研究論文

NBU- t_0 Class 에 대한 검정법 연구[1)]

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A Study on Test for New Better than Used of an unknown specified age

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Abstract

A survival variable is a non-negative random variable X with distribution function $F(t)$ satisfying $F(0)=0$ and a survival function $\overline{F}(t)=1-F(t)$. This variable is said to be New Better than Used of specified age t_0 if $\overline{F}(x+t_0)\le\overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge0$ and a fixed t_0. We propose the test for H_0 : $\overline{F}(x+t_0)=\overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge0$ against H_1 : $\overline{F}(x+t_0)\le\overline{F}(x)\cdot\overline{F}(t_0)$ for all $x\ge0$ when the specified age t_0 is unknown but can be estimated from the data when $t_0=\xi_p$, the pth percentile of F. This test statistic, which is based on the normalized spacings between the ordered observations, is readily applied in the case of small sample. Also, our test is more simple than Ahmad's test (1998). Finally, the performance of our test is presented.

Key Words : order statistic, new better than used at t_0, survival function, percentiles, test statistic, performance of the test, normalized spacing.

1. Introduction

A life is represented by a non-negative random variable X with distribution function F satisfying $F(0)=0$ and a survival function $\overline{F}=1-F$. Then, F is 'new better than used' (NBU) if, for all

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x, $t \geq 0$, $\overline{F}(x+t) \leq \overline{F}(x) \cdot \overline{F}(t)$. Hollander, Park & Proschan (1986) introduced a larger class than NBU class. The distribution F is said to be new better than used of a specified age t_0 ($NBU-t_0$) if, for all $x \geq 0$, $\overline{F}(x+t_0) \leq \overline{F}(x) \cdot \overline{F}(t_0)$. While $\overline{F}(x) = \exp(-\lambda x)$ is the only distribution such that $\overline{F}(x+t) = \overline{F}(x) \cdot \overline{F}(t)$ for all $x, t \geq 0$ (Barlow & Proschan, 1981), Hollander et al. (1986) showed that the family A of distributions such that $\overline{F}(x+t_0) = \overline{F}(x) \cdot \overline{F}(t_0)$ for all $x \geq 0$ and a fixed $t_0 \geq 0$ includes precisely the following members:

(i) $\overline{F}_1(x) = \exp(-\lambda x)$ for all $x \geq 0$, $\lambda > 0$;

(ii) all life distributions $\overline{F}_2$ such that $\overline{F}_2(t_0) = 0$;

(iii) $\overline{F}_3(x) = \overline{G}(x)$ for $0 \leq x < t_0$ and $\overline{F}_3(x) = \overline{G}^j(t_0)\overline{G}(x - jt_0)$ for $jt_0 \leq x < (j+1)t_0$ $(j = 1, 2, \ldots)$, where G is a life distribution.

Hollander et al. (1986) proposed a test for $H_0: F \in$ A against $H_1: F$ is $NBU-t_0$, where t_0 is a known value. This test was extended into a class of test by Ebrahimi & Habbibullah (1990). Also Ahmad (1998) proposed a test for $H_0: F \in$ A against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value.

In practice, however, one might be interested in the new better than used behavior at an unknown but estimable point t_0. Such points would be, e.g. the p th percentile of F, denoted by ξ_p, or the mean life μ of F. The choice $t_0 = \xi_p$ may appear naturally in manufacturing where there is the concept of 'infant mortality', where items may improve over $(0, \xi_p)$ and then begin to decay, where p, the percentage of the item's life, may be known but ξ_p often not. Therefore we investigate the testing of $NBU-t_0$ alternatives when t_0 is not known but is estimable from the data.

When testing for $H_0: F \in$ A against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value, Ahmad(1998) obtained the test statistic, $\hat{T}_k$, for the $NBU-t_0$ class based on U-statistic and discussed its properties. In this thesis, we propose a test statistic for $H_0: F \in$ A against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value. Our test statistic, which is based on the normalized spacings between the ordered observations, is readily applied in the case of small sample. Also, our test statistic is simpler than the test statistic of Ahmad.

Table 2.1 Critical values of the H_n^1 in the case of p=0.05

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
20	-0.0495	-0.1934	-0.0446	0.0633	0.0937	0.1616
25	-0.0496	-0.1717	-0.0458	0.0412	0.0674	0.1232
30	-0.0497	-0.1618	-0.0464	0.0266	0.0486	0.0989
35	-0.0497	-0.1536	-0.0469	0.0155	0.0340	0.0752
40	-0.0459	-0.1353	-0.0360	0.0459	0.0663	0.1085
45	-0.0466	-0.1240	-0.0380	0.0348	0.0529	0.0886
50	-0.0469	-0.1209	-0.0391	0.0272	0.0426	0.0778
55	-0.0471	-0.1169	-0.0401	0.0207	0.0356	0.0690
60	-0.0430	-0.1097	-0.0313	0.0374	0.0530	0.0835

Table 2.2 Critical values of the H_n^1 in the case of p=0.10

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.0940	-0.0891	-0.0880	0.1208	0.1820	0.2884
15	-0.0993	-0.0964	-0.0925	0.0524	0.0952	0.1773
20	-0.0918	-0.0810	-0.0721	0.0887	0.1243	0.1977
25	-0.0930	-0.0845	-0.0777	0.0531	0.0835	0.1477
30	-0.0855	-0.0717	-0.0619	0.0735	0.1024	0.1653
35	-0.0874	-0.0759	-0.0671	0.0495	0.0751	0.1337
40	-0.0783	-0.0638	-0.0542	0.0632	0.0854	0.1327
45	-0.0806	-0.0682	-0.0597	0.0454	0.0667	0.1097
50	-0.0732	-0.0585	-0.0494	0.0569	0.0777	0.1222
55	-0.0757	-0.0635	-0.0547	0.0427	0.0615	0.0962
60	-0.0691	-0.0548	-0.0455	0.0515	0.0706	0.1061

Table 2.3 Critical values of the H_n^1 in the case of p=0.25

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.2329	-0.2087	-0.1889	0.1175	0.1740	0.2868
15	-0.2175	-0.1890	-0.1673	0.0848	0.1303	0.2268
20	-0.1745	-0.1387	-0.1143	0.1282	0.1669	0.2405
25	-0.1697	-0.1366	-0.1129	0.1025	0.1420	0.2059
30	-0.1630	-0.1334	-0.1119	0.0870	0.1188	0.1863
35	-0.1574	-0.1292	-0.1080	0.0709	0.1024	0.1659
40	-0.1366	-0.1048	-0.0852	0.0888	0.1163	0.1684
45	-0.1351	-0.1049	-0.0848	0.0797	0.1056	0.1540
50	-0.1319	-0.1026	-0.0846	0.0700	0.0952	0.1463
55	-0.1324	-0.1013	-0.0844	0.0607	0.0835	0.1298
60	-0.1133	-0.0859	-0.0694	0.0733	0.0953	0.1352

In, section 2, we propose the test statistic for $H_0: F \in A$ against $H_1: F$ is $NBU-t_0$, where t_0 is a unknown value and selected critical values are tabulated for sample sizes $n=10$ to $n=60$.

Finally in section 3, Monte Carlo simulations are conducted to evaluate the performance of the test for small sample size and to compare the powers of the our test and Ahmad's test and we give some conclusions and remarks for further researches.

2. Testing for New Better than Used at t_0

Ahmad proved the following simple characterization of the $NBU-t_0$ class plays a major role in our developments.

Lemma 1 We have $\overline{F}$ is $NBU-t_0$ if and only if, for all integers $k \geq 1$,

$$\overline{F}(x+kt_0) \leq \overline{F}(x) \cdot \overline{F}^k(t_0).$$

Let ξ_p denote the pth percentile of F, that is, $\overline{F}(\xi_p) = 1-p$ for $0 \leq p \leq 1$. Consider the measure of departure from H_0, defined by

$$T_k^1(F) = \frac{1}{\mu}\int_0^\infty \{\overline{F}(x)\,\overline{F}^k(t_0) - \overline{F}(x+kt_0)\}\,dx$$

$$= \frac{1}{\mu}\overline{F}^k(t_0)\int_0^\infty \overline{F}(x)\,dx - \frac{1}{\mu}\int_0^\infty \overline{F}(x+kt_0)\,dx$$

When $t_0 = \xi_p$, we obtain

$$T_k^1(F) = (1-p)^k - \overline{G}(k\xi_p),$$

where $\overline{G}(x) = \int_x^\infty \overline{F}(y)\,dy/\mu$ and $G(x)$ is called as the renewal or equilibrium distribution corresponding to $F(x)$. Under H_0, $T_k^1(F) = 0$ and under H_1, $T_k^1(F) > 0$, since $F(x)$ is continuous.

Let $X_1, X_2, \ldots, X_n$ denote random sample from F. In the usual, way we estimate ξ_p by $\hat{\xi}_p = X_{([np])}$, where $X_{(r)}$ denotes the rth order statistic in the sample and $[x]$ means the largest integer less than or equal to x. The empirical distribution function $F_n(X_{(i)}) = i/n$, $i = 0, 1, \ldots, n$ where $X_{(0)} = 0$. Also the empirical survival function is $\overline{F}_n(X_{(i)}) = (n-i)/n$, $i = 0, 1, \ldots, n$.

The normalized sample spacing $D_1, D_2, \cdots, D_n$ are defined by $D_j = (n-j+1) \cdot (X_{(j)} - X_{(j-1)})$, $j = 1, 2, \ldots, n$. Now,

$$\overline{G}_n(X_{(i)}) = \sum_{j=i+1}^n D_j \Big/ \sum_{j=1}^n D_j,\ i = 1, \ldots, n-1$$

and $\overline{G}_n(X_{(n)}) = 0$. Thus, we estimate $T_k^1(F)$ by

$$H_n^k = \hat{T}_k^1(F_n) = (1-p)^k - \overline{G}(kX_{([np])}).$$

If $X_{(i)} \leq kX_{([np])} < X_{(i+1)}$, we can rewrite $\overline{G}(kX_{([np])})$ as followings;

$\overline{G}(kX_{([np])}) =$

$\left\{\sum_{h=i+2}^{n} D_h + (n-i)(X_{(i+1)} - kX_{([np])})\right\} / \sum_{j=1}^{n} D_j$

Thus,

$H_n^k = (1-p)^k -$

$\left\{\sum_{h=i+2}^{n} D_h + (n-i)(X_{(i+1)} - kX_{([np])})\right\} / \sum_{j=1}^{n} D_j.$

We use H_n^k to test H_0: $F \in$ A against H_1: F is $NBU-\xi_p$ for $0 \le p \le 1$.

In this thesis, we will discuss the properties of H_n^k in the case of $k=1$. If $k=1$ and $[np]=i$, then $X_{([np])} = X_{(i)}$. Therefore, we can rewrite H_n^1 as followings;

$H_n^1 = (1-p) - \left\{\sum_{h=i+1}^{n} D_h\right\} / \sum_{j=1}^{n} D_j.$

To make the proposed test practical, Tables 2.1 through 2.4, based on Monte Carlo sampling, give lower and upper points of H_n^1 in the $\alpha = 0.01, 0.05$ and 0.10 regions for $n = 10(5)60$ in the case of $p = 0.05, 0.10, 0.25$ and 0.50. Each value is based on 10,000 replications.

3. Powers of the New Test Statistic

In practice we often have small sample.

In this chapter, we carry out to estimate

H_n^1 by comparing with Ahmad's test $\widehat{T}_k$ at the significance levels $\alpha = 0.05$ and $\alpha = 0.10$ in the case of $k=1$ for Weibull, gamma, makeham, linear failure rate alternatives given by

(a) Weibull distribution

$F_1(x) = 1 - \exp(-x^{(1+\theta)}), \quad x \ge 0, \ \theta \ge 0$

(b) Gamma distribution

$F_2(x) = \int_0^x (1/\Gamma(1+\theta)) e^{-t} t^{\theta} dt, \quad x \ge 0, \ \theta \ge 0$

(c) Makeham distribution

$F_3(x) = 1 - \exp(-[x + \theta\{x + \exp(-x) - 1\}]),$

$x \ge 0, \theta \ge 0$

(d) Linear failure rate distribution

$F_4(x) = 1 - \exp(-\{x + \theta x^2/2\}), \quad x \ge 0, \theta \ge 0$

The random numbers for the four alternatives are generated from the IMSL subroutines. This study is done for n = 20(5)60 in the case of $k=1$ at p=0.05, and for n = 10(5)60 at p=0.10, p=0.25 and p=0.50. In this simulation, 10,000 replications are performed for each value of design constants.

The following tables contain a comparative study of the sample powers for $\widehat{T}_k$ and H_n^k in the case of $k=1$ for $NBU-t_0$ alternatives.

Table 2.4 Critical values of the H_n^1 in the case of p=0.50

	Lower Tail			Upper Tail		
n	$\alpha=0.01$	$\alpha=0.05$	$\alpha=0.10$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.01$
10	-0.3243	-0.2474	-0.1972	0.1998	0.2475	0.3308
15	-0.3061	-0.2358	-0.1976	0.1313	0.1733	0.2523
20	-0.2507	-0.1791	-0.1411	0.1415	0.1801	0.2436
25	-0.2441	-0.1793	-0.1458	0.1068	0.1415	0.2022
30	-0.2086	-0.1480	-0.1176	0.1158	0.1467	0.2018
35	-0.2055	-0.1526	-0.1231	0.0911	0.1204	0.1766
40	-0.1790	-0.1307	-0.1014	0.0976	0.1253	0.1773
45	-0.1774	-0.1343	-0.1071	0.0810	0.1088	0.1596
50	-0.1623	-0.1166	-0.0912	0.0903	0.1144	0.1624
55	-0.1627	-0.1196	-0.0953	0.0764	0.1015	0.1455
60	-0.1469	-0.1063	-0.0837	0.0801	0.1054	0.1471

Table 3.1 Small sample powers in Weibull at $\theta = 0.5$

	p = 0.05				p = 0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
10					.4265	.2407	.2744	.1224	.4399	.2441	.2970	.1983	.3935	.2579	.2465	.1913
15					.4808	.2486	.3372	.1566	.5708	.3392	.4192	.2243	.5278	.3686	.3712	.2257
20	.5361	.2272	.4071	.1417	.6069	.2897	.4623	.1920	.6613	.3793	.5172	.2601	.6213	.3947	.4479	.3161
25	.5670	.2436	.4308	.1482	.6511	.2939	.5107	.2072	.7405	.3983	.5883	.2809	.7191	.4618	.5564	.3278
30	.5933	.2329	.4656	.1517	.7320	.3516	.6008	.2433	.7926	.4715	.6649	.3445	.7698	.5147	.6236	.3792
35	.6138	.2364	.4951	.1495	.7653	.3703	.6377	.2561	.8525	.5123	.7350	.3818	.8317	.5794	.7079	.4388
40	.7405	.3234	.6150	.2179	.8243	.4209	.7191	.3000	.8881	.5402	.7943	.4138	.8753	.6002	.7617	.4766
45	.7628	.3231	.6473	.2165	.8423	.4270	.7487	.3057	.9098	.5797	.8293	.4414	.9067	.6427	.8105	.5031
50	.7726	.3221	.6689	.2222	.8857	.4695	.7942	.3514	.9326	.6105	.8610	.4721	.9227	.6727	.8411	.5414
55	.7844	.3190	.6813	.2166	.8950	.4709	.8148	.3377	.9506	.6284	.9006	.4889	.9434	.7063	.8750	.5793
60	.8589	.3878	.7684	.2693	.9272	.5126	.8556	.3840	.9658	.6684	.9214	.5460	.9601	.7352	.8795	.6170

Table 3.2 Small sample powers in Weibull at $\theta = 1.0$

	p = 0.05				p = 0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
					.7114	.4152	.5547	.2642	.7544	.4566	.6226	.3975	.7058	.4959	.5303	.4092
					.7796	.4556	.6601	.3381	.8939	.6226	.7971	.4960	.8787	.6928	.7601	.5266
20	.8331	.4515	.7378	.3359	.9056	.5836	.8303	.4568	.9554	.7256	.9041	.6010	.9462	.7588	.8637	.6855
25	.8562	.4836	.7739	.3552	.9357	.6021	.8739	.5066	.9805	.7772	.9440	.6760	.9791	.8406	.9404	.7408
30	.8722	.4877	.8029	.3787	.9695	.7022	.9380	.5960	.9915	.8394	.9737	.7555	.9895	.8947	.9693	.8153
35	.8926	.4915	.8257	.3860	.9798	.7285	.9539	.6252	.9970	.8842	.9903	.8103	.9962	.9309	.9887	.8701
40	.9725	.6657	.9401	.5481	.9929	.8020	.9817	.7082	.9992	.9130	.9965	.8529	.9987	.9493	.9951	.9106
45	.9750	.6725	.9489	.5661	.9958	.8209	.9865	.7240	.9998	.9418	.9984	.8912	.9998	.9680	.9983	.9372
50	.9779	.6826	.9579	.5872	.9981	.8643	.9946	.7947	.9997	.9574	.9987	.9192	.9999	.9799	.9993	.9552
55	.9809	.6846	.9631	.5766	.9992	.8763	.9959	.7916	.9999	.9654	.9998	.9297	1.000	.9848	.9996	.9697
60	.9944	.7974	.9856	.7006	.9993	.9154	.9985	.8549	1.000	.9799	1.000	.9581	1.000	.9922	.9999	.9807

Table 3.3 Small sample powers in gamma at $\theta = 0.5$

	p = 0.05				p = 0.10				p = 0.25				p = 0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
10					.2943	.1910	.1649	.0882	.2796	.1760	.1729	.1362	.2239	.1631	.1232	.1140
15					.3374	.1905	.2049	.1109	.3473	.2295	.2171	.1373	.2846	.2322	.1705	.1157
20	.3809	.1726	.2554	.1002	.4095	.2028	.2673	.1192	.3878	.2355	.2516	.1436	.3245	.2242	.1923	.1616
25	.4157	.1881	.2764	.1046	.4480	.2038	.2996	.1354	.4514	.2386	.2797	.1512	.3724	.2550	.2374	.1558
30	.4374	.1732	.3036	.1017	.5045	.2258	.3465	.1308	.4964	.2753	.3454	.1793	.4065	.2821	.2666	.1728
35	.4704	.1834	.3328	.1031	.5386	.2526	.3873	.1500	.5581	.3128	.3849	.2011	.4628	.3282	.3082	.2094
40	.5499	.2224	.3935	.1327	.5935	.2719	.4490	.1744	.5844	.3134	.4347	.2020	.4984	.3189	.3401	.2161
45	.5818	.2314	.4343	.1429	.6258	.2814	.4690	.1772	.6191	.3436	.4559	.2216	.4463	.3452	.2883	.2315
50	.5952	.2282	.4550	.1477	.6589	.2998	.5040	.1976	.6523	.3606	.4961	.2416	.5435	.3633	.3868	.2366
55	.6108	.2254	.4635	.1365	.6898	.3060	.5445	.1860	.7012	.3736	.5532	.2437	.5884	.3772	.4232	.2557
60	.6769	.2685	.5316	.1684	.7301	.3311	.5781	.2162	.7177	.3971	.5763	.2729	.6207	.4050	.4406	.2800

Table 3.4 Small sample powers in gamma at $\theta = 1.0$

	p = 0.05				p = 0.10				p = 0.25				p = 0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
10					.4848	.2583	.3136	.1325	.4865	.2537	.3175	.2049	.3553	.2424	.2136	.1770
15					.5792	.2854	.4177	.1802	.6117	.3683	.4498	.2489	.4889	.3694	.3298	.2106
20	.6495	.2675	.5132	.1673	.7072	.3283	.5582	.2175	.6937	.3972	.5446	.2687	.5735	.3714	.3886	.2918
25	.6965	.2925	.5573	.1793	.7601	.3375	.6211	.2476	.7730	.4159	.6101	.2969	.6654	.4325	.4903	.3039
30	.7338	.2835	.6011	.1932	.8356	.4027	.7071	.2798	.8262	.4892	.7049	.3586	.7198	.4910	.5583	.3538
35	.7589	.2939	.6434	.1943	.8747	.4290	.7637	.3076	.8836	.5439	.7708	.4026	.7882	.5509	.6487	.4156
40	.8680	.4017	.7636	.2770	.9100	.4903	.8267	.3567	.9101	.5713	.8230	.4346	.8301	.5837	.7005	.4453
45	.8910	.4108	.8038	.2914	.9299	.5180	.8595	.3731	.9298	.6280	.8604	.4859	.8686	.6260	.7511	.4903
50	.9055	.4043	.8267	.2954	.9546	.5460	.8928	.4175	.9533	.6530	.8908	.5117	.8791	.6522	.7813	.5148
55	.9125	.4119	.8434	.2893	.9627	.5654	.9174	.4196	.9714	.6692	.9222	.5318	.9151	.6801	.8157	.5570
60	.9544	.5039	.9024	.3683	.9740	.6102	.9379	.4771	.9776	.7134	.9423	.5865	.9321	.7138	.8434	.5886

Table 3.5 Small sample powers in L.F.R. at $\theta = 0.5$

	p=0.05				p=0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
					.1710	.1385	.0932	.0552	.1837	.1191	.1112	.0890	.1848	.1269	.1028	.0858
					.1593	.1346	.0890	.0701	.2044	.1652	.1240	.0889	.2375	.1786	.1350	.0822
20	.1663	.1106	.0959	.0554	.1926	.1213	.1116	.0665	.2392	.1540	.1425	.0848	.2747	.1641	.1576	.1154
25	.1626	.1177	.0924	.0582	.1904	.1171	.1124	.0667	.2608	.1462	.1490	.0824	.3059	.1842	.1884	.1069
30	.1615	.1055	.0948	.0563	.2197	.1191	.1331	.0637	.2759	.1597	.1755	.0930	.3434	.1983	.2185	.1159
35	.1638	.1096	.0946	.0561	.2149	.1334	.1253	.0693	.3042	.1825	.1890	.1048	.3757	.2320	.2469	.1425
40	.1987	.1217	.1192	.0669	.2375	.1349	.1480	.0760	.3306	.1761	.2203	.1015	.4103	.2245	.2758	.1421
45	.1974	.1260	.1187	.0670	.2418	.1428	.1491	.0770	.3367	.1955	.2192	.1120	.4385	.2498	.2911	.1506
50	.2008	.1178	.1239	.0614	.2599	.1425	.1586	.0770	.3514	.1933	.2309	.1124	.4419	.2479	.3067	.1545
55	.1991	.1098	.1178	.0579	.2647	.1329	.1632	.0683	.3810	.1885	.2622	.1029	.4677	.2468	.3192	.1587
60	.2272	.1267	.1367	.0700	.2767	.1445	.1727	.0791	.3935	.2025	.2741	.1189	.5044	.2726	.3384	.1736

Table 3.6 Small sample powers in L.F.R. at $\theta = 1.0$

	p=0.05				p=0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
10					.2088	.1553	.1207	.0652	.2381	.1441	.1469	.1098	.2439	.1583	.1370	.1073
15					.1985	.1480	.1181	.0794	.2736	.1940	.1741	.1136	.3212	.2260	.1983	.1118
20	.2084	.1210	.1274	.0663	.2535	.1416	.1587	.0814	.3279	.1904	.2176	.1132	.3750	.2159	.2380	.1603
25	.2052	.1286	.1265	.0653	.2546	.1364	.1578	.0822	.3708	.1887	.2334	.1144	.4284	.2454	.2871	.1520
30	.2015	.1135	.1248	.0637	.2954	.1467	.1915	.0814	.3892	.2099	.2702	.1283	.4859	.2769	.3396	.1711
35	.2086	.1174	.1285	.0624	.2964	.1560	.1876	.0855	.4364	.2412	.2974	.1462	.5363	.3213	.3862	.2077
40	.2620	.1361	.1713	.0779	.3311	.1641	.2266	.0965	.4799	.2369	.3455	.1468	.5811	.3213	.4268	.2149
45	.2669	.1412	.1686	.0795	.3372	.1745	.2267	.0979	.4933	.2683	.3559	.1637	.6137	.3563	.4592	.2374
50	.2669	.1346	.1731	.0721	.3712	.1780	.2519	.1039	.5180	.2625	.3775	.1694	.6412	.3569	.4891	.2392
55	.2662	.1236	.1704	.0674	.3742	.1673	.2596	.0898	.5505	.2605	.4175	.1578	.6716	.3643	.5133	.2467
60	.3086	.1463	.2054	.0852	.3998	.1861	.2742	.1092	.5786	.2898	.4429	.1862	.7107	.3976	.5486	.2780

Table 3.7 Small sample powers in Makeham at $\theta = 0.5$

	p=0.05				p=0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
					.1470	.1305	.0759	.0514	.1521	.1074	.0849	.0815	.1410	.1050	.0748	.0695
					.1371	.1295	.0723	.0653	.1643	.1480	.0924	.0765	.1681	.1460	.0891	.0654
20	.1425	.1075	.0793	.0520	.1569	.1152	.0863	.0595	.1765	.1335	.0985	.0705	.1862	.1274	.0942	.0884
25	.1415	.1131	.0777	.0556	.1547	.1087	.0858	.0625	.1892	.1252	.1013	.0653	.2019	.1439	.1091	.0777
30	.1387	.1025	.0805	.0543	.1755	.1097	.0994	.0579	.1980	.1358	.1155	.0768	.2185	.1467	.1237	.0819
35	.1398	.1069	.0795	.0539	.1689	.1224	.0950	.0629	.2177	.1539	.1269	.0848	.2387	.1790	.1416	.1009
40	.1672	.1158	.0929	.0635	.1847	.1238	.1112	.0672	.2341	.1456	.1399	.0791	.2539	.1665	.1497	.0950
45	.1634	.1201	.0944	.0634	.1882	.1328	.1098	.0702	.2317	.1600	.1386	.0886	.2695	.1781	.1575	.1030
50	.1629	.1125	.0950	.0581	.1938	.1282	.1131	.0678	.2403	.1589	.1456	.0869	.2644	.1783	.1599	.0986
55	.1637	.1049	.0933	.0537	.2015	.1214	.1158	.0611	.2617	.1533	.1663	.0786	.2760	.1773	.1608	.1046
60	.1820	.1204	.1049	.0642	.2078	.1302	.1215	.0702	.2660	.1632	.1647	.0916	.2927	.1903	.1707	.1120

Table 3.8 Small sample powers in Makeham at $\theta = 1.0$

	p=0.05				p=0.10				p=0.25				p=0.50			
	$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$		$\alpha = 0.10$		$\alpha = 0.05$	
n	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$	H_n^1	$\widehat{T}_k$
10					.1793	.1439	.0981	.0576	.1907	.1249	.1149	.0937	.1800	.1267	.0973	.0849
15					.1700	.1384	.0962	.0726	.2124	.1716	.1303	.0951	.2269	.1790	.1279	.0823
20	.1788	.1139	.1029	.0581	.2064	.1279	.1217	.0721	.2458	.1611	.1482	.0890	.2555	.1642	.1425	.1139
25	.1751	.1212	.1027	.0608	.2064	.1237	.1229	.0716	.2705	.1558	.1543	.0891	.2881	.1843	.1711	.1060
30	.1748	.1095	.1042	.0596	.2378	.1280	.1446	.0712	.2882	.1697	.1817	.1001	.3184	.1966	.1968	.1142
35	.1806	.1130	.1057	.0588	.2365	.1413	.1398	.0745	.3177	.1960	.1968	.1128	.3503	.2317	.2245	.1424
40	.2178	.1283	.1333	.0711	.2596	.1444	.1665	.0823	.3460	.1889	.2303	.1099	.3809	.2236	.2481	.1413
45	.2190	.1323	.1324	.0719	.2680	.1543	.1656	.0850	.3517	.2114	.2318	.1222	.4073	.2507	.2628	.1493
50	.2216	.1239	.1388	.0655	.2870	.1541	.1773	.0876	.3688	.2101	.2420	.1237	.4089	.2460	.2748	.1541
55	.2187	.1159	.1320	.0629	.2911	.1449	.1865	.0760	.3979	.2043	.2757	.1143	.4336	.2461	.2844	.1562
60	.2517	.1355	.1555	.0749	.3075	.1596	.1953	.0914	.4116	.2226	.2898	.1323	.4630	.2718	.3015	.1738

Tables 3.1 and 3.2 show that our proposed test H_n^1 performs better than $\widehat{T}_k$ in all cases in Weibull distributions.

Tables 3.3 and 3.4 show that proposed test H_n^1 performs better than $\widehat{T}_k$ in all cases in gamma distributions.

Tables 3.5 and 3.6 show that proposed test H_n^1 performs better than $\widehat{T}_k$ in all cases in linear failure rate distributions.

Tables 3.7 and 3.8 show that proposed test H_n^1 performs better than $\widehat{T}_k$ in all cases in makeham distributions.

Therefore we recommend our proposed statistic H_n^1 as test statistic for test whether the survival distribution is new better than used of an unknown specified age for $p \leq 0.5$.

We will derive the limiting distribution and consistency of H_n^k which is based on the order statistics from the sample in further study.

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