

■ 연구논문

구매비용에 따라 차등적으로 신용기간이 허용되는 상황하의
최적재고모형

An Optimal Inventory Model when the Supplier Permits Day-terms Credit
Depending on the Amount of Purchase

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요 지

최근의 생산환경 변화에 따라 기업은 경쟁력 강화 및 매출 신장을 목적으로 고객의 구매비용에 대하여 일정기간 외상을 허용하고 있으며, 또한 경쟁기업간에 제품 수요 증대를 통한 경쟁우위를 위하여 고객의 구매비용에 따라 차별적으로 신용기간이 적용되는 경우를 흔히 볼 수 있다. 본 연구는 이와 같이 구매비용에 따라 차등적으로 신용기간이 허용되는 상황에 대한 재고모형을 분석하였고, 최적재고정책 결정을 위한 해법을 개발하였다.

1. Introduction

In today's business transactions, it is more and more common to see that the customers are permitted some grace period before they settle the account with the supplier. And generally, the customer can earn interest on the sales of the inventory depending on the length of that payment period. Trade credit would play an important role in the conduct of business for many reasons. For a supplier who offers trade credit, it is an effective means of price discrimination, which circumvents antitrust measures and is also an efficient method to stimulate the demand of the product. For a customer, it is an efficient method of bonding a supplier when the customer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stock. As implicitly stated by Mehta [8], a major reason for the supplier to offer a credit period to the customers is to stimulate the demand for the product that he produces. Also, Fewings [4] stated that the advantage of trade credit for the supplier is substantial in terms of influence on the customer's purchasing and marketing decisions. The supplier usually expects that the increased sales volume can compensate the capital losses incurred during the credit period. In this regard, a number of research papers appeared which deal with the inventory model under a fixed credit period. Chapman *et al.*

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[1], Chung [3], Goyal [5], and Jaggi and Aggarwal [6] analyzed the effects of trade credit on the optimal inventory policy. Recently, Chu *et al.* [2] also examined the inventory model for deteriorating items under the condition of permissible delay in payments. Also, Kim *et al.* [7] introduced the model determining an optimal credit policy to increase supplier's profits with price dependent demand functions. Note that a common assumption of the above research works is the availability of a certain length of credit period that is offered by the supplier.

Generally, a number of factors determine the length of the credit period in a given line of business or for a given creditor. Considering that the trade credit may be used as an effective means of price discrimination in customer's order to increase the sales of the product, the quantity involved in transaction is considered as one of the important factors determining the length of credit period. In this regard, a number of suppliers would consider the length of the credit period as supplier's dominant strategy against the competitive suppliers in expectation of increasing the sales volume. In Korea, some department stores, pharmaceutical companies and agricultural machinery manufacturers associate the length of the credit period with the customer's total amount of purchase, i.e., they offer a longer credit period for larger amount of purchase. This kind of commercial practice is based on the principle of economies of scale from the supplier's point of view and tends to make customer's order size large enough to qualify a certain credit period break.

This paper deals with the problem of determining the customer's order size with permissible delay in payments where the length of delay is a function of the amount purchased by the customer. In the next section, we formulate a relevant mathematical model. For the model developed, the properties of an optimal solution are discussed and its solution algorithm is presented in section 3. A numerical example is provided in section 4, which is followed by concluding remarks.

2. Mathematical formulation

In deriving the mathematical model, the following notations and assumptions are used.

Notations:

- D : annual demand
- Q : order size per each order
- T : replenishment cycle time
- C : unit purchase cost
- S : cost of placing one order
- H : unit inventory holding cost per item per year excluding the capital opportunity cost.
- R : capital opportunity cost(as a percentage) per investment in inventory per year.
- I : interest rate(as a percentage) which can be earned in a year.
- tc_j : permissible delay in settling accounts for the amount purchased CQ ,
 $v_{j-1} \leq CQ (= CDT) < v_j$, where $tc_{j-1} < tc_j$, $j = 1, 2, \dots, m$ and $v_0 < v_1 < \dots < v_m$, $v_0 = 0$, $v_m = \infty$.

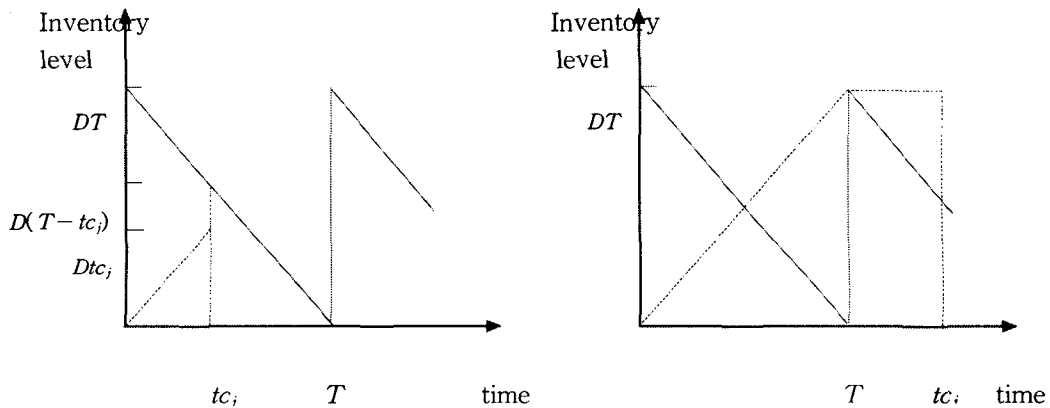
Assumptions:

- 1) Replenishments are instantaneous with a known and constant lead-time.
- 2) No shortages are allowed.
- 3) The demand for the item is constant with time.
- 4) The supplier allows a delay in payments for the items supplied where the length of delay is a function of the customer's total amount of purchase.
- 5) The purchasing cost of the products sold during the credit period is deposited in an interest bearing account with rate I . At the end of the period, the credit is settled and the customer starts paying the capital opportunity cost for the items in stock with rate R ($R \geq I$).

And the annual total cost consists of the following four elements.

- 1) Annual ordering cost = $\frac{S}{T}$.
- 2) Annual purchasing cost = $\frac{CQ}{T} = CD$.
- 3) Annual inventory carrying cost = $\frac{HDT}{2}$.
- 4) Annual capital opportunity cost for $v_{j-1} \leq CDT < v_j$:

(i) Case 1 ($tc_j \leq T$): (see figure 1 (a)) As products are sold, the purchasing cost of the products is used to earn interest with annual rate I during the credit period tc_j . And the average number of products in stock earning interest during time $(0, tc_j)$ is $\frac{Dtc_j}{2}$ and the interest earned per order becomes $CI\left(\frac{Dtc_j}{2}\right)tc_j$. When the credit is settled, the products still in stock have to be financed with annual rate R . Since the average number of products during time (tc_j, T) becomes $\frac{D}{2}(T - tc_j)$, the interest payable per order can be expressed as $CR\frac{D}{2}(T - tc_j)(T - tc_j)$. Therefore,



condition ($R \geq D$) as stated by Goyal[5], $TC_{i,j}(T)$ is a convex function for every i and j . Thus, there exists a unique value $T_{i,j}$, which minimizes $TC_{i,j}(T)$ as follows:

$$T_{1,j} = \sqrt{\frac{2S + C(R-D)Dtc_j^2}{D(H+CR)}} \quad (3)$$

$$T_{2,j} = \sqrt{\frac{2S}{D(H+CI)}} \quad (4)$$

Note that the value of $T_{1,j}$ is strictly increasing as j increases and the value of $T_{2,j}$ is identical for every j .

Also, from equation (3), $T_{1,j} \geq tc_j$ can be rewritten as

$$\sqrt{\frac{2S + C(R-D)Dtc_j^2}{D(H+CR)}} \geq tc_j \quad (5)$$

Squaring both sides of equation (5) and rearranging, we have

$$\sqrt{\frac{2S}{D(H+CI)}} \geq tc_j \quad (6)$$

Therefore, for any j , if $T_{1,j} \geq tc_j$, then $T_{2,j} \geq tc_j$, which implies that $TC_{2,j}(T)$ is decreasing in T for $T < tc_j$. And if $T_{2,j} < tc_j$, then $T_{1,j} < tc_j$, which implies that $TC_{1,j}(T)$ is increasing in T for $T \geq tc_j$.

Now, from the above results, we can make the following useful properties about the characteristics of the customer's annual total cost function for T , $T \in I_j = \{T \mid v_{j-1}/DC \leq T < v_j/DC\}$, $j = 1, 2, \dots, m$. (The proofs is given in Appendix.) These properties simplifies our search process such that only a finite number of candidate values of T needs to be considered to find an optimal value T^* . Let k be the smallest index such that $T_{2,j} < tc_j$.

Property 1.

For $T \in I_j$, $j \geq k$, we can consider the following three cases for $T_{2,j}$: $T_{2,j} < \frac{v_{j-1}}{CD}$,

$$\frac{v_{j-1}}{CD} \leq T_{2,j} < \frac{v_j}{CD}, \text{ and } \frac{v_j}{CD} \leq T_{2,j}$$

(i) If $T_{2,j} < \frac{v_{j-1}}{CD}$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in I_j$.

(ii) If $\frac{v_{j-1}}{CD} \leq T_{2,j} < \frac{v_j}{CD}$, then $T = T_{2,j}$ yields the minimum total cost for $T \in I_j$.

(iii) If $\frac{v_j}{CD} \leq T_{2,j}$, then we do not need to consider T for $T \in I_j$ to find T^* .

Property 2.

For $T \in I_j$, $j < k$, we can consider the following four cases for $T_{1,j}$; $T_{1,j} < \frac{v_{j-1}}{CD}$, $\frac{v_{j-1}}{CD} \leq T_{1,j} < \frac{v_j}{CD}$, $tc_j < \frac{v_j}{CD} \leq T_{1,j}$ and $\frac{v_j}{CD} \leq tc_j \leq T_{1,j}$.

- (i) If $T_{1,j} < \frac{v_{j-1}}{CD}$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in I_j$.
- (ii) If $\frac{v_{j-1}}{CD} \leq T_{1,j} < \frac{v_j}{CD}$, then $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$.
- (iii) If $tc_j < \frac{v_j}{CD} \leq T_{1,j}$, then $T = \frac{v_j^-}{CD}$, where $v_j^- = v_j - \epsilon$ and ϵ is a very small positive number, yields the minimum total cost for $T \in I_j$.
- (iv) If $\frac{v_j}{CD} \leq tc_j \leq T_{1,j}$, then we do not need to consider T for $T \in I_j$ to find T^* .

Property 3.

- (i) If $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$, then $T^* \geq T_{1,j}$.
- (ii) If $T = \frac{v_j^-}{CD}$ yields the minimum total cost for $T \in I_j$, then $T^* \geq \frac{v_j^-}{CD}$.

Based on the above properties, we develop the following solution procedure to determine an optimal replenishment cycle time T^* .

Solution algorithm

- Step 1: Compute $T_{2,j} = \sqrt{\frac{2S}{D(H+CD)}}$ and let k be the smallest index such that $T_{2,j} < tc_j$.
 If $T_{2,j} \geq tc_j$ for all $1 \leq j \leq m$, then set $k = m + 1$ and go to Step 3.
 Otherwise, find the index l satisfying $v_{l-1} \leq T_{2,l}CD < v_l$ and go to Step 2.
- Step 2: 2.1 If $k > l$, then go to Step 2.3. Otherwise, go to Step 2.2.
 2.2 Compute the annual total cost for $T = T_{2,l}, \frac{v_l}{CD}, \frac{v_{l+1}}{CD}, \dots, \frac{v_{m-2}}{CD}$ and $\frac{v_{m-1}}{CD}$, and go to Step 3.
 2.3 Compute the annual total cost for $T = \frac{v_{j-1}}{CD}$, $j = k, k+1, \dots, m$ and go to Step 3.
- Step 3: 3.1 Set $j = k - 1$.
 3.2 If $tc_j \geq \frac{v_j}{CD}$, then go to Step 3.4. Otherwise, go to Step 3.3.
 3.3 In case
 i) $T_{1,j} < \frac{v_{j-1}}{CD}$, compute the annual total cost for $T = \frac{v_{j-1}}{CD}$ and go to Step 3.4.
 ii) $T_{1,j} < \frac{v_j}{CD}$, compute the annual total cost for $T = T_{1,j}$ and go to Step 4.
 iii) $T_{1,j} \geq \frac{v_j}{CD}$, compute the annual total cost for $T = \frac{v_j^-}{CD}$, $v_j^- = v_j - \epsilon$ where ϵ

is a very small positive number and go to Step 4.

3.4 Reset $j = j - 1$ and go to Step 3.2.

Step 4: Select the one that yields the minimum total cost as T^* and stop.

4. Numerical example

To illustrate the solution algorithm, the following problem is considered.

Total amount of purchase	Credit period
$0 \leq CQ < \$1500$	$tc_1 = 0.1$
$\$1500 \leq CQ < \3000	$tc_2 = 0.2$
$\$3000 \leq CQ$	$tc_3 = 0.3$

(1) $D = 2500$, $S = \$ 70$, $C = \$ 5$, $H = \$ 1$, $R = 0.10(= 10\%)$, $I = 0.06(= 6\%)$.

(2) Supplier's credit policy for the customer's amount purchased:

The optimal solution can be obtained through the following steps.

Step 1. Since $T_{2,j}(=0.2075) < tc_3(=0.3)$, $k = 3$.

Since $T_{2,j}CD = 2594 \in [v_1, v_2]$, $l = 2$ and go to Step 2.

Step 2.

2.1. Since $k(=3) > l(=2)$, go to Step 2.3.

2.3. Since $\frac{v_2}{CD}(=0.24) < tc_3(=0.3)$, compute $TC_{2,3}(v_2/CD)$ with equation (2) for $\frac{v_2}{CD}$ and go to Step 3.

Step 3.

3.1. Set $j = 3 - 1 = 2$.

3.2. Since $tc_2 = 0.2 < \frac{v_2}{CD}(=0.24)$, go to Step 3.3.

3.3. Since $T_{1,2} = 0.2066 \in [v_2/CD, v_2/CD]$, compute $TC_{1,2}(T_{1,2})$ with equation (1) and go to Step 4.

Step 4. Since $TC_{2,3}(0.24) = 12957 = \min.\{TC_{2,3}(0.24), TC_{1,2}(0.2066)\}$, an optimal replenishment cycle time becomes 0.24 with its minimum total cost \$12957.

5. Concluding remarks

This paper dealt with the problem of determining the optimal order quantity for customer when the supplier permits delay in payments. Assuming that the length of delay is a function of the customer's total amount of purchase, the relevant mathematical model was developed. The availability of the credit period according to the total purchased volume can be justified by the principle of economies of scale. The credit policy tends to

make the customer's order size larger by inducing him to qualify for a longer credit period in his payments. After formulating the customer's inventory model, we proposed the solution procedure, which leads to an optimal ordering policy. With an example problem, the validity of the algorithm was illustrated. The model developed in this paper may help customers find a theoretical optimum ordering policy when the order-size-dependent delay in payments is available.

Appendix

Proof of Property 1.

(i) In this case, two possible cases occur; $T_{2,j} < tc_j < \frac{v_{j-1}}{CD}$ and $T_{2,j} < \frac{v_{j-1}}{CD} \leq tc_j$. For the first case, $T_{2,j} < tc_j$ implies that $T_{1,j} < tc_j$. So, $TC_{1,j}(T)$ is increasing in T for $T \in I_j$. Thus,

$$TC_{1,j}(v_{j-1}/CD) \leq TC_{1,j}(T) \text{ for } T \in I_j. \quad (A1)$$

Hence, if $T_{2,j} < tc_j < \frac{v_{j-1}}{CD}$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in I_j$. For the second case, since $T_{2,j} < \frac{v_{j-1}}{CD}$, $TC_{2,j}(T)$ is increasing in T for $T \geq \frac{v_{j-1}}{CD}$. Also, $TC_{1,j}(T)$ is increasing in T for $T \geq tc_j$. Since the total cost function is continuous at $T = tc_j$, we have

$$TC_{2,j}(v_{j-1}/CD) \leq TC_{2,j}(T) \text{ for } \frac{v_{j-1}}{CD} \leq T < tc_j, \quad (A2)$$

$$< TC_{2,j}(tc_j) \quad (A3)$$

$$\leq TC_{1,j}(T) \text{ for } tc_j \leq T. \quad (A4)$$

Hence, if $T_{2,j} < \frac{v_{j-1}}{CD} \leq tc_j$, then $T = \frac{v_{j-1}}{CD}$ yields the minimum total cost for $T \in I_j$.

Therefore, if $T_{2,j} < \frac{v_{j-1}}{CD}$, then the total cost becomes the minimum at $T = \frac{v_{j-1}}{CD}$ for $T \in I_j$.

(ii) By definition of $T_{2,j}$

$$TC_{2,j}(T_{2,j}) \leq TC_{2,j}(T) \text{ for any } T. \quad (A5)$$

Since $T_{2,j} < tc_j$ and the total cost function is continuous at $T = tc_j$,

$$TC_{2,j}(T_{2,j}) \leq TC_{2,j}(T) \text{ for } \frac{v_{j-1}}{CD} \leq T < tc_j, \quad (A6)$$

$$< TC_{2,j}(tc_j) \quad (A7)$$

$$\leq TC_{1,j}(T) \text{ for } tc_j \leq T. \quad (A8)$$

Hence, the total annual cost becomes the minimum at $T = T_{2,j}$ for $T \in I_j$.

(iii) Since $T_{2,j} \geq v_j/CD$, $TC_{2,j}(T)$ is decreasing in T for $T \in I_j$ and, thus,

$$TC_{2,j}(v_j/CD) \leq TC_{2,j}(T) \text{ for } T \in I_j. \quad (A9)$$

Also,

$$TC_{2,j}(T) > TC_{2,j}(v_j/CD) \text{ for } T \in I_j. \quad (A10)$$

$$> TC_{2,j+1}(v_j/CD). \tag{A11}$$

Hence, we do not need to consider T to find T^* for $T \in I_j$. Q.E.D.

Proof of Property 2.

(i) Since $T_{1,j} < v_{j-1}/CD$,

$$TC_{1,j}(v_{j-1}/CD) \leq TC_{1,j}(T) \text{ for } T \in I_j. \tag{A12}$$

So, the total cost becomes the minimum at $T = \frac{v_{j-1}}{CD}$ for $T \in I_j$.

(ii) In this case, two possible cases occur; $tc_j < \frac{v_{j-1}}{CD} \leq T_{1,j}$ and $\frac{v_{j-1}}{CD} \leq tc_j \leq T_{1,j}$. For the first case, by definition of $T_{1,j}$,

$$TC_{1,j}(T_{1,j}) \leq TC_{1,j}(T) \text{ for } T \in I_j. \tag{A13}$$

So, if $tc_j < \frac{v_{j-1}}{CD} \leq T_{1,j}$, then $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$. For the second case, by definition of $T_{1,j}$,

$$TC_{1,j}(T_{1,j}) \leq TC_{1,j}(T) \text{ for any } T. \tag{A14}$$

Also, $T_{2,j} \geq tc_j$ and $TC_{2,j}(T)$ is decreasing in T for $T < tc_j$. Since the total cost function is continuous at $T = tc_j$, we have

$$TC_{1,j}(T_{1,j}) \leq TC_{1,j}(T) \text{ for } tc_j \leq T < \frac{v_j}{CD}. \tag{A15}$$

$$< TC_{2,j}(T) \text{ for } \frac{v_{j-1}}{CD} \leq T < tc_j. \tag{A16}$$

So, if $\frac{v_{j-1}}{CD} \leq tc_j \leq T_{1,j}$, $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$.

(iii) Since $T_{1,j} \geq v_j/CD$,

$$TC_{1,j}(v_j^-/CD) \leq TC_{1,j}(T) \text{ for } tc_j \leq T < \frac{v_j}{CD}. \tag{A17}$$

Also, $TC_{2,j}(T)$ is decreasing in T for $T \leq tc_j$. Since the total cost function is continuous at $T = tc_j$, we have

$$TC_{1,j}(v_j^-/CD) \leq TC_{1,j}(T) \text{ for } tc_j \leq T < \frac{v_j}{CD}. \tag{A18}$$

$$< TC_{2,j}(T) \text{ for } T < tc_j. \tag{A19}$$

So, the total cost becomes the minimum at $T = \frac{v_j^-}{CD}$ for $T \in I_j$.

(iv) $\frac{v_{j-1}}{CD} \leq tc_j \leq T_{1,j}$ implies that $\frac{v_j}{CD} \leq T_{2,j}$ and $\frac{v_j}{CD} \leq tc_j$. So, by the result of Property

1 (iii), we do not need to consider T to find T^* for $T \in I_j$. Q.E.D.

Proof of Property 3.

(i) From Property 2 (ii), if $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$, then

$$\frac{v_{j-1}}{CD} \leq T_{1,j} < \frac{v_j}{CD} \text{ and } tc_j \leq T_{1,j}. \tag{A20}$$

By definition of $T_{1,j}$, we have

$$TC_{1,j}(T_{1,j}) \leq TC_{1,j}(T) < TC_{1,l}(T), \quad l < j \text{ for any } T. \tag{A21}$$

Also, by definition of the index k , $TC_{2,n}(T)$ is decreasing in T for $T < tc_n, n \leq j$. Hence, if $T = T_{1,j}$ yields the minimum total cost for $T \in I_j$, then $T^* \geq T_{1,j}$.

(ii) From Property 2 (iii), if $T = \frac{v_j^-}{CD}$ yields the minimum total cost for $T \in I_j$, then

$$tc_j < \frac{v_j^-}{CD} \leq T_{1,j}. \tag{A22}$$

By definition of $T_{1,j}$, we have

$$TC_{1,j}(v_j^- / CD) \leq TC_{1,j}(T) \text{ for } T < \frac{v_j^-}{CD}, \tag{A23}$$

$$TC_{1,j}(T) < TC_{1,l}(T), \quad l < j \text{ for any } T. \tag{A24}$$

Also, by definition of the index k , $TC_{2,n}(T)$ is decreasing in T for $T < tc_n, n \leq j$. Hence, if $T = \frac{v_j^-}{CD}$ yields the minimum total cost for $T \in I_j$, then $T^* \geq \frac{v_j^-}{CD}$. Q.E.D.

References

1. Chapman, C.B., Ward, S.C., Cooper, D.F. and Page, M.J.; "Credit policy and inventory control," *Journal of Operational Research Society*, 35(12): 1055-1065, 1985.
2. Chu, P., Chung, K.J. and Lan, S.P.; "Economic order quantity of deteriorating items under permissible delay in payments," *Computers and Operations Research*, 25(10): 817-824, 1998.
3. Chung, K.J.; "A theorem on the determination of economic order quantity under conditions of permissible delay in payments," *Computers and Operations Research*, 25(1): 49-52, 1998.
4. Fewings, D.R.; "A credit limit decision model for inventory floor planning and other extended trade credit arrangement," *Decision Science*, 23(1): 200-220, 1992.
5. Goyal, S.K.; "Economic order quantity under conditions of permissible delay in payments," *Journal of Operational Research Society*, 36(4): 335-338, 1985.
6. Jaggi, C.K. and Aggarwal, S.P.; "Credit financing in economic ordering policies of deteriorating items," *International Journal of Production Economics*, 34: 151-155, 1994.
7. Kim, J.S., Hwang, H. and Shinn, S.W.; "An optimal credit policy to increase supplier's profits with price dependent demand functions," *Production Planning and Control*, 6(1): 45-50, 1995.
8. Mehta, D. "The formulation of credit policy models," *Management Science*, 15(2): B30-B50. 1968.