to simulate a complete stress-strain curve of a confined concrete in this study (Fig. 1).

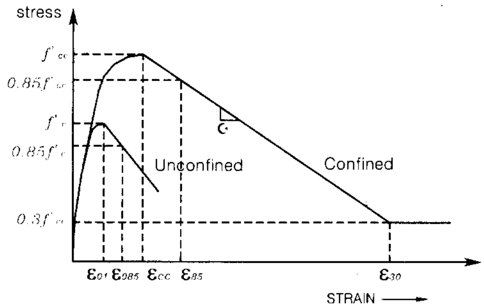


Fig. 1 Model of stress-strain relation

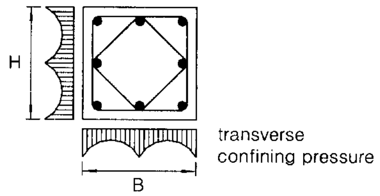
Let confining factor K_s be expressed as

$$K_s = 1.0 + \frac{b \cdot h}{P_{occ}} \cdot K_a \cdot K_p \quad (1)$$

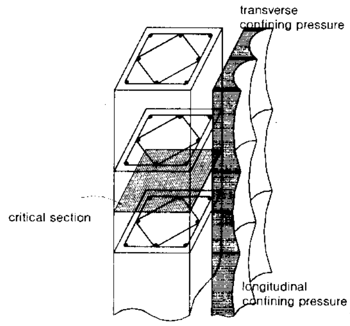
where, $K_a =$ factor representing portion of effectively confined area; $K_p =$ factor representing portion of longitudinal pressure; b and $h =$ center-to-center distance of perimeter tie of rectangular core; $P_{occ} = 0.85 f'_c (A_{co} - A_{st})$; $A_{co} =$ confined concrete area; and $A_{st} =$ total area of longitudinal steel.

2.1 Effect of the Effective Concrete Area (K_a)

In this research, the K_a factor is the one adopted from Sheikh's Model[2] (see Fig. 2).



(a) Transverse distribution of confining pressure



(b) Longitudinal distribution of confining pressure

Fig. 2 Distributions of confining pressure

where, n_B and $n_H =$ the number of arcs in B and H directions; c_B and $c_H =$ the center-to-center distances between longitudinal bars in B and H directions ; and $s =$ longitudinal spacing of transverse reinforcement.

$$K_a = \left(1.0 - \frac{\sum_{i=1}^{n_B} c_{B,i}^{\ 2} + \sum_{i=1}^{n_H} c_{H,i}^{\ 2}}{\alpha \cdot A_{co}} \right) \cdot \left(1.0 - \frac{0.5s}{b} \cdot \tan\theta \right) \left(1.0 - \frac{0.5s}{h} \cdot \tan\theta \right) \quad (2)$$

2.2 Effect of Confining Pressure(K_p)

The K_p factor is developed in this study with reference to Fig. 2 in order to take into account the distributions of the confining pressures. The K_p is estimated by considering the distributions of confining pressure in transverse direction (p_c) and in longitudinal direction (p_s). The transverse reinforcement yield, the average confining pressure acting on the concrete underneath the transverse reinforcement can be assumed to be equal to the yielding strength of the transverse reinforcements divided by the area of the transverse reinforcement in direct contact with the concrete surface.

where, $A_{sh} =$ cross-sectional area of transverse reinforcement; f_{yh} = the strength in the lateral reinforcement;

α_B and α_H = the angles between the transverse reinforcement in B and H directions; α_B or α_H is equal to 90° if the transverse reinforcement is perpendicular to B; ns_B and ns_H = number of steel at one side of column sections; d_t = diameter of hoop bar; and c_o = the minimum center-to-center distance between longitudinal bars.

$$p_c = \left(\frac{\sum_{i=1}^{ns_B} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{B,i} + \sum_{i=1}^{ns_H} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{H,i}}{d_t \cdot (b+h)} \right) \cdot \left(\frac{2c_o}{c_B + c_H} \right)^{\eta} \tag{3}$$

Note that the expression $\left(\frac{2c_o}{c_B + c_H} \right)^{\eta}$ in the above eq. (3) represents the effect of geometrical arrangement of longitudinal reinforcements on the distribution of confining pressure in a section. It is reasonable assumption that more closely spaced longitudinal bars would produce less varying pressure distribution along the horizontal transverse reinforcement level (see Fig. 2). The influence of confining pressure will be reduced between the transverse steels. Fig. 2 schematically represents these pressure changes in the longitudinal direction. At the mid-height between the transverse reinforcements, the pressure and also the effectively confined area will be the minimum, which implies that this section will be the most susceptible section to failure when a column is subject to its maximum strength.

The changes in the confining pressures can be reasonably approximated with the assumption that if a concrete is continuously confined by transverse reinforcement (i.e, $s = 0$ ideally), then p_c expressed by eq. (3) acts on the column, but if the spacings between transverse reinforcements are much larger (i.e, $s = \infty$), then p_c eventually diminishes. These extreme boundary conditions can be met if a function is assumed in the form of

$$\left(1 + \frac{2s}{b+h} \right)^{-\omega} \tag{4}$$

where, ω is a correcting coefficient. Note that the function p_s gets close to 1.0(0.0) as the spacing between transverse steels, approaches to zero (infinity).

Therefore, at mid-height, the factor representing confining pressure will be

$$K_p = p_c \cdot p_s = \left(\frac{\sum_{i=1}^{ns_B} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{B,i} + \sum_{i=1}^{ns_H} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{H,i}}{d_t \cdot (b+h)} \right) \cdot \left(\frac{2c_o}{c_B + c_H} \right)^{\eta} \cdot \left(1 + \frac{2s}{b+h} \right)^{-\omega} \tag{5}$$

Substituting eq. (2) and eq. (5) into eq. (1), the eq. (1) becomes

$$\begin{aligned} K_s &= 1.0 + \frac{b \cdot h}{P_{occ}} \cdot K_a \cdot K_p \\ &= 1.0 + \beta \cdot \frac{b \cdot h}{P_{occ}} \cdot \left(1.0 - \frac{\sum_{i=1}^{n_B} c_{B,i}^{\,2} + \sum_{i=1}^{n_H} c_{H,i}^{\,2}}{\alpha \cdot A_{co}} \right) \cdot \left(1.0 - \frac{0.5s}{b} \cdot \tan\theta \right) \left(1.0 - \frac{0.5s}{h} \cdot \tan\theta \right) \cdot \\ &\quad \left(\frac{\sum_{i=1}^{ns_B} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{B,i} + \sum_{i=1}^{ns_H} A_{sh} \cdot f_{yh} \cdot \sin\alpha_{H,i}}{d_t \cdot (b+h)} \right) \cdot \left(\frac{2c_o}{c_B + c_H} \right)^{\eta} \cdot \left(1 + \frac{2s}{b+h} \right)^{-\omega} \end{aligned} \tag{6}$$

in which the multiplier β is a correcting coefficient.

2.3 Other Characteristic Points

It is suggested in Ref.[1] that all the remaining characteristic points on stress-strain curve can be reasonably approximated as a function of K_s. The following expressions are used for these characteristic points. These are empirically determined by trial and error in this research.

$$\varepsilon_{cc} = \varepsilon_{01} \cdot (1.0 + k_1 \cdot K^*) \tag{7}$$

$$\varepsilon_{85} = \varepsilon_{085} \cdot \left(1.0 + k_2 \cdot K^* \cdot \frac{\rho_{sh}}{\sqrt{f'_c}} \right) \tag{8}$$

where, ε_{cc} = strain values corresponding to the maximum stress f'_{cc} ; $K^* = K_s - 1.0$; $\varepsilon_{01} = 0.002$; and $\varepsilon_{085} = \frac{0.29 f'_c - 0.5}{145 f'_c - 1000}$.

Note that expressions for the plain concrete strains at peak (ε_{01}) and 85% of the peak in the descending branch (ε_{85}) are adopted from Ref.[20] and the developed model satisfies their values if $K_s = 1.0$ for the unconfined concrete.

2.4 Identification of Parameters

All the previous theoretical constitutive models[1-7] developed for the confined concrete have taken the individual characteristic points independently without considering the interacting effects between each other, by simply choosing the best fitting value for a certain criterion. The best fitting value of a main parameter related to a confined concrete, peak compressive strength for example, was obtained by performing a simple regression analysis only with test results on the peak compressive strength of the confined concrete, without referring to their overall effect on the whole stress-strain curve.

As far as one of the particular values from characteristic points such as f'_{cc}, ε_{cc} or ε_{85} is concerned, such a best fitting analysis can be justified. But when a reasonable model describing a complete stress-strain relationships is needed, the model dealing with a total minimization of the differences resulting from the pre-peak and the post-peak region of stress-strain curves is preferred.

System identification approach therefore, is taken in this research in order to determine the characteristic points of constitutive model of the confined concrete. The main parameters constituting the model (α, θ, η, ω, β, k_1 and k_2 in eq. (6) through eq. (8)) are chosen to minimize the sum of weighted average of squares of relative errors between model predictions and test results on confined concrete peak compressive strength (f'_{cc}) and corresponding strain (ε_{cc}), compressive strength at compressive strain equal to 0.015, and total area up to compressive strain equal to 0.015 under confined concrete compressive stress-strain curve. The strain of 0.015 is chosen as one of the characteristic points determining the post-peak behavior. Since it is observed from most of test results that columns could sustain large fraction of resistance until strains equal to or greater than this value.

The error function which measures the degree of differences between model predictions and test results for the particularly selected values of main parameters, is defined as follows(see Fig. 3):

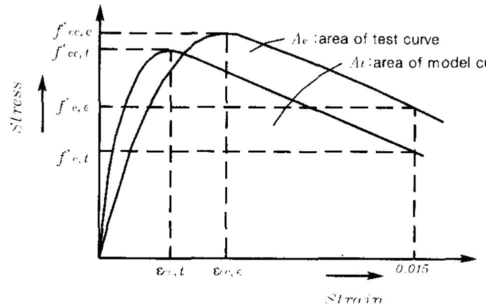


Fig. 3 Terms used for determination of total error

$$E_p = E_{pr} + E_{po} \tag{9}$$

where,

E_{pr} = Error resulting from pre-peak region

$$= E_{pr,f} + E_{pr,\varepsilon}$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \left\{ W_f \left[\frac{f'_{cc,e} - f'_{cc,t}}{f'_{cc,e}} \right]_i^2 + W_\varepsilon \left[\frac{\varepsilon_{cc,e} - \varepsilon_{cc,t}}{\varepsilon_{cc,e}} \right]_i^2 \right\} \tag{10a}$$

E_{po} = Error resulting from post-peak region

$$= E_{po,f} + E_{po,A}$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} \left\{ W_e \left[\frac{f'_{e,e} - f'_{e,t}}{f'_{e,e}} \right]_i^2 + W_A \left[\frac{A_e - A_t}{A_e} \right]_i^2 \right\} \tag{10b}$$

where, $E_{pr,f}$ = error function of peak stress; $E_{pr,\varepsilon}$ = error function of strain at peak stress; $E_{po,f}$ = error function of post-peak stress; $E_{po,A}$ = error function of total area under stress up to strain equal to 0.015; n = number of tests; W_f, W_ε, W_e, and W_A = weights corresponding to relative errors on peak stress strain at peak stress, stress at strain equal to 0.015, and area under stress-strain curve up to the strain equal to 0.015, respectively. Subscripts e and t represent experimental and theoretical results, respectively

Equal weights of 1.0 are assigned to all weights, W_f, W_ε, W_e, and W_A. Table 1 summarizes value of main parameters in the developed model (eq.(6) through eq.(8)) which best fit the developed model to the sixty one test results from system identification. Table 2 lists the specimen details adopted from different researchers.

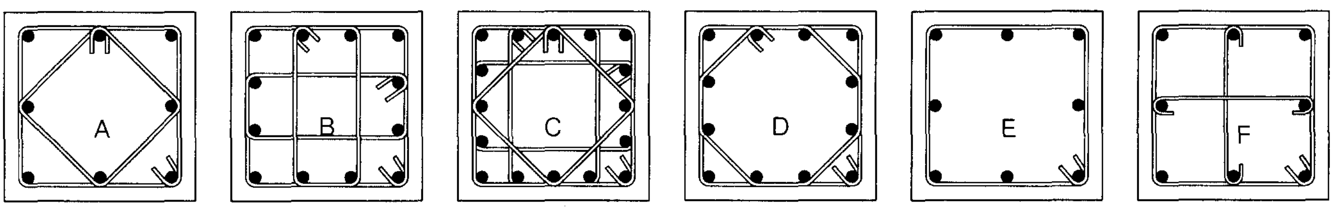


Fig. 4 Lateral steel type used in various tests

Table 1 Values of main parameters determined from comparisons with the sixty-one test results

Values of main parameters						
α	θ	η	ω	β	k_1	k_2
8.1	0.174	0.01	0.4	0.31	4.0	3950

Table 2 Test specimens used for identification of main parameters

Ref	No	Speci-men	Concrete cross section			Longitudinal reinforcement steel			Lateral steel				
			f'_c	B	H	f_y	n_s	d_b	f_{yh}	s	ρ_{sh}	d_t	Type
8	1	4C1-3	36.38	305	305	371.3	16	16	400	50.8	0.0076	3.2	C
	2	4C6-5	34.93	305	305	371.3	16	16	371	38	0.0227	4.8	C
	3	4C6H-6	34.3	305	305	371.3	16	16	262	38	0.0227	4.8	C
	4	4A4-8	40.8	305	305	384.5	8	22	420	28.7	0.0159	4.8	A
	5	4A6-10	40.65	305	305	384.5	8	22	455	35	0.0232	6.35	A
	6	4C3-11	40.65	305	305	406.5	16	16	358.3	95.25	0.0162	6.35	C
	7	4C4-12	40.79	305	305	406.5	16	16	468.5	25.4	0.0152	3.2	C
	8	2A5-14	31.5	305	305	403	8	16	427	76.2	0.0239	9.5	A
	9	2A6-15	31.7	305	305	403	8	16	379	35	0.0232	6.35	A
	10	2C1-16	32.5	305	305	413.4	16	13.4	589	50.8	0.0076	3.2	C
	11	2C5-17	32.8	305	305	413.4	16	12.8	348	102	0.0237	7.9	C
	12	2C6-18	33	305	305	413.4	16	12.8	551	38	0.0227	4.8	C
	13	4B3-19	33.4	305	305	391.3	12	19	400	102	0.018	7.9	B
	14	4B4-20	34.7	305	305	391.3	12	19	544	38	0.017	4.8	B
	15	4B6-21	35.48	305	305	391.3	12	19	489	47.75	0.024	6.35	B
	16	4D3-22	35.48	305	305	391.3	12	19	386	83	0.016	7.9	D
	17	4D4-23	35.48	305	305	391.3	12	19	530	29	0.017	4.8	D
	18	4D6-24	35.48	305	305	391.3	12	19	475	38	0.023	6.35	D
9	19	RC1	25.3	450	450	434	12	20	309	72	0.0182	10	D
	20	RC2	25.3	450	450	394	8	24	309	72	0.0174	10	A
	21	RC3	25.3	450	450	434	12	20	309	72	0.0115	8	D
	22	RC4	25.3	450	450	434	12	20	309	72	0.0258	12	D
	23	RC5	25.3	450	450	434	12	20	309	116.8	0.012	10	D
	24	RC6	25.3	450	450	434	12	20	309	144.7	0.01	10	D
	25	RC7	25.3	450	450	434	12	20	309	289.5	0.005	10	D
	26	RC8	25.3	450	450	434	12	20	309	386.0	0.0036	10	D
11	27	L8S-C	21.57	200	200	441	8	13	694	30	0.023	6	E
	28	L8C-C	21.57	200	200	441	8	13	694	45	0.023	6	F
	29	L8S-N	21.57	178	178	441	8	13	694	30	0.023	6	E
	30	L8C-N	21.57	178	178	441	8	13	694	45	0.023	6	F
	31	L0S-C	21.57	200	200	441	0	0	694	30	0.023	6	E
	32	L4S-C	21.57	200	200	441	4	13	694	30	0.023	6	E
	33	L8S-H	21.57	200	200	441	8	13	694	30	0.023	6	E
	34	H8S-C	47.46	200	200	441	8	13	694	30	0.023	6	E
	35	H8C-C	47.46	200	200	441	8	13	694	45	0.023	6	F
	36	H8S-N	47.46	178	178	441	8	13	694	30	0.023	6	E
	37	H8C-N	47.46	178	178	441	8	13	694	45	0.023	6	F
	38	H0S-C	47.46	200	200	441	0	0	694	30	0.023	6	E
	39	H4S-C	47.46	200	200	441	4	13	694	30	0.023	6	E
	40	H8S-H	47.46	200	200	441	8	13	694	30	0.023	6	E
10	41	A1	38	305	305	490	8	19.1	440	381	0.0207	3.15	A
	42	A2	38	305	305	490	8	19.1	440	304.8	0.0207	3.15	A

Table 2 Test specimens used for identification of main parameters (continued)

Ref	No	Specimen	Concrete cross section			Longitudinal reinforcement steel			Lateral steel				
			f'_c	B	H	f_y	n_s	d_b	f_{yh}	s	ρ_{sh}	d_t	Type
10	43	B1	38	305	305	490	8	19.1	440	381	0.0182	3.15	F
	44	C1	38	305	305	490	8	19.1	440	381	0.0182	3.15	F
	45	D2	38	305	305	490	8	19.1	440	381	0.0121	3.15	E
13	46	L-6L	19.6	200	200	441	8	13	559	40	0.0291	6	A
	47	L-6H	19.6	200	200	441	8	13	1275	40	0.0291	6	A
	48	L-8H	19.6	200	200	441	8	13	1275	40	0.0514	8	A
	49	M-6H	39.2	200	200	441	8	13	1275	40	0.0291	8	A
12	50	L8D-30	21.28	200	200	386.4	8	13	566	30	0.023	5	A
	51	L8C-30	21.28	200	200	386.4	8	13	566	30	0.023	5	F
	52	L8D-50	21.28	200	200	386.4	8	13	558	50	0.023	6	A
	53	L12R-70	21.28	200	200	400.5	12	10	580	70	0.0402	8	B
	54	M8D-30	34.9	200	200	386.4	8	13	566	30	0.023	5	A
	55	M8C-30	34.9	200	200	386.4	8	13	566	30	0.023	5	F
	56	M8D-50	34.9	200	200	386.4	8	13	558	50	0.023	6	A
	57	M12R-70	34.9	200	200	400.5	12	10	580	70	0.0402	8	B
	58	H8D-30	56.4	200	200	386.4	8	13	566	30	0.023	5	A
	59	H8C-30	56.4	200	200	386.4	8	13	566	30	0.023	5	F
	60	H8D-50	56.4	200	200	386.4	8	13	558	50	0.023	6	A
	61	H12R-70	56.4	200	200	400.5	12	10	580	70	0.0402	8	B

Table 3 Value of error functions and values of main parameters for different models

(a) Values of main parameters for different models

Authors	Model status	Value of main parameters							
		κ_1	κ_2	κ_3	κ_4	κ_5	κ_6	κ_7	κ_8
Soliman & Yu (1)	Original	1.4	0.45	0.0028	0.05	0.797	0.85	-	-
	Refined	1.6	0.01	0.0001	0.02	-	2.1	-	-
Sheikh & Uzumeri(2)	Original	140	5.5	80	0.75	0.15	248	-	-
	Refined	180	80	80	1.8	0.05	15	-	-
Modified Kent & Park(3)	Original	1.0	0.75	-	-	-	-	-	-
	Refined	0.92	0.29	-	-	-	-	-	-
Fafitis & Shah(4)	Original	1.15	21	0.1488	0.029	0.0019	24.7	-1.45	1.15
	Refined	1.0	6	0.2	0.007	0.0025	3	-0.03	1.3
Saatcioglu & Razvi(5)	Original	0.26	5	260	-	-	-	-	-
	Refined	0.31	2.5	157	-	-	-	-	-
Model	Original	$\alpha = 8.1$, $\theta = 0.174$, $\beta = 0.31$, $\omega = 0.4$, $\eta = 0.01$, $k_1 = 4$, $k_2 = 3950$							

Note : Definitions on κ_1 though κ_8 are given in the appendix.

(b) Values of error functions for different models for different models

Authors	Model status	Value of error function (E_p) and relative value								Total Error E_p	Relative value
		Pre-peak (E_{pr})				Post-peak (E_{po})					
		Value of $E_{pr,f}$	Rela-tive	Value of $E_{pr,\varepsilon}$	Rela-tive	Value of $E_{po,f}$	Rela-tive	Value of $E_{po,A}$	Rela-tive		
Soliman & Yu (1)	Original	0.107	8.23	0.33	2.75	0.45	4.5	0.14	4.67	1.03	3.99
	Refined	0.077	5.90	0.31	2.58	0.28	2.8	0.07	2.33	0.74	2.85
Sheikh & Uzumeri(2)	Original	0.025	1.92	0.21	1.75	0.19	1.9	0.10	3.33	0.52	2.03
	Refined	0.022	1.69	0.21	1.75	0.10	1.0	0.02	0.67	0.35	1.36
Modified Kent & Park(3)	Original	0.012	0.92	0.29	2.42	0.19	1.9	0.06	2.00	0.56	2.16
	Refined	0.011	0.85	0.30	2.5	0.13	1.3	0.04	1.33	0.48	1.86
Fafitis & Shah(4)	Original	0.035	2.69	0.66	5.5	0.42	4.2	0.13	4.30	1.25	4.84
	Refined	0.032	2.46	0.14	1.17	0.08	0.8	0.03	1.00	0.28	1.09
Saatcioglu & Razvi(5)	Original	0.031	2.38	0.27	2.25	0.16	1.6	0.04	1.33	0.50	1.95
	Refined	0.25	1.92	0.15	1.25	0.13	1.3	0.04	1.33	0.35	1.35
Model	Original	0.013	1.0	0.12	1.0	0.10	1.0	0.03	1.0	0.26	1.00

3. Comparisons of Constitutive Model Predictions with Test Results

In this section, the developed model is compared statistically with test results and with those predictions from the other models for concentric compression. A Total of six constitutive models for confined concrete under concentric compression are compared with the sixty one test results reported by different researchers [1-5, 8-13]. The accuracy of each model is examined through evaluating the error function (E_p), defined as eqs. (9) and (10). Two different approaches are made in order to examine the accuracy of each predictive model. First, the models as suggested are used and their predictions are entered in error functions. Second, the models from different researchers are refined by adjusting the values of main parameters so that these refined parameters can represent the best fit for the test results adopted in this study. The refined values were obtained by the use of error function given by eqs.(9) and (10)

Table 3 summarizes the values of error functions for each criterion and the sum of these errors for all criteria. The parameters ($\kappa_1 \sim \kappa_8$) refined in each model are given in the appendix. It can be shown that the developed model is superior in its accuracy to other models except for the peak compressive strength. The modified model by Kent and Park[3] has better prediction than the developed model but the difference between this model and the developed model seems to be marginal. Note that the smallest value for the total accumulated error is obtained for the developed model

Fig. 5 (a) through (d) shows the typical comparisons between model predictions and test results. It can be seen that the developed model can reasonably simulate the relatively reasonable stress-strain curves from test results.

4. Conclusion

In constitutive modeling of a confined concrete under concentric load, both concepts on the effectively confined area and the pressure distributions in horizontal and longitudinal directions with respect to the column cross sections are incorporated into the model equations. Only one of these concepts has been used in the previously suggested models. The main parameters in the model are identified using the error function which minimizes the total errors induced both in the pre-peak and the post-peak regions between the model predictions and test results. A Total of sixty-one test results are used. The developed model is shown to be superior to other models in predicting test results of the compressive stress-strain curves of the confined concrete under concentric load.

Acknowledgments

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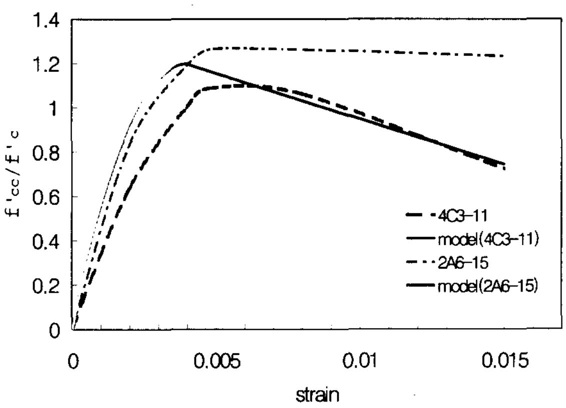
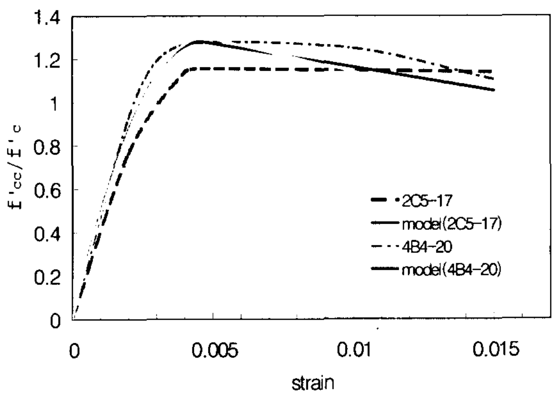
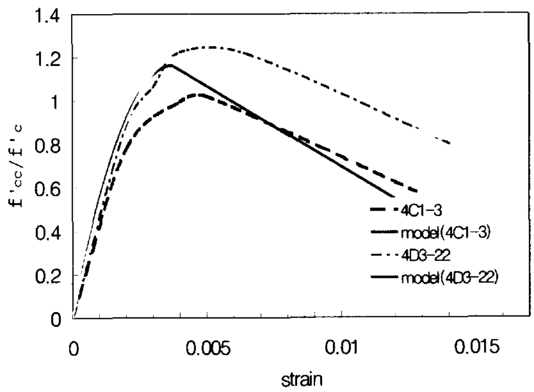


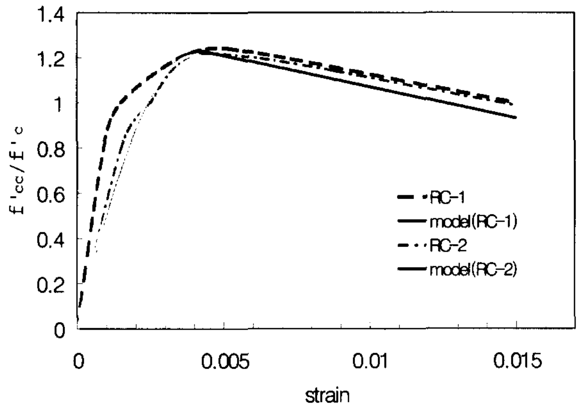
(a) 4C3-11 and 2A6-15 [8]



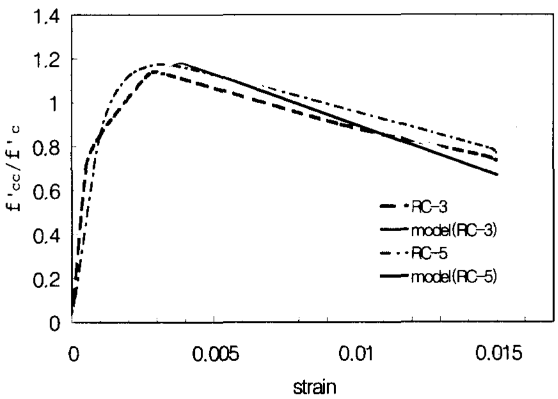
(b) 2C5-17 and 4B4-20 [8]



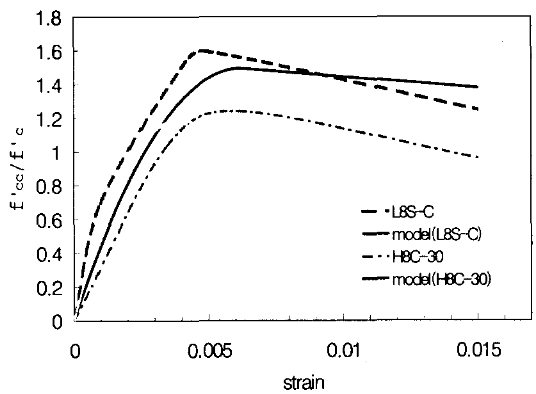
(C) 4C1-3 and 4D3-22 [8]



(d) RC-1 and RC-2 [9]



(e) RC-3 and RC-5 [9]



(f) L8S-C and H8C-30 [12]

Fig. 5 Comparisons between model predictions and test results on confined concrete under concentric load

References

1. Soliman, M.T.M., and Yu, C.W., "The Flexural Stress-Strain Relationship for Concrete Confined in Rectangula Transverse Reinforcement," Magazine of Concrete Research (London), Vol.19, No. 61, Dec., 1967, pp.223-238.
2. Shamin, A. S., and Uzumeri, S. M., M., "Analytical Model For Concrete Confined In Tied Columns," ASCE, Vol. 108, No. ST12, Decemver, 1982, pp.2703-2722.
3. Park, R., M., Nigel Priestley, M. J., and Gill, W. D., "Ductility of Square-Confined Concrete Columns", ASCE, Vol.108, No. ST4, April, 1982.
4. Fafitis, A., and Shah, S. P., "Predictions of Ultimate Behavior of Confined Columns Subjected to Large Deformations", ACI J., 82(4), 423-433.
5. Saaticioglu, M., Member, " Strength and Ductility of

Confined Concrete", ASCE, Vol. 118, No. 6, June, 1992.

6. Mander, J. B., Priestley, M.J.N., and Park, R., "The Theoretical Stress-Strain Model for Confined Concrete", ASCE, Journal of structural Engineering, Vol 114, No. 8, August, 1988.
7. Razvi S., and Saatcioglu, M., "Confinement Model for High-strength Concrete", ASCE, Journal of structural Engineering, Vol. 125, No. 3, March, 1999.
8. Sheikh, S. A., and Uzumeri, S. M., M. ASCE, "Strength And Ductility of Tied Concrete Column", ASCE, Vol. 106, No. ST5, May, 1980, pp.1079--1104.
9. Mohamed, A. H., Halim, A., Associate Member, ASCE and T. M., Abu-Lebdeh, "Analytical Study for Concrete Confined In Tied Columns", ASCE, Journal of Structural, Vol.115, No.11, November 1989.
10. Moehle, J. P., A. M. ASCE and Cavanagh, T., "Confinement Effectiveness of Cross ties in RC", ASCE, Journal of structural Engineering, Vol. 111, No. 10, October, 1985.
11. Joo, J. W., and Chung, H. S., Ph.D, "An Experimental Study on the Strength and the Ductility of Concentrically Loaded High-Strength Reinforced Concrete Columns", Dept. of Architectural Engineering The Graduate School Chungang University.
12. Lee, Y. I., and Chung, H. S., Ph.D, "An Experimental Study on the Strength and the Ductility of High-Strength Reinforced Concrete Column According to the Configuration of Ties", Dept. of Architectural Engineering, The Graduate School, Chung-Ang University.
13. Chung, B. H., and Chung, H. S., Ph.D, "A Study on Confinement Effectiveness of Lateral Ties in R.C. Columns under Axial Loads", Dept. of Architectural Engineering The Graduate School Chung-Ang University.
14. Mander, J. B., Priestley, M.J.N., and Park, R., "Observed Stress-Strain Behavior of Confined Concrete", ASCE, Journal of structural Engineering, Vol. 114, No. 8, August, 1988.
15. Cusson, D., and Paultre, P., "High-Strength Concrete Columns Confined by Rectangular Ties", ASCE, Journal of structural Engineering, Vol. 120, No. 3, March, 1994.
16. Dilger, W. H., Koch, R., and Kowalczyk, R., "Ductility of Plain and Confined Concrete under Different Strain Rates", ACI Journal, January-February, 1984.
17. Attard, M. M., and Setunge, S., "Stress-Strain Relationship of Confined and Unconfined Concrete", ACI Materials Journal, September-October, 1996.
18. Cusson, D., Francois de Larrard, Boulay, C., and Paultre, P., "Strain Localization in Confined High-Strength Concrete Columns", ASCE, Journal of structural Engineering, Vol. 122, No. 9, September, 1996.
19. Scott, B. D., Park, R., and Priestley, M.J.N., "Stress-Strain Behavior of Concrete Confined By Overlapping Hoops at Low and High Strain Rates", ACI Journal, January-February, 1982.
20. Kent, D. C., and Park, R., "Flexural Member with Confined Concrete", Journal of the Structural Division, Vol. 97, No.ST7, July, 1971.

Appendix

The existing constitutive models on confined concrete under compression are given below with best fitting parameters replaced by one of the κ_1 though κ_8. The parameters are found to minimize the error function, E_p, defined in eq.(9) and (10). Numerical values for these parameters are given in Table 3. The readers can refer to the corresponding articles for the meaning of each variables.

(A) Model by M. T. M. Soliman, and C. W. Yu[1]

$$f = \frac{f'_{cc}}{e_{ce}}(\varepsilon - e_{ce})^2 + f'_{cc} \quad \text{for } 0 \le \varepsilon < e_{ce} \tag{A·1a}$$

$$f = f'_{cc} \quad \text{for } e_{ce} \le \varepsilon < e_{cs} \tag{A·1b}$$

$$f = \frac{0.2f'_{cc}}{e_{cs} - e_{cf}} \cdot \varepsilon + f'_{cc} - \frac{0.2f'_{cc}}{e_{cs} - e_{cf}} \cdot e_{cs} \quad \text{for } e_{cs} \le \varepsilon \tag{A·1c}$$

where, $f'_{cc} = 0.9f'_c(1 + \kappa_4 \cdot q'')$;

$$q'' = (\kappa_1 \cdot \frac{A_c}{A_g} - \kappa_2)\frac{A_{sh}(s_o - s)}{A_{sh}s + \kappa_3 \cdot B_{SY} \cdot s^2} .$$

$$e_{ce} = \kappa_5 \cdot f'_{cc} \times 10^{-6} \tag{A·2a}$$

$$e_{cs} = 0.0025 \cdot (1 + q'') \tag{A·2b}$$

$$e_{cf} = 0.0045 \cdot (1 + \kappa_6 \cdot q'') \tag{A·2c}$$

(B) Model by S. A. Sheikh and S. M. Uzumeri, M.[2]

$$f = \frac{f'_{cc}}{\varepsilon_{s1}}(\varepsilon - \varepsilon_{s1})^2 + f'_{cc} \quad \text{for } 0 \le \varepsilon < \varepsilon_{s1} \tag{B·1a}$$

$$f = f'_{cc} \quad \text{for } \varepsilon_{s1} \le \varepsilon < \varepsilon_{s2} \tag{B·1b}$$

$$f = \frac{0.2f'_{cc}}{\varepsilon_{s2} - \varepsilon_{s85}} \cdot \varepsilon + f'_{cc} - \frac{0.2f'_{cc}}{\varepsilon_{s2} - \varepsilon_{s85}} \cdot \varepsilon_{s1} \quad \text{for } \varepsilon_{s1} \le \varepsilon < \varepsilon_{30} \tag{B·1c}$$

$$f = 0.3f'_{cc} \quad \text{for } \varepsilon_{30} \le \varepsilon \tag{B·1d}$$

where,

$f'_{cc} = K_s \cdot f_{cp}$; $f_{cp} = 0.85 \cdot f'_c$;

$K_s = 1.0 + \frac{b^2}{\kappa_1 \cdot P_{occ}}\left[\left(1-\frac{nC^2}{\kappa_2 \cdot bh}\right)\left(1-\frac{s}{2b}\right)\left(1-\frac{s}{2h}\right)\right]\sqrt{\rho_{sh} \cdot f_{yh}}$.

$$\varepsilon_{s1} = \kappa_3 \cdot K_s \cdot f'_c \times 10^{-6}, \quad f'_c \text{ in Mpa.} \qquad \text{(B·2a)}$$

$$\frac{\varepsilon_{s2}}{\varepsilon_{00}} = 1 + \frac{\kappa_6}{C}\left(1-5.0\left(\frac{s}{b}\right)^2\right)\frac{\rho_{sh} f_{yh}}{\sqrt{f'_c}} \qquad \text{(B·2b)}$$

$$\varepsilon_{s85} = \frac{\kappa_5}{Z} + \varepsilon_{S2} \qquad \text{(B·2c)}$$

where, $Z = \frac{0.5}{\kappa_4 \cdot \rho_{sh}\sqrt{\frac{b}{s}}}$.

(C) Model by R. Park, M. J. N. Priestley, and W. D. Gill[3]

$$f = K \cdot f'_c \left[\frac{2\varepsilon}{0.002K} - \left(\frac{\varepsilon}{0.002K}\right)^2\right]$$

for $0 \le \varepsilon < 0.002K$ (C·1a)

$$f = K \cdot f'_c \cdot \left[1 - Z_m(\varepsilon - 0.002K)\right]$$

for $0.002K \le \varepsilon$ (C·1b)

but not less than $0.2K \cdot f'_c$,

where, $K = 1 + \kappa_1 \cdot \frac{\rho_{sh} f_{yh}}{f'_c}$; and

$$Z_m = \frac{0.5}{\frac{3 + 0.29 f'_c}{145 f'_c - 1000} + \kappa_2 \cdot \rho_{sh}\sqrt{\frac{h'}{s}} - 0.002K} .$$

(D) Model by A. Fafitis and S. P. Shah[4]

$$f = f'_{cc}\left[1 - \left(1 - \frac{\varepsilon}{\varepsilon_{cc}}\right)^A\right]$$

for $0 \le \varepsilon < \varepsilon_{cc}$ (D·1a)

$$f = f'_{cc} \cdot \exp\left[-k(\varepsilon - \varepsilon_{cc})^{\kappa_8}\right]$$

for $\varepsilon_{cc} \le \varepsilon$ (D·1b)

where, $f'_{cc} = f'_c + \left(\kappa_1 + \frac{\kappa_2}{f'_c}\right) \cdot f_r$;

$\varepsilon_{cc} = \kappa_3 \times 10^{-4} f'_c + \kappa_4 \cdot \frac{f_r}{f'_c} + \kappa_5$;

$A = \frac{E_c \varepsilon_{cc}}{f'_{cc}}$; $k = \kappa_6 \cdot f'_c \exp[\kappa_7 \cdot f_r]$; and

$f_r = \frac{A_{sh} f_{yh}}{d_e s}$.

(E) Model by M. Saatcioglu and S. R. Razvi [5]

$$f = \frac{f'_{cc}}{\varepsilon_1}(\varepsilon - \varepsilon_1)^2 + f'_{cc}$$

for $0 \le \varepsilon < \varepsilon_1$ (E·1a)

$$f = \frac{0.2 f'_{cc}}{\varepsilon_1 - \varepsilon_{85}} \cdot \varepsilon + f'_{cc} - \frac{0.2 f'_{cc}}{\varepsilon_1 - \varepsilon_{85}} \cdot \varepsilon_1$$

for $\varepsilon_1 \le \varepsilon < \varepsilon_{20}$ (E·1b)

$$f = 0.2 f'_{cc}$$

for $\varepsilon_{20} \le \varepsilon$ (E·1c)

The characteristic points are expressed as follows :

$$f'_{cc} = f'_c + k_1 \cdot f_{le} \qquad \text{(E·2a)}$$

$$\varepsilon_1 = \varepsilon_{01} \cdot (1 + \kappa_2 \cdot K) \qquad \text{(E·2b)}$$

$$\varepsilon_{85} = \kappa_3 \cdot \rho\varepsilon_1 + \varepsilon_{085} \qquad \text{(E·2c)}$$

where,

$K = \frac{k_1 f_{le}}{f'_c}$; $k_1 = 6.7(f_{le})^{-0.17}$;

$f_{le} = \frac{f_{lex} \cdot b + f_{ley} \cdot h}{b + h}$;

$f_{lex} = k_{2x} \cdot f_{lx}$; $f_{ley} = k_{2y} \cdot f_{ly}$;

$k_{2x} = \kappa_1 \cdot \sqrt{\left(\frac{b}{s}\right)\left(\frac{h}{C_B}\right)\left(\frac{1}{f_{lx}}\right)} \le 1.0$;

$k_{2y} = \kappa_1 \cdot \sqrt{\left(\frac{b}{s}\right)\left(\frac{h}{C_H}\right)\left(\frac{1}{f_{ly}}\right)} \le 1.0$;

$f_{lx} = \frac{\sum A_{sB} \cdot f_{yh} \cdot \sin\alpha_i}{s \cdot b}$; $f_{ly} = \frac{\sum A_{sH} \cdot f_{yh} \cdot \sin\alpha_i}{s \cdot h}$.

$\sum A_s$ = the total area of transverse reinforcement in two directions, crossing b and h ; and

$\rho = \frac{\sum A_s}{s \cdot (b_{cx} + b_{cy})}$.