

## Effective Beam Width Coefficients for Lateral Stiffness in Flat-Plate Structures

Jung-Wook Choi,<sup>1)</sup> Chul-Soo Kim,<sup>1)</sup> Jin-Gyu Song,<sup>1)</sup> and Soo-Gon Lee<sup>1)</sup>

<sup>1)</sup> Department of Architectural Engineering, Chonnam National University, Korea

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### Abstract

Flat-plate buildings are commonly modeled as two-dimensional frames to calculate unbalanced moments, lateral drift, and shear at slab-column connections. The slab-column frames under lateral loads are analyzed using effective beam width models, which is convenient for computer analysis. In this case, the accuracy of this approach depends on the exact values of effective beam width to account for the actual behavior of slab-column connections. In this parametric study, effective beam width coefficients for wide range of the variations are calculated on the several types of slab-column connections, and the results are compared with those of other researches. Also the formulas for effective beam width coefficients are proposed and verified by finite element analysis. The proposed formulas are founded to be more suitable than others for analyzing flat-plate buildings subjected to lateral loading.

**Keywords :** *Flat-Plates, Effective Beam Width Model, Effective Beam Width Coefficients, Slab-Column Connections*

### 1. Introduction

Flat-plate buildings are commonly modeled as two-dimensional plane frames for structural analysis. The ACI Building Code(ACI 318-99)<sup>(1)</sup> recommends bending finite element models, equivalent frame models, and effective beam width models for analyzing flat-plate structures under lateral loads. The finite element models are powerful to study elastic behavior of flat plates. However, analysis of a complete building by this models involves relatively high cost and intense labor. The equivalent frame models and the effective beam width models can provide good estimates of lateral stiffness for flat-plate structures. In these models, the effective beam width models are simpler than another models to be modeled in structural analysis programs. Therefore the application of the effective beam width models can be appropriate for flat-plates subjected to lateral loading in design practice.

The concept of the effective beam width models is a beam approach, where columns are modeled conventionally and slabs are modeled as the beams with effective width. The effective width can be defined as reduction factor×effective beam width coefficient×total slab width. To determine the accurate coefficients and reduction factor by crack, which is not referred, is very important for this models.

In this study, the effective beam width coefficients for wide range of the variations are studied on the several types of slab-column connections, and the results are compared with those of other researches. The formulas for effective beam width coefficients are proposed and verified by using the finite element technique.

### 2. Previous Researches

Two limits on the rotational stiffness of slab-column connections are reported by Aalami<sup>(2,3)</sup> using the finite element and the finite difference methods, the upper limit represents rigid joint behavior and the lower limit has a flexible joint region.

In 1975, Pecknold<sup>(4)</sup> proposed an equivalent effective slab width model, in which the effective slab width coefficient was derived using elastic plate theory and Levy type solution in Eq. (1). The effect of "rigid column" was modeled by introducing the load distribution, which was expanded in a Fourier series.

$$\alpha = \frac{1}{1-\mu^2} \cdot \frac{c_2}{l_1} \cdot \frac{l_1}{l_2} / (f_b + 6 \sum_{m=1}^{\infty} (\frac{1}{m\pi})^3 Q_m \cdot A_m) \quad (1)$$

where,

- $1 / 1 - \mu^2$  : the effect of the Poisson's ratio
- $Q_m$  : the factor for the load distribution,
- $f_b$  : the factor for the decrease in rotational flexibility,
- $A_m$  : the factor for the geometries and the boundaries.
- $c_1, c_2, l_1, l_2$  : the geometries of connections (see Fig. 4)

In 1977, Allen and Darvall<sup>(5-7)</sup> used a Fourier series technique and published the effective beam width values that are identical with Pecknold's results. In a later study (this reference was not available, but was reported by Vanderbilt<sup>(8)</sup> and Moehle<sup>(9)</sup>), Allen compared the finite element analysis results with his previous values, and concluded that his previously published values were too high. Then he developed a refined Fourier technique to generate effective beam width values. These revised values are up to 13% smaller than the original ones.

The study of effective beam width coefficients to cover several geometries was reported by Banchik.<sup>(10)</sup> He employed the finite element method to calculate the effective beam width coefficients. The variation of the coefficients for an interior frame, which includes interior and edge connections with bending perpendicular to the edge, was represented in Eq. (2). The coefficients for an exterior frame, which includes corner and edge connections bending parallel to the edge, was represented in Eq. (3).

$$\alpha = 5 \cdot \frac{c_1}{l_1} + \frac{1}{4} \cdot \frac{l_1}{l_2} \quad (\text{for interior frame}) \quad (2)$$

$$\alpha = 3 \cdot \frac{c_1}{l_1} + \frac{1}{8} \cdot \frac{l_1}{l_2} \quad (\text{for exterior frame}) \quad (3)$$

where,  $c_1, c_2, l_1, l_2$  : the geometries of connections (see Fig. 4)

In 1995, Durrani and Luo<sup>(11,12)</sup> proposed effective width coefficients for interior connections and edge (bending perpendicular to the edge) connections. Eq. (4) for the interior connections was suggested by calibrating the results of Pecknold's elastic solution. Eq. (5) for the exterior connections was established by modifying the equivalent frame method.

$$\alpha = \frac{R_{12} \cdot \left(\frac{c_2}{l_2}\right)}{0.05 + 0.002 \cdot \left(\frac{l_1}{l_2}\right)^4 - 2 \cdot \left(\frac{c_1}{l_1}\right)^3 - 2.8 \cdot \left(\frac{c_1}{l_1}\right)^2 + 1.1 \cdot \left(\frac{c_1}{l_1}\right)} \quad (4)$$

where,

$$R_{12} = -0.221 \cdot \left(\frac{c_1}{c_2}\right)^4 + 0.0281 \cdot \left(\frac{c_1}{c_2}\right)^3 + 0.1535 \cdot \left(\frac{c_1}{c_2}\right)^2 + 0.773 \cdot \left(\frac{c_1}{c_2}\right) + 0.0845$$

$$\alpha = \frac{K_t}{K_t + K_s} \quad (5)$$

where,

- $K_t$  : the torsional stiffness of the torsional member
- $K_s$  : the flexural stiffness of the slab

### 3. Calculating Process for Effective Beam Width Coefficients

The effective beam width coefficients of slab-column connections in a flat-plate are calculated using a general purpose program named MIDAS-GENw as shown in Fig. 1.

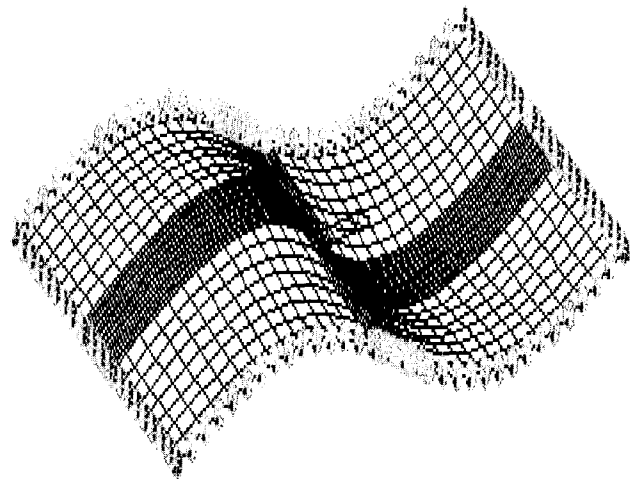


Fig. 1 Deformation of a slab-column connection

#### 3.1 Basic Assumptions

The basic assumptions to calculate effective beam width coefficients are as follows ;

- (1) The lines of inflection are assumed to occur along slab mid-spans perpendicular to the bending direction.<sup>(4-12)</sup>
- (2) The slab-column interface is infinitely stiff in bending, such that an applied moment at the column-slab junction will result in a rigid body rotation of the interface, which is termed a "stiffening effect" herein.<sup>(4)</sup>
- (3) In considering the deformation of the slab under lateral loads, the boundary conditions assume sway conditions such that boundaries perpendicular to loading direction are restrained against displacement and the other boundaries deflect freely with zero rotation about the axes along the boundaries.<sup>(2,3)</sup>
- (4) The nonlinearity of material such as the effects of the slab cracking, creep, and steel ratio of slab is not considered.
- (5) The gravity loads including the self-weight of the slab are not considered and the basic Poisson's ratio is assumed to be zero.

### 3.2 Modeling

For flat-plates, the four types of the slab-column connections are classified, as shown in Fig. 2 ; interior connections, edge connections with bending perpendicular to the edge, edge connections with bending parallel to the edge, and corner connections (which are termed "INT", "PER", "PAR", and "COR" herein). A set of different aspect ratios  $c_1/l_2$ ,  $l_1/l_2$ , and  $c_2/c_1$  are selected to encompass the ratios commonly used in practice. Aspect ratios ranged from  $0.06 \leq c_1/l_2 \leq 0.12$ ,  $0.5 \leq l_1/l_2 \leq 2$  and  $0.5 \leq c_1/c_2 \leq 2$ .

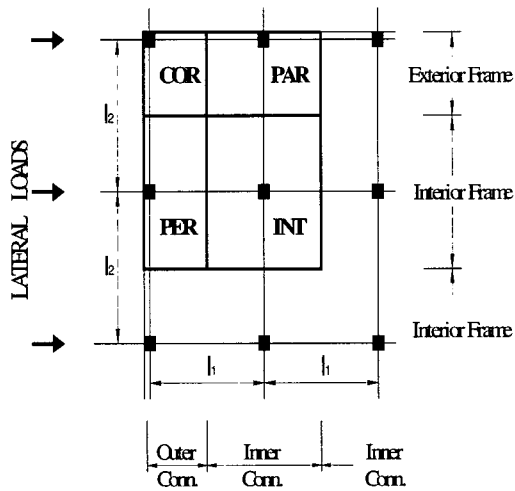
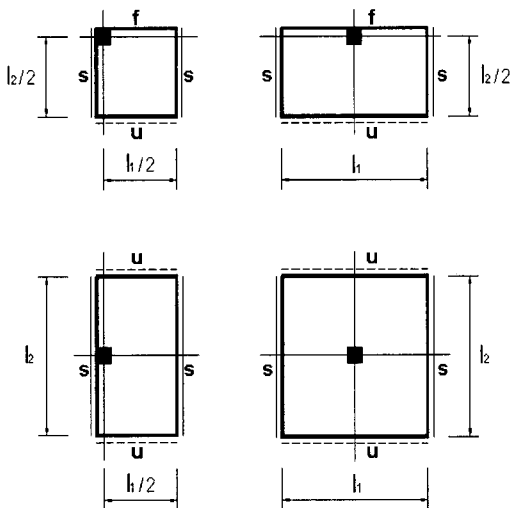


Fig. 2 Four types of connections in flat-plates



f : free, s : simple support, u : unsupported zero rotation

Fig. 3 Boundary conditions on the types

In the finite element model, the slab is modeled by four-node QUAD plate elements. The element mesh is different according to each type and slab aspect ratios. For example,

the most number of elements,  $56 \times 36$ , is used for interior connections with  $l_1/l_2 = 2$  and the least number of elements,  $16 \times 21$ , is used for corner connections with  $l_1/l_2 = 0.5$ . The slab is meshed finely near joint regions in which contact areas are made effectively rigid, having a stiffness of eight orders of magnitude greater than the slab outside the joints, to explain a stiffening effect of interface.

### 3.3 Calculation of Effective Beam Width Coefficients

The unit moment load is applied at the center of the joint, and a slab-column connection is analyzed. For each analysis, the rotation at the center of the column is obtained. The rotational stiffness of the assemblage is thus determined as  $M/\theta$ . In using given rotational stiffness, the effective beam width coefficients,  $\alpha$ , are obtained.

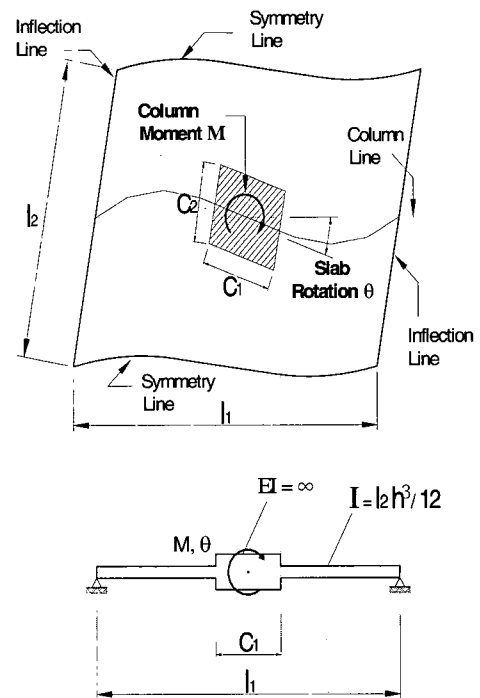


Fig. 4 Relations between column moments and slab rotations

For the interior connections (INT) and the edge connections with bending parallel to the edge (PAR), the coefficients are defined in Eq. (6). For the corner connections (COR) and the edge connections bending perpendicular to the edge (PER), the coefficients are defined in Eq. (7).

$$M (= 2M_{ba}) = 2 \times \frac{2EI}{l_1} (2\theta_b + \theta_a - 3R) \pm C_{ba}$$

$$= \frac{12EI}{l_1} \theta \quad (\theta = \theta_a = \theta_b, R = 0, C_{ba} = 0)$$

$$= \frac{\alpha l_2 E h^3}{l_1} \theta \quad (I = \frac{l_2' h^3}{12}, l_2' = \alpha \cdot l_2)$$

$$\therefore \alpha = \frac{M}{\theta} \frac{l_1}{l_2} \frac{1}{E h^3} \quad (\text{for INT, PAR}) \quad (6)$$

$$M (= M_{ab}) = \frac{2EI}{l_1} (2\theta_a + \theta_b - 3R) \pm C_{ab}$$

$$= \frac{12EI}{l_1} \theta = \frac{\alpha l_2 E h^3}{l_1} \theta$$

$$\therefore \alpha = 2 \frac{M}{\theta} \frac{l_1}{l_2} \frac{1}{E h^3} \quad (\text{for PER, COR}) \quad (7)$$

where,

$M/\theta$ : the rotational stiffness,  $l_1/l_2$ : the slab aspect ratio

$EI$ : the flexural stiffness,  $h$ : slab thickness

$l_2'$ : equivalent effective width(not considering reduction factor)

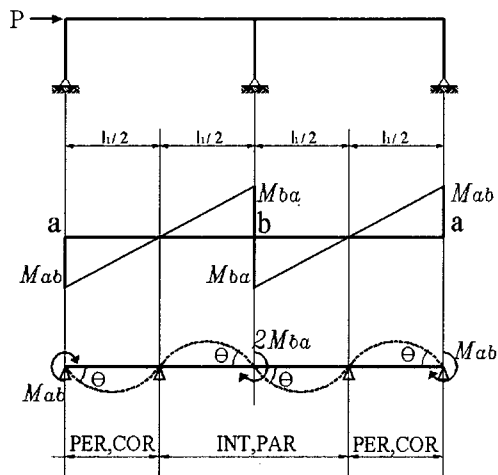


Fig. 5 Derivation of the equations (6) and (7)

The equations as defined above are derived by a slope-deflection method shown in Fig. 5 on the assumption that the slab-column frame is to be the effective beam-column frame, in which equivalent effective beam has the same longitudinal slope throughout a given cross section.

#### 4. Parametric Studies

The parameters affecting the effective beam width can be divided into geometries ( $l_1/l_2$ ,  $c_2/c_1$ , and  $c_1/l_2$ ), gravity loads, and material properties (crack, creep, and poisson's ratio). The geometric parameters are only considered in this study.

For each parameters, analysis are performed through "Calculating process for effective beam width coefficients" in chapter 3. The effective beam width coefficients are obtained and compared to the results of previous research in this chapter.

Table 1 Effective Beam width coefficients on the  $c_1/l_2$  and  $l_1/l_2$

(a) INT(interior connection)

$l_1/l_2 \backslash c_1/l_2$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.06	0.431	0.480	0.528	0.570	0.606	0.637	0.644	0.687
0.08	0.531	0.571	0.614	0.653	0.686	0.714	0.737	0.758
0.10	0.642	0.668	0.703	0.736	0.764	0.787	0.806	0.823
0.12	0.771	0.773	0.795	0.820	0.841	0.859	0.873	0.885

(b) PER(edge connection)

$l_1/l_2 \backslash c_1/l_2$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.06	0.472	0.523	0.585	0.612	0.647	0.677	0.702	0.724
0.08	0.580	0.621	0.663	0.700	0.731	0.756	0.777	0.795
0.10	0.699	0.724	0.756	0.786	0.810	0.830	0.846	0.860
0.12	0.834	0.834	0.852	0.871	0.888	0.901	0.912	0.920

(c) PAR(edge connection)

$l_1/l_2 \backslash c_1/l_2$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.06	0.262	0.285	0.309	0.330	0.348	0.364	0.377	0.388
0.08	0.330	0.347	0.367	0.385	0.401	0.415	0.426	0.434
0.10	0.411	0.414	0.427	0.442	0.454	0.464	0.473	0.479
0.12	0.505	0.489	0.493	0.501	0.508	0.515	0.520	0.524

(d) COR(corner connection)

$l_1/l_2 \backslash c_1/l_2$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.06	0.282	0.306	0.329	0.350	0.367	0.382	0.394	0.405
0.08	0.355	0.371	0.390	0.407	0.422	0.434	0.444	0.452
0.10	0.439	0.442	0.453	0.465	0.476	0.484	0.491	0.496
0.12	0.537	0.519	0.520	0.525	0.530	0.534	0.537	0.540

#### 4.1 Effects of $c_1/l_2$ and $l_1/l_2$

The effective beam width coefficients on the  $c_1/l_2$ ,  $l_1/l_2$  are indicated in Table 1. Aspect ratios ranged from  $0.06 \leq c_1/l_2 \leq 0.12$ ,  $0.5 \leq l_1/l_2 \leq 2$ , when  $c_2/c_1$  is to be 1.

In considering the effect of  $c_1/l_2$ , the values are increased by increasing  $c_1/l_2$  shown in Fig. 6. The effect of the slab aspect ratio on the values is decreased when  $c_1/l_2$  is increased. Fig. 6 reveals the effect of  $c_1/l_2$  for INT only, however, the results of the other connections are similar to INT.

As can be seen in Fig. 7, the effective beam width coefficients are increased as  $l_1/l_2$  increases. The increasing rate of coefficients for interior frames (INT and PER), is larger than that for exterior frames (PAR and COR).

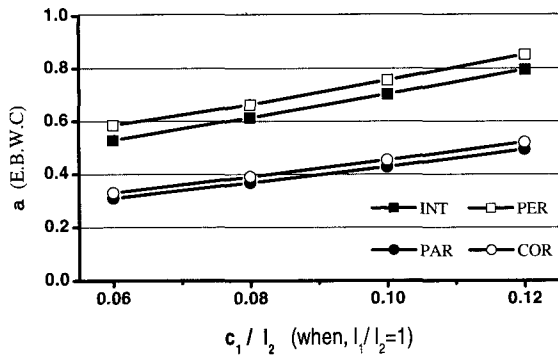


Fig. 6 Effect of the  $c_1/l_2$

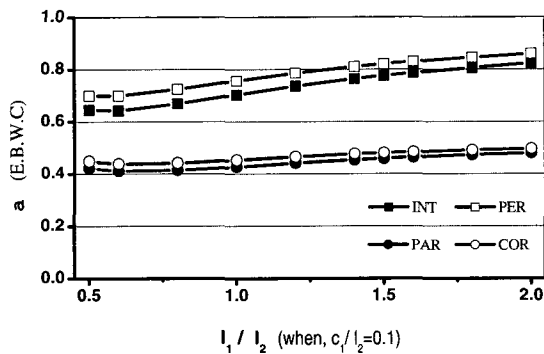


Fig. 7 Effect of the  $l_1/l_2$

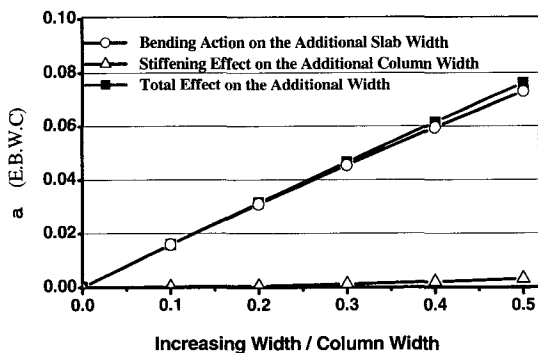


Fig. 8 The reason of the increasing coefficients for PAR

It is of interest to note that the theoretical effective beam width coefficients of connections in the exterior frame are approximately 60% of the values in the interior frame on the same conditions. Provided that the widths of slab and column in the exterior frame are precisely half of those in the interior frame, the effective beam width coefficients of the exterior frame must be exactly half of the values of the interior frame. Therefore the increasing portion (about 10%) can be the effect of additional bending action of slab, which is dominant in Fig. 8, and the additional stiffening effect of column from center line to the edge line on the exterior column.

The effective beam width coefficients of outer connec-

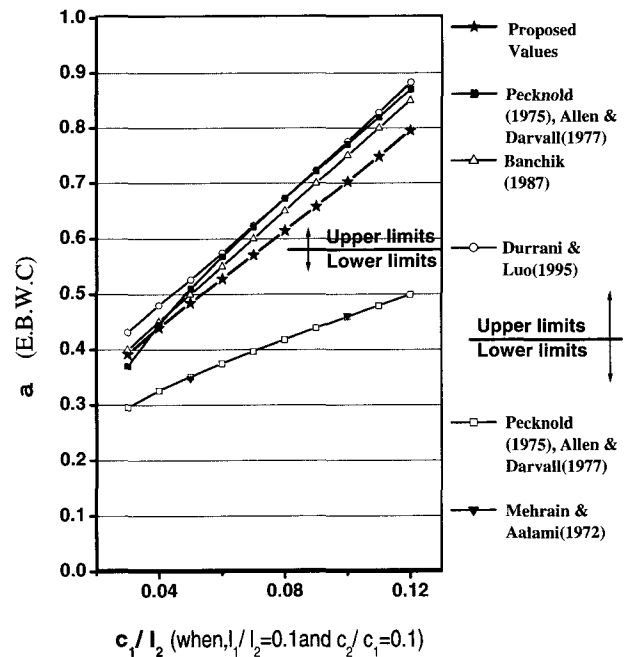


Fig. 9 Comparison between proposed values and past researches for INT

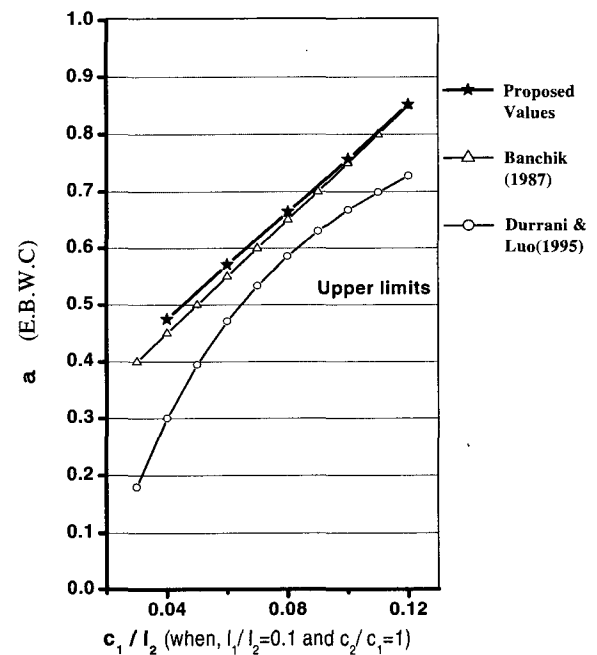


Fig. 10 Comparison between proposed values and past researches for PER

tions (PER and COR) are bigger, about 7.5%, than the values of inner connections (INT and PAR) on the same condition, which can be the same reason as above.

$c_1/l_2$  is founded as the most important parameter for evaluating the effective beam width coefficients, which has been identified by Mehraïn and Aalami, Pecknold, and Al-

len and Darvall. Another geometric parameter of relatively low importance is the slab aspect ratio  $l_1/l_2$ .

Fig. 9 reveals the relations between the proposed values and the results of the previous researches for INT. Discrepancies are founded among the values for the effective beam width coefficients. It is considered that the proposed values might be similar to the modified values of Allen and Darvall. For PER, the proposed values are similar to Banchik's results whereas they are different from Luo and Durrani's results as shown in Fig. 10.

The comparison in Fig. 9 and 10 is concerned with square slabs only, however, the results of the connections with rectangular slabs are similar to square ones.

#### 4.2 Effect of $c_2/c_1$

The above mentioned,  $c_1/l_2$  is the most important parameter and  $l_1/l_2$  is a relatively low important one for evaluating the effective beam width coefficients. For another geometric parameter  $c_2/c_1$ , the effective beam width coefficients are calculated, shown in Fig. 11. The values are increased with increasing  $c_2/c_1$ . For the increasing rate of coefficients, it is similar between INT and PER, as well as

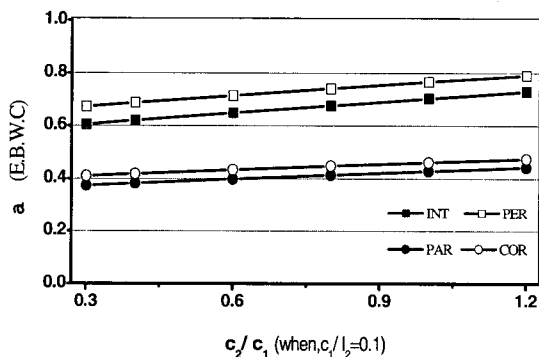


Fig. 11 Effect of the  $l_1/l_2$

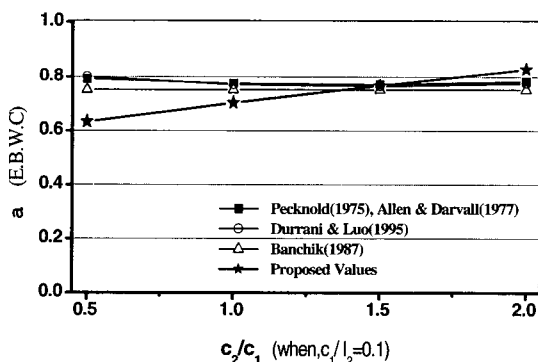


Fig. 12 Comparison between proposed values and past researches for INT

between PAR and COR.

Table 2 indicates the modification factors of the effective beam width coefficients with respect to column aspect ratios. It is founded that the rate of the modification factors on the  $c_2/c_1$  are increased when  $c_1/l_2$  are increased. The parameter  $c_2/c_1$  is less important than the  $c_1/l_2$ , but it can be considered, as shown in Table 2, whereas it has been ignored by the results of previous research.

Fig. 12 reveals the relations between the proposed values and the result of the reported research for INT. Big discrepancies are found among the values of the effective beam width coefficients. It seems that the effective beam width coefficients can be increased with increasing  $c_2$  on the same size of  $c_1$ .

Therefore, the proposed values can be better than the results from previous research for the effect of the  $c_2/c_1$ .

Table 2 Modified factors on the  $c_2/c_1$  for coefficients

(a) INT(interior connection)

$c_1/l_2 \backslash c_2/c_1$	0.5	1.0	1.5	2.0
0.06	0.922	1	1.067	1.142
0.08	0.913	1	1.083	1.160
0.10	0.900	1	1.090	1.181
0.12	0.893	1	1.102	1.202

(b) PER(edge connection)

$c_1/l_2 \backslash c_2/c_1$	0.5	1.0	1.5	2.0
0.06	0.932	1	1.065	1.127
0.08	0.922	1	1.074	1.146
0.10	0.913	1	1.083	1.163
0.12	0.905	1	1.091	1.178

(c) PAR(edge connection)

$c_1/l_2 \backslash c_2/c_1$	0.5	1.0	1.5	2.0
0.06	0.847	1	1.139	1.270
0.08	0.832	1	1.155	1.303
0.10	0.818	1	1.170	1.333
0.12	0.806	1	1.182	1.359

(d) COR (corner connection)

$c_1/l_2 \backslash c_2/c_1$	0.5	1.0	1.5	2.0
0.06	0.864	1	1.126	1.244
0.08	0.849	1	1.140	1.275
0.10	0.836	1	1.156	1.303
0.12	0.823	1	1.166	1.327

## 5. Formulas for the Coefficients

As mentioned in the introduction, effective beam width models is a beam approach, where slabs in the flat-plate are substituted for the beams with effective width. The effective width can be defined as reduction factor  $\times$  effective beam width coefficient  $\times$  total slab width. The accuracy of this approach depends on the exact values of effective beam width coefficients and reduction factor by the effect of the crack.

The effective beam width coefficients are proposed with respect to the parameters ( $l_1/l_2$ ,  $c_2/c_1$ , and  $c_1/l_2$ ) in Table 1, Table 2, which can be indicated in Eq. (8).

$$\alpha = \frac{l_2'}{l_2} = \frac{1}{1 - \mu^2} \times \alpha_r \times \alpha_s \quad (8)$$

where,

$l_2'$ ,  $l_2$  : equivalent effective width, slab width

$1 / 1 - \mu^2$  : the effect of the Poisson's ratio

$\alpha_r$  : modified factor on the  $c_2/c_1$

$\alpha_s$  : the effective beam width coefficients on the  $c_1/l_2$  and  $l_1/l_2$

However, it is inconvenient to use the table for calculating the effective beam width coefficients. Therefore, the formulas are proposed by the regression analysis using the SAS program as shown in Eq. (9) to Eq. (12).

$$\alpha_{INT} = \frac{1}{1 - \mu^2} \cdot (0.83 + 0.17 \frac{c_2}{c_1}) \cdot (5.36 \frac{c_1}{l_2} + \frac{1}{6} \frac{l_1}{l_2}) \quad (9)$$

$$\alpha_{PER} = \frac{1}{1 - \mu^2} \cdot (0.83 + 0.17 \frac{c_2}{c_1}) \cdot (5.88 \frac{c_1}{l_2} + \frac{1}{6} \frac{l_1}{l_2}) \quad (10)$$

$$\alpha_{PAR} = \frac{1}{1 - \mu^2} \cdot (0.69 + 0.31 \frac{c_2}{c_1}) \cdot (3.54 \frac{c_1}{l_2} + \frac{1}{13} \frac{l_1}{l_2}) \quad (11)$$

$$\alpha_{COR} = \frac{1}{1 - \mu^2} \cdot (0.69 + 0.31 \frac{c_2}{c_1}) \cdot (3.79 \frac{c_1}{l_2} + \frac{1}{13} \frac{l_1}{l_2}) \quad (12)$$

where,

$\alpha_{INT}$ ,  $\alpha_{PER}$ ,  $\alpha_{PAR}$ , and  $\alpha_{COR}$  are the effective beam width coefficients on the each types.

For the proposed formulas, the second terms are modification factors of the coefficients on the column aspect ratio. It can be one when  $c_2/c_1$  is equal to be one. The values of second terms for INT and PER are the same between PAR and COR. The third terms are the effective beam width coefficients on the parameter  $c_1/l_2$  and  $l_1/l_2$ . It can be founded that  $c_1/l_2$  is more important than  $l_1/l_2$ .

## 6. Verifications

Two three-story flat-plate buildings are modeled, as shown in Fig. 13, 14, and analyzed to verify the proposed formulas as described previously.

First, the standard values are established by analyzing slab-column plane frames using the finite element technique. In the finite element method, the slab is modeled by QUAD elements with the MIDAS-GENw computer package. Poisson's ratio is taken to be equal to zero. The plate in the joint region is made effectively rigid, having a stiffness eight orders of magnitude greater than plates outside the joints.

Secondly, the comparative values are established by analyzing effective beam-column plane frames by the effective beam width technique. In this case, the effective beam widths, not considering the reduction factor by the effect of the crack, are calculated by the proposed formulas in Eq. (8) to Eq. (12), Banchick's formulas as given in Eq. (2), (3), and Durrani & Luo's formulas in Eq. (4), (5).

Finally, the result of the standard values are assumed to be the actual behavior of flat-plate buildings, and the errors of the comparative values with regard to the standard values are indicated, as shown in table 3, 4.

In considering the results of analysis for the case 1 in table 3, the proposed formulas give a very good approximation, except for rotations and lateral drifts in the X-direction. Durrani and Luo's formulas are good with respect to rotations and lateral drifts, but the unbalanced moments are evaluated so high in the INT and too low in the PER. Banchick's formulas are good for the interior frames, but rotations and lateral drifts in the exterior frame have about a 10% error.

For the case 2 to consider the effect of column aspect ratio in table 4, the comparative values give stiff behavior in the X-direction and flexible behavior in the Y-direction. The proposed formulas give a very good approximation. Durrani and Luo's formulas become wide on variations of the unbalanced moments and the errors of Banchick's formulas become wide for the rotations and lateral drifts in the exterior frame.

In considering the above results, the proposed formulas are accurately evaluated to explain the lateral behavior of flat-plates, especially for the interior connection.

The analyses are performed for the single-story and proto-type buildings as well, The results of it are similar to three-story buildings. The suggested equations by Pecknold Allen & Darvall, and Aalami cannot be applied here, because their research regions are limited for INT only. However, it can be judged, because the results of their research for INT are similar to the results of Luo and Durrani.

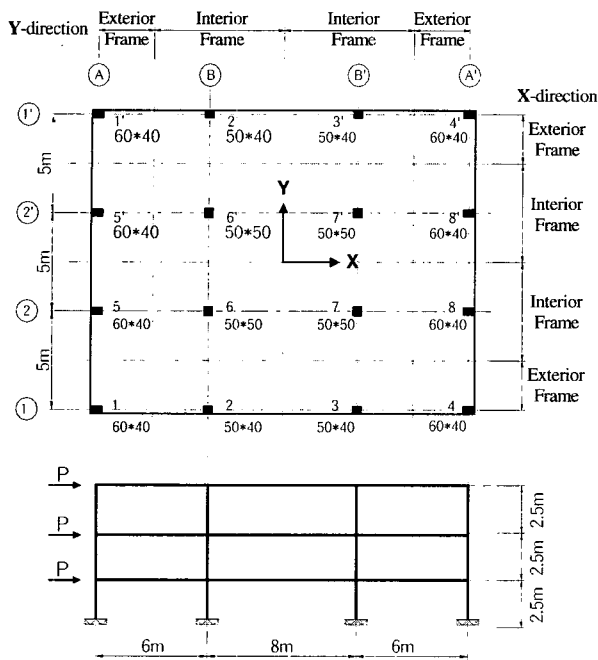


Fig. 13 Plan and section in flat-plate (CASE1)

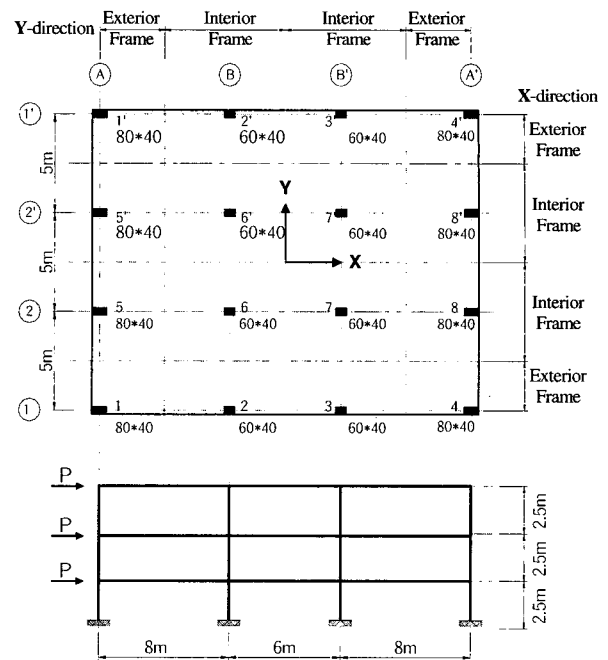


Fig. 14 Plan and section in flat-plate (CASE2)

Table 3 The errors of comparative values (CASE1)

(a) X-direction

Object	Comparativity	Interior Frame(%)		Exterior Frame(%)	
		INT	PER	PAR	COR
Unbalanced Moment	Pro.Val.	0.5	2.4	0.1	-0.1
	Banchik	3.8	2.8	4.2	0.5
	Durrani	4.9	-5.5	-	-
Rotation	Pro.Val.	-0.9	-0.9	-5.3	-7.6
	Banchik	-6.8	-6.1	-9.5	-10.6
	Durrani	-1.0	-0.1	-	-
Lateral Drift	Pro.Val.	-0.5	-0.5	-6.7	-6.9
	Banchik	-4.2	-4.3	-9.6	-9.7
	Durrani	-0.1	-0.1	-	-

(b) Y-direction

Object	Comparativity	Interior Frame(%)		Exterior Frame(%)	
		INT	PER	PAR	COR
Unbalanced Moment	Pro.Val.	0.1	-0.8	1.8	1.2
	Banchik	3.2	-3.5	-0.4	-4.6
	Durrani	5.9	-9.6	-	-
Rotation	Pro.Val.	3.2	3.4	-0.1	-0.1
	Banchik	-0.2	0.8	12.0	-11.4
	Durrani	0.2	2.5	-	-
Lateral Drift	Pro.Val.	2.3	2.3	0.3	0.1
	Banchik	0.4	0.3	8.1	7.9
	Durrani	0.9	0.8	-	-

Table 4 The errors of comparative values (CASE2)

(a) X-direction

Object	Comparativity	Interior Frame(%)		Exterior Frame(%)	
		INT	PER	PAR	COR
Unbalanced Moment	Pro.Val.	1.2	5.4	4.8	4.3
	Banchik	7.6	3.9	10.6	3.8
	Durrani	9.9	-7.0	-	-
Rotation	Pro.Val.	-2.0	-1.9	-2.1	-2.3
	Banchik	-7.0	-6.4	-5.1	-4.9
	Durrani	-5.1	-3.7	-	-
Lateral Drift	Pro.Val.	-1.3	-1.3	-1.3	-1.7
	Banchik	-4.9	-4.9	-3.5	-3.8
	Durrani	-2.9	-3.0	-	-

(b) Y-direction

Object	Comparativity	Interior Frame(%)		Exterior Frame(%)	
		INT	PER	PAR	COR
Unbalanced Moment	Pro.Val.	0.6	-0.5	3.0	0.7
	Banchik	1.7	-3.3	-3.3	-9.0
	Durrani	4.2	-8.2	-	-
Rotation	Pro.Val.	5.0	4.8	-0.7	-0.5
	Banchik	6.5	6.7	21.6	-20.6
	Durrani	6.7	8.0	-	-
Lateral Drift	Pro.Val.	3.3	3.3	0.0	-0.1
	Banchik	4.3	4.3	14.5	14.4
	Durrani	4.8	4.7	-	-



## 7. Conclusions

This research is dealing with the effective beam width coefficients for flat-plate buildings under lateral loads. Based on the results of this study, the following conclusions are presented.

- (1) The geometric parameters affecting the effective beam width are  $c_1/l_2$ ,  $l_1/l_2$  and  $c_2/c_1$ . The effect of  $c_1/l_2$  is dominant, whereas the effects of  $l_1/l_2$  and  $c_2/c_1$  are less important respectively, but cannot be ignored.
- (2) For the effect of the  $c_1/l_2$ , and  $l_1/l_2$ , discrepancies are founded among the values of the effective beam width coefficients. The results of the proposed values seem to be similar to the modified values of Allen and Darvall.
- (3) For the effect of the  $c_2/c_1$ , it is judged that the proposed values are more suitable than the results in previously reported researches, because the effective beam width coefficients can be increased with increasing  $c_2$  on the same size of  $c_1$ .
- (4) The formulas are proposed by the regression analysis using the SAS program, as shown in Eq. (9) to Eq. (12). In the formulas, the second term is concerned with the modification factor on column aspect ratio and the third term is concerned with the effective beam width coefficients on the  $c_1/l_2$ , and  $l_1/l_2$ .
- (5) For the result of the verifications with three-story plane frames, the values of the proposed formulas are accurately predicted within 5% error with respect to unbalanced moments, rotations, and lateral drifts of the slab-column connections. The results of this are more accurate than the ones of past researches for analyzing flat-plate buildings.
- (6) The lateral stiffness of flat-plate is affected not only by geometric parameters but also by cracking. Therefore the stiffness reduction by crack must be considered to evaluate precise behavior.

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