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Evaluation of Neutron Cross Sections of Dy Isotopes in the Resonance Region

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Abstract

The neutron cross sections of ^{160}Dy, ^{161}Dy, ^{162}Dy, ^{163}Dy, and ^{164}Dy have been evaluated in the resonance region of which upper energy is set to several tens of keV. The cross sections are formulated with resonance parameters in the energy region under consideration. In the resolved resonance region, the positive-energy resonance parameters were adopted from the BNL compilation published in 1984 with slight, if any, modifications. A bound level resonance for each isotope except ^{162}Dy was invoked to reproduce the reference 2200 m/s cross sections and the bound coherent scattering length. Subsequently, the statistical behavior of the resolved resonance parameters was analyzed, and thus obtained *s*-wave average parameters were adopted in the unresolved resonance region. In addition, recent measurements of the capture cross sections in the unresolved region were taken into account in adjusting the average resonance parameters for high orbital angular momentum resonances. The present evaluation resulted in large improvements in the cross sections over the ENDF/B-VI release 6.

Key Words : dysprosium, neutron resonance parameter, resolved resonance, unresolved resonance, evaluation, cross section

1. Introduction

Dysprosium (Dy, atomic number 66), which is a strong neutron absorber, is used as the poison material for control rods as well as burnable poisons in a nuclear reactor. The natural Dy element consists of ^{156}Dy (0.057%), ^{158}Dy (0.10%), ^{160}Dy (2.3%), ^{161}Dy (19.0%), ^{162}Dy (25.5%), ^{163}Dy (24.9%), and ^{164}Dy (28.1%). Because of their low natural abundance and absorption cross sections, isotopes 156 and 158 are beyond the scope of the present evaluation and the discussion hereafter. The nuclear characteristics of Dy isotopes are as follows. The thermal (*i.e.*, 2200 m/s) capture cross sections of the Dy isotopes range from ~60 b (^{160}Dy) to 2,500 b (^{164}Dy). These values are lower than or comparable with those of other neutron poison materials such as

^{155}Gd (~60,000 b), ^{157}Gd (~250,000 b), ^{152}Eu (~13,000 b), ^{167}Er (~650 b), and ^{10}B (σ(n,α) ~3,800 b). However, their capture resonance integrals (300 ~ 2,800 b) are comparable with or larger than those of others, such as ^{155}Gd (~1,500 b) and ^{157}Gd (~750 b). On the other hand, here is the other important characteristic; the neutron absorption capability of a poison containing the natural Dy element is degraded slowly because all the Dy isotopes is more or less strong neutron absorber. Let' s consider a ^{A}Dy(n,γ)$^{A+1}$Dy reaction. The disappearance of ^{A}Dy nuclei by absorbing neutrons does not necessarily result in the decrease of reactivity worth, because such absorption supplements $^{A+1}$Dy, which is also a strong absorber, so that the total absorption capability is, roughly speaking, conserved. Keeping the absorption capability is indispensable for a control rod material, especially in the case of a soluble boron free reactor in which the control rods have to be inserted to the effective core during even normal operation. Meanwhile, the cross sections of Dy, which is one of the rare-earth elements, are important to the study of nucleo-synthesis in astrophysics[1]. That study requires the capture cross sections in the keV to several hundreds keV region.

The ENDF/B-VI[2] release 6 contains the resolved resonance data of $^{160\sim164}$Dy, which were merely imported from the previous version V evaluated in 1974. The ENDF/B-V had adopted the data in the report BNL-325 (the so-called "barn book"), 3rd edition published in 1973[3] or 2nd edition in 1966 (^{164}Dy case). No unresolved region data for Dy isotopes are provided in the ENDF/B-VI.6, except for ^{164}Dy. The European library JEF-2[4] also took the ENDF/B-V data with only a modification to the Multi-Level Breit-Wigner formula, instead of the Single Level formula in ENDF/B-V. Therefore, in principle, there must be no difference between the capture cross sections calculated using the resonance data in JEF-2 and ENDF/B-VI.6. The Japanese library JENDL-3.2[5] does not contain any data on Dy isotopes. In 1975, Liou *et al.* reported a comprehensive measurement of resolved resonances[6], which became the major source of the revised Mughabghab' s BNL compilation published in 1984[7]. In the energy region of a few to several hundreds keV, several capture cross section measurements have been reported[1,8~10] after the compilation, and large deviations in the capture cross sections of ENDF/B-VI.6 from the measured values were found. Therefore, a necessity arose for revising the resonance data in both the resolved and unresolved resonance regions by taking account of new measurements after the evaluation for the ENDF/B-V in 1974.

The present evaluation concerns the neutron resonance parameters of Dy isotopes up to the incident neutron energy of several tens of keV where the first inelastic scattering channel opens. In the energy region under consideration, called the resonance region, two reaction channels, the capture and the elastic scattering, are opened in the case of Dy isotopes. In the resolved resonance region, detailed resonance parameters such as the resonance energy, the neutron width, and the radiative width are provided for each individual resonance. In the unresolved region typically above a few keV, no measurements of individual resonances are available, so 'average' parameters are provided for that region. The cross sections are calculated according to some formulae[11] from the resonance parameters. Reflecting the importance of Dy isotopes as neutron absorbers, the present evaluation is focused on the neutron capture cross section. For the energy region above the resonance region to 20 MeV, a new evaluation[12] has been performed in the Nuclear Data Evaluation

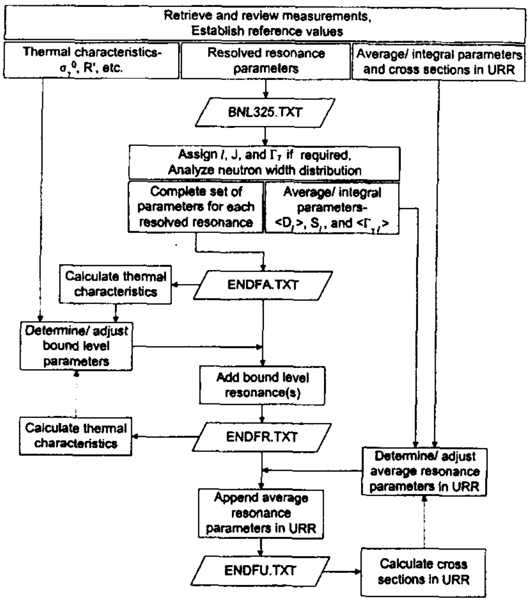

Fig. 1. Evaluation Procedure

Laboratory of the Korea Atomic Energy Research Institute (KAERI). In Nov. 1999, during the course of the present evaluation, the Cross Section Evaluation Working Group of the USA decided to include Wright's new evaluation of Dy isotopes[13] in the ENDF/B-VI release 7. Concerning the resolved resonance parameters, because the present and his evaluations adopted the same source of parameters, *i.e*, the BNL compilation[7], the cross sections in the thermal region are almost same. However, some discrepancies in the capture cross sections in the unresolved resonance region are found. Details will be pointed out in Section 3.3.

2. Evaluation Method and Procedure

Fig. 1 shows the evaluation procedure, which is basically the same with the procedure established during the evaluation work of the cross sections of fission products[14]. The overall procedure is summarized below. Details of the physics models and computational methods are found in Ref. 14 or relevant references cited.

For the first step of the evaluation, bibliographic information is retrieved from CINDA, the Computer Index to Nuclear Data[15]. In addition, the measured resonance parameters, as well as other data such as the capture cross sections in the thermal and keV energy regions, are obtained from the EXFOR database[16] or from the literature. After reviewing the available measurements, numerical data of the resolved resonance parameters are prepared as an electronic file, BNL325.TXT. The capture and scattering cross sections at 0.0253 eV, the bound coherent scattering length (b_{coh}), and the capture resonance integral are the quantities to be reproduced from the resolved resonance parameters. The reference thermal characteristics were adopted from the BNL compilation[7] in the present evaluation. No new measurements of the thermal characteristics of Dy isotopes after the Compilation are found.

In the second step, the orbital angular momentum (*l*) and the resonance spin (J) of a resonance, which have not been determined from the measurements, are determined. The Bayesian method[14,17] was applied to distinguish the *p*-wave (*l*=1) from the *s*-wave (*l*=0) resonances. For a resonance with a neutron width of $g\Gamma_n$, the probability that this resonance is a *p*-wave resonance is calculated from Bayes' theorem of conditional probability. Present evaluation adopted rather rigorous probability density functions (pdf's) for the distributions of *s*- and *p*-wave neutron widths. Those pdf's had been derived[18] from the χ^2 distribution proposed by Porter and Thomas[19]. On the other hand, we note that the resonance spins for weak and/or high-energy resonances are seldom determined experimentally. For those resonances, a method of random assignment[14] was applied with the pdf for the J distribution calculated by using the Bethe formula for the level density. The Bethe formula itself and necessary values of physical quantities are found in Ref. 20. The file ENDFA.TXT is prepared at the end of this step. In this file, a set of parameters for a resonance consists of the orbital angular momentum (l), resonance energy (E_0), resonance spin (J), neutron width (Γ_n), and radiative width (Γ_γ; capture width in other word). The fission width and competition width are zero for Dy isotopes. Multi-level Breit-Wigner formalism[11] was adopted in the present evaluation.

In the same step, the distribution of the reduced neutron widths of the resolved resonances is fitted to a theoretical distribution to obtain the average level spacing and the neutron strength functions. This is called the Porter-Thomas (P-T) analysis in the present article, because the Porter-Thomas distribution[19] was adopted as the theoretical distribution of the reduced neutron widths. The Levenberg-Marquardt method[21] was applied to this non-linear fitting problem. The resulting average parameters are those for the unresolved region.

In the third step, it is checked whether the current positive-energy resolved resonances produce the reference thermal cross sections and scattering lengths. If the resonance parameters fail to reproduce reference values, one or two bound level (i.e. negative-energy) resonances are invoked. For Dy isotopes, only one bound level resonance was invoked for each isotope except ^{162}Dy, for which no bound level resonance was required. Sometimes the potential (or effective) scattering length (R') is adjusted if no reasonable bound level has been obtained. The file ENDFR.TXT is prepared when this step is finished.

The last step is the step evaluating the average resonance parameters for the unresolved resonance region and appends them to the resolved resonance data file. The statistical analysis of neutron widths in the second step is usually restricted to *s*-wave resonances (and *p*-wave resonances in some nuclides) due to the lack of measurements of resolved, higher l-value resonances. Thus the average resonance parameters for higher l resonances should be evaluated via another route. In the present evaluation, the parameters for up to d-wave (l=2) resonances were evaluated because the contribution of $l \geq 3$ resonances to the cross section is negligible below a few hundreds keV. To obtain the average resonance parameters, a method of parameter adjustment based on the Bayes' theorem was applied. The general formalism for the parameter adjustment is presented in the Appendix. In the present evaluation, the model parameter vector consists of the neutron strength functions for *s*-, *p*-, and *d*-wave resonances, the average *s*-wave level spacing, and the average radiative widths for *s*-, *p*-, and *d*-waves. However, because the *s*-wave parameters are usually determined from the resolved resonances and they are rather reliable, only the parameters for *p*- and *d*-waves are subjected to adjustment in most cases. The function vector consists of the capture cross sections at different energy points calculated from the unresolved resonance parameters, and vector $\mathbf{y}$ is the measured capture cross sections. The file ENDFU.TXT including the unresolved region data is prepared at the end of the step. The point-wise cross sections in the unresolved region are calculated from the ENDFU.TXT file and the results are compared with the measured cross sections. If the discrepancies between the current cross sections and the measured values are not acceptable, the unresolved parameters are further adjusted.

Table 1. Bound Level Resonance Parameters

Isotope	ENDF/B-VI.6				Present			
	E_0 (eV)	J	Γ_n (meV)	Γ_γ (meV)	E_0 (eV)	J	Γ_n (meV)	Γ_γ (meV)
^{160}Dy	-	-	-	-	−58.2	0.5	555.	105.8
^{161}Dy	−2.38	2.5	20.98	110	−1.89	3	10.8	106.8
^{162}Dy	-	-	-	-	-	-	-	-
^{163}Dy	−0.517	2.5	0.0338	100	−1.61	3	0.255	108.6
^{164}Dy	−1.89	0.5	56.3	55.1	−1.88	0.5	51.9	61.4

Table 2. Potential Scattering Lengths, R' (fm)

Isotope	BNL Compilation[7]	ENDF/B-VI.6	Present
^{160}Dy	-	6.55	7.46
^{161}Dy	-	6.56	7.47
^{162}Dy	7.5±0.5	6.58	5.90, 7.48*
^{163}Dy	-	6.59	7.50
^{164}Dy	7.5±0.5	8.05, 7.10*	7.51

* In the unresolved resonance region.

Table 3. 2200 m/s Cross Sections

Isotope	Capture Cross Section (barn)				Scattering Cross Section (barn)			
	BNL Compilation[7]	LIPAR-5 [23]	ENDF/B-VI.6*	Present	BNL Compilation[7]	LIPAR-5 [24]	ENDF/B-VI.6*	Present
^{160}Dy	56±5	57.0	61.25	56.0	5.6±0.7	5.14	1.96	5.72
^{161}Dy	600±25	600	586.1	600.3	16.5±1.5	16.4	24.4	17.5
^{162}Dy	194±10	193.8	200.4	193.9	0.28±0.20	0.005	− 1.34	0.16
^{163}Dy	124±7	124.1	134.6	123.5	3.2±0.5	3.39	1.83	3.28
^{164}Dy	2650±100	2653	2527	2654	319±10	329	391	328

* Data taken from Ref. 31.

3. Results and Discussion

3.1. Thermal Characteristics

To reproduce the reference thermal characteristics, one bound level resonance was invoked for each isotope except for ^{162}Dy. The present bound level resonance parameters and the potential (or effective) scattering lengths are summarized in Tables 1 and 2, respectively. In the present evaluation, the potential scattering lengths were calculated from a systematic, $R' = 1.23A^{1/3}+0.80$ fm, where A is the atomic mass. These values were applied to both resolved and unresolved resonance regions with an exceptional case of ^{162}Dy at the resolved region. In the mass region where the 4S giant resonance attains a maximum, the ratio of the potential scattering radius to the nuclear radius is about 1.0[22]. For ^{162}Dy, however, it was impossible to reproduce the reference b_{coh} with $R' = 7.48$ fm. Since neither invoking a bound level nor adjusting the partial

Table 4. Capture Resonance Integrals (barn)

Isotope	BNL Compilation[7]	LIPAR-5[a] [23]	ENDF/B-VI.6[b] [31]	Present[c]
^{160}Dy	1160±130	1102	1676	1107
^{161}Dy	1200±100	1058	1217	1073
^{162}Dy	2755±270	2744	2787	2747
^{163}Dy	1470±100	1475	1468	1488
^{164}Dy	340±20	342.2	329	342.9

Integrated up to (a) the upper energy of the resolved resonance region, (b) 100 keV, and (c) the upper energy of the unresolved region.

Table 5. Bound Coherent Scattering Lengths (fm)

Isotope	BNL Compilation[7]	ENDF/B-VI.6	Present
^{160}Dy	6.7±0.4	4.2	6.7
^{161}Dy	10.3±0.4	12.5	−10.4
^{162}Dy	−1.4±0.5	1.3	−1.2
^{163}Dy	5.0±0.4	4.2	−4.9
^{164}Dy	49.4±0.5	56	51

widths of the first resonance could reproduce the references, the potential scattering length was adjusted without a bound level.

Tables 3 through 6 summarize the thermal characteristics calculated with the bound level parameters in Table 1, the positive-energy resolved resonance parameters adopted from the BNL compilation[7] (see Section 3.2), and the scattering lengths in Table 2. The Doppler broadening at 300 K was taken into account in the cross section and capture resonance integral calculations. The present values in Tables should be close to the values of the BNL compilation that are regarded as the reference values in most cases. The values of Russian evaluation, LIPAR-5[23], are presented in Tables for the comparison with the present evaluation.

As shown in Table 3, the present evaluation reproduced the reference capture cross sections. Especially we note that the ENDF/B-VI.6 capture cross section of ^{164}Dy is much lower than the reference value. The present scattering cross sections are also consistent with those in the BNL compilation as well as the LIPAR-5. Note that the ENDF/B-VI.6 adopts the single level Breit-Wigner formula so that a negative scattering cross section is possible even though it is not physical.

Integrating the capture cross sections from 0.5 eV with a weighting function of the 1/E spectrum results in the resonance integrals in Table 4. The present results are consistent with those in the BNL compilation, except the case of ^{161}Dy. However, the BNL value for ^{161}Dy seems too large. The measured value by Lucas *et al.*[24] is 1150 b, and that by Dobrozemsky *et al.*[25] is 1060±80 b. The present value (1073 b) is consistent with the average of these two available measurements (1105 b) and with the value in LIPAR-5 (1058 b) within the uncertainty of 3 %. This is a special case: the BNL value is not regarded as the reference.

Concerning the bound coherent scattering length in Table 5, the recommended BNL values came from the measurement of Child *et al.* at

Table 6. Westcott Factors for Capture at 2200 m/s

Isotope	LIPAR-5 [23]	ENDF/B-VI.6*	Present
160Dy	1.0058	1.0094	1.0063
161Dy	0.9900	0.9936	0.9895
162Dy	1.0047	1.0028	1.0050
163Dy	1.0112	1.0051	1.0109
164Dy	0.9875	0.9873	0.9880

* Calculated from the thermal and Maxwellian-average capture cross sections in Ref. 31.

Table 7. Status of the Resolved Resonance Data

Isotope	Highest Resonance Energy (eV)			No. of Positive-energy Resonances		
	BNL Compilation[7]	ENDF/B-VI.6*	Present	BNL Compilation[7]	ENDF/B-VI.6*	Present
^{160}Dy	1994.3	20.5	1994.3	65	3	65 (0)
^{161}Dy	996.2	66.4	996.2	253	27	253 (0)
^{162}Dy	15814	409	4844.6	142	8	75 (7)
^{163}Dy	996.6	483.6	996.6	114	60	114 (0)
^{164}Dy	21151	145.5	6996.3	116	1	69 (28)

No. of *p*-wave resonances in parentheses.

0.07 eV[26]. A more recent compilation of scattering lengths by Koester *et al.*[27] also recommends Child' s data. Because the first resonances of all Dy isotopes are far from the energy of 0.0253 eV, no significant differences are expected between the lengths at 0.0253 eV and 0.07 eV. The present calculated lengths at 0.0253 eV are, of course, consistent with those of Child.

Meanwhile, the present Westcott factors for capture, presented in Table 6, show good agreement with those of other evaluations and suggest that the Dy isotopes are 1/v absorbers.

As a summary, the thermal characteristics that are calculated from the present resolved resonance parameters are satisfactory. Improvements over ENDF/B-VI.6 are found especially in the thermal capture cross sections of ^{163}Dy and ^{164}Dy, the thermal scattering cross sections of all Dy isotopes, and the capture resonance integral of ^{160}Dy.

3.2. Resolved Resonance Parameters

The status of the present evaluation is summarized in Table 7. The positive-energy resonance parameters in the BNL compilation[7] have been adopted. However, some adjustments of the parameters and/or some decision making were necessary. Those are explained below for every individual cases.

After the BNL compilation was published, Beer *et al.* reported the capture areas of ^{160}Dy resonances measured at ORELA in the energy range of 2.5 ~ 4 keV[8]. However, Beer' s data were not integrated into the present evaluation. The reasons are as follows. The neutron widths, which were deduced in the present evaluation from Beer' s capture areas with an average Γ_γ of 106 meV, are too small and result in a very small neutron strength function. The *s*-wave strength function Beer himself reported is only about 1/5 of the strength function in the energy region of 0

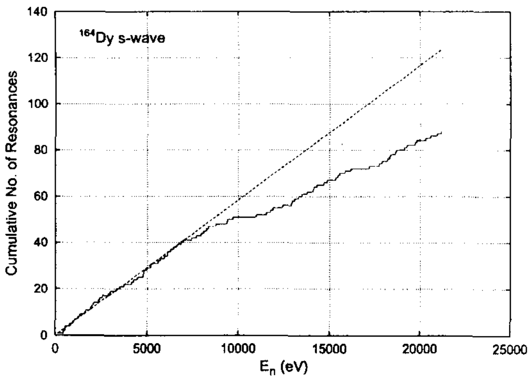

Fig. 2. Staircase Plot of the Number of *s*-wave Resonances(Dy-164)

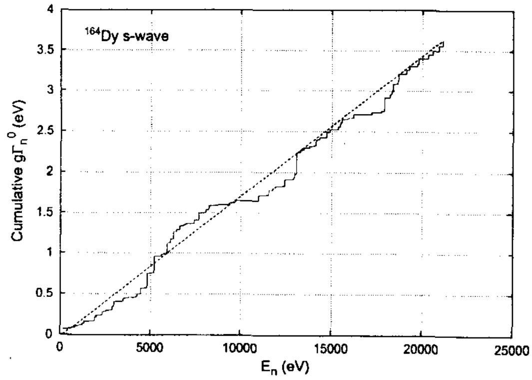

Fig. 3. Staircase Plot of the *s*-wave Reduced Neutron Widths(Dy-164)

~ 2 keV. In addition, because the BNL compilation contains data up to 2 keV, there is an empty energy range of 2 ~ 2.5 keV.

On the other hand, as shown in Table 7, the resonances of ^{162}Dy and ^{164}Dy in the high energy region were excluded from the present sets of the resolved region data. Followings are the bases for this decision: (i) recent measurements of average capture cross sections above several keV[1,8~10] are available so that the cross sections in that energy region can be described with reliable average resonance parameters in the unresolved region, and (ii) many weak resonances in the high energy region seem to be missed. Figs. 2~5 have been prepared to backup the latter basis of such decision. In these figures, dashed lines are the linear-fit of actual data and solid, staircase lines are for the actual data. Fig. 2 shows the cumulative number of measured resonances of ^{164}Dy up to ~20 keV as well as a linear fit in the range of 0 ~ 7 keV. The slope in the figure is the level density (*i.e.*, the number of resonances per unit energy interval) and the energy-dependence of the level density is negligible at the energy region under present consideration. Thus this figure suggests many resonances were not measured in the high energy region, while Fig. 3 suggests that the missed resonances should be weak resonances (*i.e.*, resonances having small Γ_n) because the linearity is maintained. We recall that the slope in Fig. 3 (as well as in Fig. 5) is the *s*-wave neutron strength function. Similar plots are shown in Figs. 4 and 5 for ^{161}Dy for the comparison with the ^{164}Dy case. Note the linearity in Fig. 4, which shows that the number of missed ^{161}Dy resonances is small enough. As a summary, the available set of ^{164}Dy (and ^{162}Dy) resonances seems to be incomplete at the high energy region.

The orbital angular momenta of the resonances, which had not been determined experimentally, were determined from the Bayesian approach as mentioned in the second step of the evaluation procedure. No *p*-wave resonances are observed for ^{160}Dy, ^{161}Dy, and ^{163}Dy. As examples concerning the analysis of the statistical behavior of reduced neutron widths, Figs. 6 and 7 show the distribution of the *s*-wave reduced neutron widths of ^{160}Dy and ^{161}Dy, respectively. The vertical axis is the cumulative number of resonances whose reduced widths are larger than a value in the horizontal axis. The fit of data to the theoretical Porter-Thomas distribution results in the average level spacing and neutron strength function of *s*-wave resonances. In the cases of ^{161}Dy, ^{163}Dy, and ^{164}Dy, the resonances with reduced widths smaller than 0.06 meV, 0.3 meV, and 9 meV,

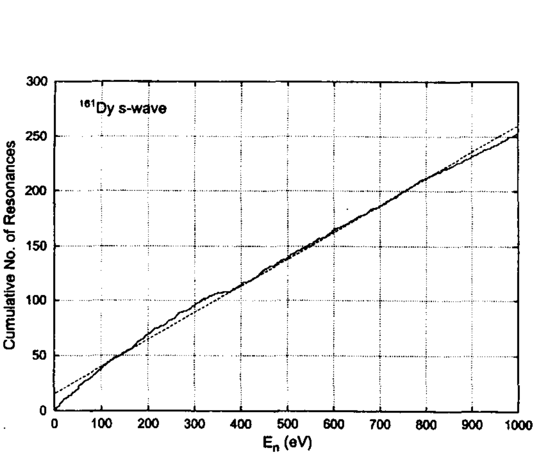

Fig. 4. Staircase Plot of the Number of s-wave Resonances(Dy-161)

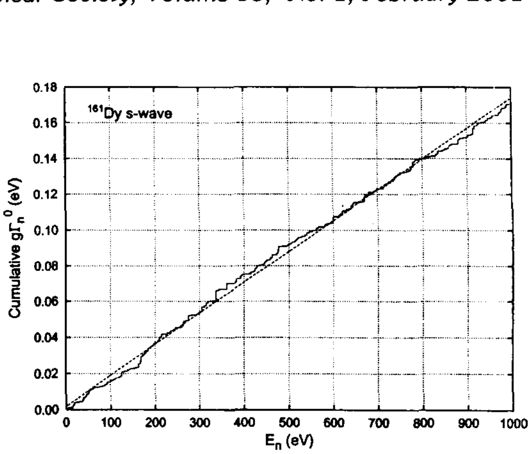

Fig. 5. Staircase Plot of the s-wave Reduced Neutron Widths (Dy-161)

respectively, were excluded from the fit. The effect of the missed weak resonances on the level spacing is then significantly reduced. Also, such exclusion results in a reasonably low x^2 value of the fitting. On the other hand, because of a small number of measured *p*-wave resonances, if any, the fitting of the *p*-wave reduced widths to the theoretical distribution did not result in reasonable values. The *s*-wave average parameters resulting from the analyses of the resolved resonances are summarized in the column 'P-T Analysis' of Table 8. This table is explained in Section 3.3.

Concerning the radiative width for each isotope, the uncertainty-weighted average width was calculated from measured several radiative widths, and then the average value was given to resonances of which the radiative widths had not been determined from the measurements. The numbers of known radiative widths are 10 (^{160}Dy), 37 (^{161}Dy), 17 (^{162}Dy), 38 (^{163}Dy), and 5 (^{164}Dy). The resulting average radiative widths are also summarized in Table 8.

Other minor modifications include such as excluding extraordinary small resonances that are presumed not to exist and increasing the uncertainty of a parameter that seemed too small.

3.3. Unresolved Resonance Parameters

The upper energy of the unresolved resonance region was set to the energy where the first inelastic scattering reaction channel opens. The upper energies are 87.336 keV (^{160}Dy), 25.812 keV (^{161}Dy), 81.162 keV (^{162}Dy), 73.895 keV (^{163}Dy), and 73.844 keV (^{164}Dy).

The average resonance parameters are listed in Table 8 and compared with those of the other evaluations. The present adopted parameters are the adjusted ones to reproduce the measured capture cross sections in the unresolved resonance region. The recent measured capture cross sections of ^{160}Dy and ^{161}Dy by Beer *et al.* at the ORNL and Karlsruhe[8], of $^{160-164}$Dy by Obninsk group of Russia[9,28] and by Voss *et al.* at Karlsruhe[1], and of $^{161-163}$Dy by Mizuno *et al.* at the Tokyo Institute of Technology[10] had been reviewed. However, only the capture cross sections by Voss *et al.* were adopted as the reference in the parameter adjustment. The measurement of Voss *et al.* is the latest one and rather complete over the energy range under consideration, and shows least uncertainties. Furthermore, their measurement technique eliminates the effect of moisture in the oxide

Table 8. Average Resonance Parameters in the Unresolved Resonance Region

Isotope	Parameter	BNL Com -pilation[7]	RIPL [29]	Bokhovko[28]	Present	
					P-T Analysis	Adopted
^{160}Dy	S_0 (10^{-4})	2.00±0.4	2.00±0.36		1.80±0.34	1.80
	$<D_0>$ (eV)	27.3±1.7	27±5	2.0	27.5±1.7	27.5
	$<\Gamma_{\gamma 0}>$(meV)	108±10	108±10		105.8±5.3	105.8
	S_1			1.5±0.4		1.65
	$<D_1>$					$<D_0>$/2.77
	$<\Gamma_{\gamma 1}>$					80.1
	S_2			2.2±0.6		1.89
	$<D_2>$					$<D_0>$/3.93
	$<\Gamma_{\gamma 2}>$					105.0
^{161}Dy	S_0	1.76±0.17	1.73±0.17		1.79±0.17	1.79
	$<D_0>$	2.67±0.13	2.40±0.20	1.75	2.95±0.09	1.73
	$<\Gamma_{\gamma 0}>$	113±10	112±10		106.8±1.4	106.8
	S_1			1.3±0.4		1.41
	$<D_1>$					$<D_0>$/1.82
	$<\Gamma_{\gamma 1}>$					84.7
	S_2			1.8±0.5		1.79
	$<S_2>$					$<D_0>$/2.34
	$<\Gamma_{\gamma 2}>$					106.8
^{162}Dy	S_0	1.8±0.3	1.88±0.25		2.0±0.37	2.01
	$<D_0>$	64.6±1.9	62.0±5.0	1.8	62.7±3.8	62.7
	$<\Gamma_{\gamma 0}>$	112±20	112±20		116.8±4.8	97.2
	S_1	1.1±0.4		1.3±0.4		1.41
	$<D_1>$					$<D_0>$/1.82
	$<\Gamma_{\gamma 1}>$					84.7
	S_2			2.0±0.6		2.00
	$<D_2>$					$<D_0>$/4.08
	$<\Gamma_{\gamma 2}>$					97.2
^{163}Dy	S_0	1.9±0.3	2.02±0.30		2.06±0.29	2.06
	$<D_0>$	6.85±0.54	6.80±0.60	1.9	7.02±0.34	7.02
	$<\Gamma_{\gamma 0}>$	113±13	113±13		108.6±3.8	108.6
	S_1			1.1±0.3		1.30
	$<D_1>$					$<D_0>$/1.84
	$<\Gamma_{\gamma 1}>$					88.6
	S_2			2.0±0.6		2.10
	$<D_2>$					$<D_0>$/2.42
	$<\Gamma_{\gamma 2}>$					108.6
^{164}Dy	S_0	1.70±0.25	1.70±0.25		1.87±0.30	1.87
	$<D_0>$	147±9	150±10	1.7	143.9±8.11	143.9
	$<\Gamma_{\gamma 0}>$	114±11	114±14		114.2±10.1	101.3
	S_1	1.3±0.3		1.1±0.3		0.99
	$<D_1>$					$<D_0>$/2.79
	$<\Gamma_{\gamma 1}>$					60.9
	S_2			1.6±0.4		1.52
	$<D_2>$					$<D_0>$/4.01
	$<\Gamma_{\gamma 2}>$					107.4

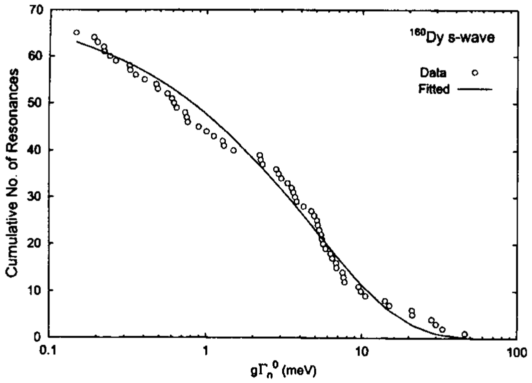

Fig. 6. Complement of the Cumulative Distribution of *s*-wave Reduced Neutron Widths (Dy-160)

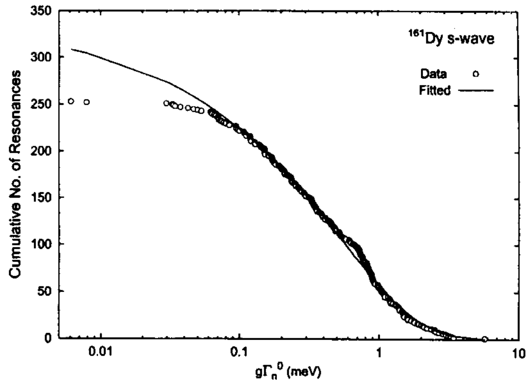

Fig. 7. Complement of the Cumulative Distribution of *s*-wave Reduced Neutron Widths (Dy-161)

sample on the capture cross section so that the results are more reliable than early measurements using oxide samples.

In Table 8, S_l is the neutron strength function in the unit of 10^{-4}, $<D_l>$ is the average level spacing in eV, and $<\Gamma_{\gamma l}>$ is the average radiative width in meV. In Bokhovko's measurement[28], they quoted the *s*-wave strength functions from Ref. 7. In the present evaluation, the neutron strength function is constant over the entire unresolved energy region, while the level spacing varies with the energy according to the Bethe formula of the level density. The present level spacing in the table is the spacing at a neutron separation energy of $^{A+1}$Dy.

For every isotope except ^{161}Dy, the adopted *s*-wave level spacing and strength function are those obtained from the P-T analyses of the resolved resonances. The *s*-wave average radiative widths for ^{162}Dy and ^{164}Dy were adjusted to obtain better agreement between the calculated capture cross sections and the measured values in the low energy region. Parameters for the *p*- and *d*-wave of all the isotopes were adjusted for better agreement in the higher energy region. Note that the radiative widths of the *p*-wave are lower than those of the *s*-wave by 20 ~ 40 %.

Concerning the *s*-wave parameters, the present evaluation is consistent with the BNL compilation as well as a recent evaluation for the Reference Input Parameter Library in 1998 [29] within their associated uncertainties. However, the case of ^{161}Dy is rather strange. It was observed that the *s*-wave parameters from the P-T analysis resulted in capture cross sections smaller than the measured values by 20 ~ 30 % in the unresolved region up to 25.8 keV. The *s*-wave level spacing was adjusted accordingly, however, the adjusted spacing is only about 60 % of that obtained from the P-T analysis. It is hard to presume that an intermediate structure may exist, because no prompt jump in the cumulative $g\Gamma_n^0$ staircase plot (Fig. 4) is observed. An analysis by Pandita and Agrawal suggested the possibility of many (102) missed levels[30], but it is equally hard to imagine that so many *s*-wave resonances are "uniformly" missed throughout the entire resolved energy region. The reason is as follows: the staircase plot of the cumulative number of resonances (Fig. 5) shows linearity, so the missed weak resonances should be uniformly distributed. However, the energy resolution of a time-of-flight measurement, with which Liou *et al.* measured the resolved resonances[6], decreases with energy so that more

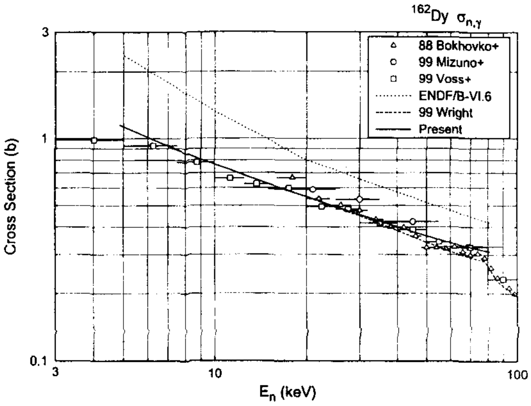

Fig. 8. Capture Cross Section in the Unresolved Resonance Region (Dy-160)

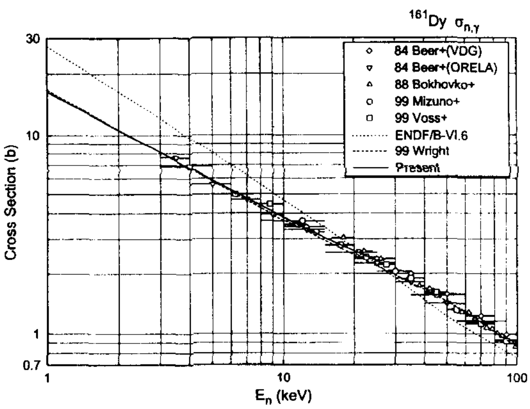

Fig. 9. Capture Cross Section in the Unresolved Resonance Region (Dy-161)

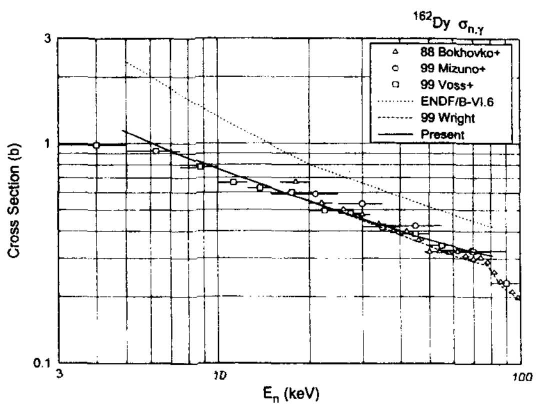

Fig. 10. Capture Cross Section in the Unresolved Resonance Region (Dy-162)

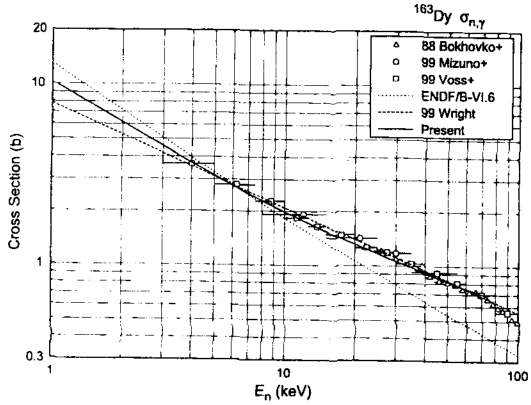

Fig. 11. Capture Cross Section in the Unresolved Resonance Region (Dy-163)

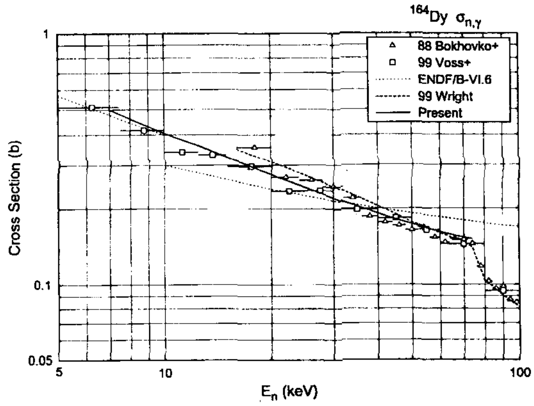

Fig. 12. Capture Cross Section in the Unresolved Resonance Region (Dy-164)

resonances shall be missed at high energies. In addition, the effect of missed weak resonances on the calculated level spacing has to be significantly reduced by applying the weak resonance cutoff during the P-T fit of reduced neutron widths. One of the possibilities to reproduce the measured capture cross sections is to increase the s-wave neutron strength function. However, a trial adjusted S_0 of 2.5E-4 deviates greatly from the general trend of S_0 in this mass region. A decision was made to adopt the values listed in the last column of Table 8, and the anomaly in the ^{161}Dy case was left unresolved.

Table 9. Capture Cross Sections Averaged over the Maxwellian Spectrum at 30 keV (mb)

Isotope	Voss [1]	Beer's Compilation[32]	Bokhovko [28]	ENDF/B-VI.6	Present
^{160}Dy	889.7±12.0	772±39	806±40	1571	846
^{161}Dy	1964 ±19.0	2006±60	1836±92	1985	*
^{162}Dy	446.0± 3.7	473±50	427±21	720	441
^{163}Dy	1112 ±11.0	1140±38	1026±51	828	1088
^{164}Dy	211.9± 2.9	268±27	209±15	242	205

* Not calculated because the upper energy of the unresolved region (25.8 keV) is too low.

Figs. 8 through 12 show the capture cross sections in the unresolved resonance region. The cross sections calculated using the present average parameters show good agreement with the measured values. Large improvements over the ENDF/B-VI.6 are obviously observed. Also, the present cross sections are superior to those of Wright[13]. It seems that Wright did not refer to the measurements by Voss *et al.*[1] and by Mizuno *et al.*[10] In addition, Wright provided average parameters that cover only a part of the unresolved region: for instance, the unresolved region data for ^{160}Dy only cover up to 20 keV and only sparse point-wise cross sections are given above that energy. From this point of view, it could be said that the present evaluation is more comprehensive than that of Wright.

Table 9 summarizes the capture cross sections averaged over the Maxwellian spectrum peaked at 30 keV. These are useful in investigating the nucleo-synthesis process. Because the upper energy covered by the resonance region is not high enough, the present average values were calculated in two parts:

$$\bar{\sigma}_\gamma = \bar{\sigma}_\gamma^R \int_{E_0}^{E_R} \omega(E)dE + \bar{\sigma}_\gamma^H \int_{E_R}^{20MeV} \omega(E)dE ,$$

where $\bar{\sigma}_\gamma^R$ is the average value calculated in the resonance region and $\bar{\sigma}_\gamma^H$ is the average value calculated for the energy range from the upper energy of the unresolved resonance region (E_R) to 20 MeV. The former was calculated from the present resonance parameters and the latter from the cross sections evaluated by Lee[12]. $\omega(E)$ is the Maxwellian spectrum and E_0 is set to 1 eV. The present average cross sections agree with Voss' measured values[1], the latest ones with smallest uncertainties, within 5 %. It is noticed in cases of $^{160,162,163}Dy$ that the values calculated from the ENDF/B-VI.6 deviate far from either Voss' new measurements or Beer's recommended values compiled in 1992[32]. These results have been foreseen qualitatively from the graphs of the capture cross sections. For instance, as shown in Fig. 8 for ^{160}Dy, the capture cross sections of ENDF/B-VI.6 are almost twice of the present cross sections around 30 keV, so that the Maxwellian average cross section based on the ENDF/B-VI.6 must be much larger than the present value.

4. Concluding Remarks

The neutron reaction data of the Dy isotopes in the resonance region have been evaluated. The present evaluation resulted in large improvements in cross sections over the ENDF/B-VI.6. Such improvements originate mainly from the fact that the present evaluation has taken recent measurements into account.

From the other view point, the importance of the established evaluation methods may not be overlooked: they make it possible, or easy at least, to convert many raw data sets in scattered literatures into a single, representative data set in a fixed format.

One of the future works is the benchmarking of the new evaluation in terms of integral quantities such as the reactivity worth of Dy control rods. Such a benchmarking work is under planning. In fact, by merging the present results in the resonance region and those of previous evaluation[12] in the region above the resonance energy, complete files for Dy isotopes in the ENDF-6 format have been already prepared. A group cross section library based on an old evaluation might be replaced with a new multi-group library generated by processing new evaluated files. On the other hand, the anomaly in the unresolved resonance parameters of ^{161}Dy should be revisited later.

Acknowledgement

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Appendix. Formalism for Parameter Adjustment[33]

Suppose we know a model parameter vector $\boldsymbol{x}_0$ that is presumed to be equal to the true vector $<\boldsymbol{x}>$, its covariance matrix $\boldsymbol{X}_0 = <(\boldsymbol{x}-\boldsymbol{x}_0)(\boldsymbol{x}-\boldsymbol{x}_0)^t>$, and a function vector $\boldsymbol{f}(\boldsymbol{x}_0)$. Assume that $\boldsymbol{f}(\boldsymbol{x})$ can be linearized by the first order Taylor expansion around $\boldsymbol{x}_0$ as

$$f(x) \cong f(x_0) + C(x - x_0)$$

where $\boldsymbol{C}$ is a sensitivity matrix of which (i,j) element is the derivative of f_i to x_j at $\boldsymbol{x}_0$. Applying Bayes' theorem on the conditional probability, we obtain the adjusted, so-called posterior, parameter vector and its covariance matrix as follows:

$$x_1 = x_0 + X_0 C^t \left(C X_0 C^t + V_y \right)^{-1} (y - f(x_0))$$

and

$$X_1 = X_0 - X_0 C^t \left(C X_0 C^t + V_y \right)^{-1} C X_0 ,$$

where $\boldsymbol{y}$ is the measured data and $\boldsymbol{V}_y$ is the covariance matrix of $\boldsymbol{y}$.

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