확률 시평에서 잔존가치를 고려한 최적의 교체 주기

Optimum Replacement Intervals Considering Salvage Values In Random Time Horizon

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ABSTRACT

An optimization problem to obtain the optimal replacement interval considering the salvage values is studied. The system is minimally repaired at failure and is replaced by new one at age T(periodic replacement policy with minimal repair of Barlow and Hunter[2]). Our model assumes that the time horizon associated with the number of replacements is random. The total expected cost considering the salvage values with random time horizon is obtained and the optimal replacement interval minimizing the cost is found by numerical methods. Comparisons between non-considered salvage values and this case are made by a numerical example.

1. Introduction

Recently, in many important classes of systems, such as computer hardware /software, robotics, automatic control, and other electronics technology, the life cycles of products become shorter and shorter, and technology companies update their product lines to take advantage of rapidly evolving new technologies out of fear of losing their markets to competitors. This can cause considerable benefits in terms of the ability to harness the latest technological advances to achieve more operationally effective systems.

However, to increase the benefits of the latest technological advances, it is also necessary for maintenance policies to be adapted to this shifting trend, because of the lack of control(e.g., in terms of configuration management and version control) and the development team.

Barlow and Hunter[2] introduced the concept of minimal repair and proposed a periodic replacement policy with minimal repair. Under this policy, if the system fails before age T, it is minimally repaired.

The system is replaced by new one at age T. Many authors(for surveys, Lam and Yeh[10], Pierskalla and Voelker[12], and Valdez-Flores

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and Feldman[14]) have studied the optimal replacement policies with minimal repair. Beichelt[4] summarized the existing policies and classified them by eleven policies.

Most of existing policies considered the infinite time horizon and obtained the optimal replacement policies minimizing the expected cost rates. The vast majority of these models assume that technology remains constant so that old systems are replaced with identical models(infinite number of replacements), a situation which seldom holds in practice.

Under the finite time horizon, Ansell et al.[1] proposed a fixed age replacement to model a system with increasing failure rate, Jack[8] studied the replacement problem involving imperfect repair, and Legát et al.[9] proposed a simple formula modifying the commonly used infinite time solution so that it gives an approximation to the exact finite time solution. Since technology forecasting is difficult and fraught with uncertainties (refer Martino[11]), it is difficult to find the exact time horizon(useful life). Therefore random time horizon is necessary for the practical replacement models(refer Yun and Choi[16]). A different approach to random time horizon takes into consideration of spares available(Derman et al.[6]), and number of repairs(Wells [15]) and timing(Hopp and Nair[7]) to replacement decision is obtained in the optimal maintenance model.

In our paper, the periodic replacement policy with minimal repair and salvage value is considered under random time horizon which also consider uncertainty of time horizon. The expected total cost is an optimization criterion and we obtain the optimal replacement interval minimizing the expected total cost. A numerical example is included.

Notation

C_1 : repair cost

C_2 : replacement cost

$f(t)$, $F(t)$: probability density function, distribution function

$h(t)$, $H(t)$: hazard function, cumulative hazard function

α , β : parameters of Weibull distribution

λ : hazard rate of time horizon

$ETCS(T)$: expected total cost considering the salvage value

$S(t)$: salvage value function with time t

Assumption

1. minimal repair is considered and repair time is negligible
2. replacement time is negligible and the system is replaced at age T(Periodic replacement)
3. time horizon follows exponential distribution
4. salvage value is exponentially decline with time t

2. Model

The system is minimally repaired at failure before age T and it is replaced at age T (Periodic replacement policy with minimal repair). The interval of the interesting time horizon is a random variable and total expected cost is an optimization criterion. First, we obtain the total expected cost. Since replacement occurs at times, $T, 2T, 3T, \ldots$, the expected maintenance cost per unit cycle(refer Barlow and Prochan[3], Beichelt[4]) is

$$C_1H(T) + C_2 - S(T) \quad . \tag{1}$$

Suppose that the time horizon t fully accommodates the first k cycles, and ends during the (k+1)th cycle, then total maintenance costs up to the beginning of the (k+1)th cycle is .

$$k[C_1H(T) + C_2 - S(T)] \quad . \tag{2}$$

and maintenance cost during last cycle, i.e., the (k+1)th cycle can be obtained as follows :

$$C_1H(t-kT) - S(t-kT) \tag{3}$$

Since the time horizon t has a pdf f(t), the expected total cost, say $ETCS(T)$, is

$$ETCS(T) = \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\{k[C_1H(T) + C_2 - S(T)] + [C_1H(t-kT) - S(t-kT)]\}f(t)dt \tag{4}$$

$$ETCS(T) = \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\{k[C_1H(T) + C_2] + C_1H(t-kT)\}f(t) - \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\{kS(T) + S(t-kT)\}f(t) \tag{5}$$

If we denote the left hand side of Eq. (5) by ETC(T), then $ETCS(T)$;

$$ETCS(T) = ETC(T) - \sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\{kS(T) + S(t-kT)\}f(t) \tag{6}$$

It is difficult to obtain the optimal replacement interval under general failure distribution, so we considered for Weibull failure distribution with $H(t) = \alpha t^{\beta}$ and exponentially distributed time horizon (λ).

Since

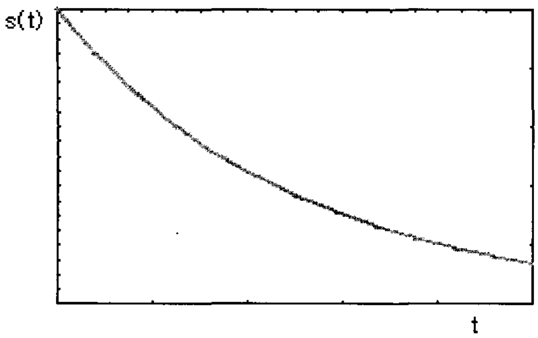
$$\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T} kf(t)dt = \frac{1}{e^{\lambda T}-1} \quad \text{and}$$

$$\sum_{k=0}^{\infty} e^{-\lambda kT} = \frac{e^{\lambda T}}{e^{\lambda T}-1} \quad ,$$

the expected total cost, ETCS(T), is

$$ETCS(T) = ETC(T) - \frac{S(T) + \lambda e^{\lambda T}\int_0^T S(t)e^{-\lambda T}dt}{e^{\lambda T}-1}. \tag{7}$$

In this paper, exponentially declining salvage value reflecting the depreciation is only considered(Fig.1). It is following salvage value function ;



[Fig. 1] Exponentially declining salvage value with time t

$$S(t) = C_2 e^{-pt} \ , \quad t \geq 0$$

where, p : declining rate

Then, after some operations, Eq.(7) is

$$ETCS(T) = ETC(T) - \frac{C_2}{(p+\lambda)} \frac{(pe^{-pT} + \lambda e^{\lambda T})}{e^{\lambda T} - 1} \qquad (8)$$

Characteristics of $ETCS(T)$ function

In Eq.(8), $ETC(T)$ is strictly convex function (refer Yun and Choi[16]). We denote the right hand side of Eq.(8) by

$$K(T) = \frac{C_2}{(p+\lambda)} \frac{(pe^{-pT} + \lambda e^{\lambda T})}{e^{\lambda T} - 1}$$

then, $\lim_{p \to \infty} K(T) = 0$,

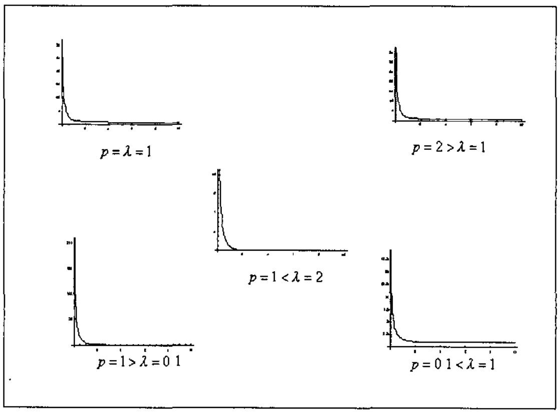
$$\frac{dK(T)}{dT} = \frac{-p(p+\lambda)e^{(p-\lambda)T} + p^2 e^{-pT} - \lambda^2 e^{\lambda T}}{(e^{\lambda T} - 1)^2}$$

and

$$\frac{d^2K(T)}{dT^2} = \frac{p(p+1)e^{(p-2\lambda)T}[(p+\lambda)e^{\lambda T} - 2]}{(e^{\lambda T} - 1)^4} + \frac{e^{\lambda T}(e^{\lambda T} - 1)(\lambda^3 - p^3 e^{-pT})}{(e^{\lambda T} - 1)^4} + \frac{e^{-pT}[p(p^2 - \lambda^2)e^{\lambda T} - p^3]}{(e^{\lambda T} - 1)^4}.$$

Also for $T > 0$, $\frac{d^2K(T)}{dT^2} > 0$. Therefore, $\frac{d^2K(T)}{dT^2}$ is an increasing function of T. Due to the parameters (λ , p) and the formidable nature of function type, no formal proof of convex or concave function is attempted. But we know that $K(T)$ is always convex function in Fig. 2. Hence, there is no evidence whether Eq.(8) is a convex function or a concave function[refer Simmons[13]).

However, numerical experience of using the MATHEMATICA software to evaluate the function type shows that Eq.(8) is convex function and unimodal.

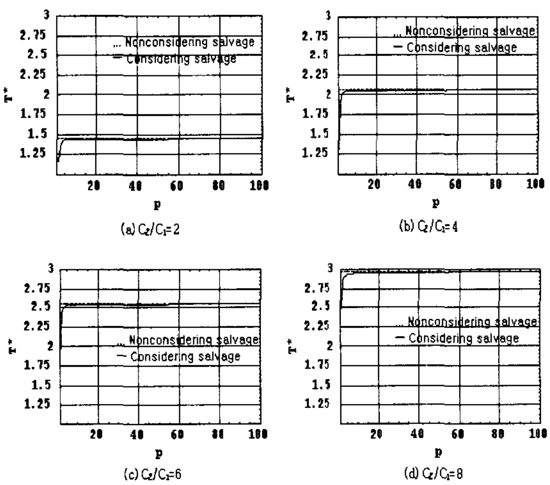


[Fig. 2] Graphically display $K(T)$

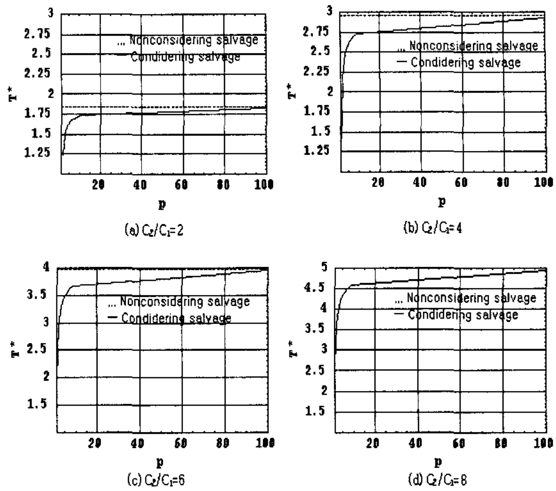
3. Numerical example

The Weibull failure distribution is assumed and the various values of the parameters, $\alpha=1$, $\beta=2$(IFR : Increasing Failure Rate) and $C_2/C_1=2,4,6,8$ are considered. We obtained the optimum replacement T^* by a numerical computation software, MATHE- MATICA.

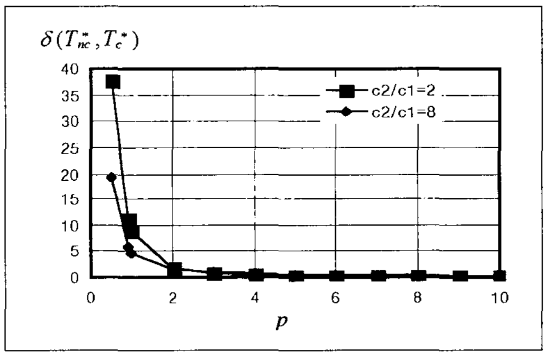
Fig. 3. and Fig. 4. graph the optimal replacement intervals as function of declining rate p with the hazard rate $\lambda=$ 0.1 and 1, respectively. In Fig. 3. and Fig. 4., we find that the optimal replacement interval considering salvage value is monotonically increasing and less than that using the non-considering salvage value. As the declining rate p increases, optimal interval converges to the optimal interval with non-considering salvage value.



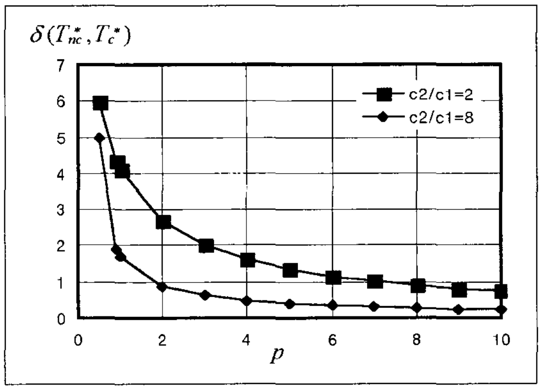
[Fig. 3] Optimal replacement intervals with declining rate $p(\lambda=0.1)$



[Fig. 4] Optimal replacement intervals with declining rate $p(\lambda=1)$



[Fig. 5] Relative deviation with declining rate $p(\lambda=0.1)$



[Fig. 6] Relative deviation with declining rate $p(\lambda=1)$

Fig. 5. and Fig. 6. show Eq.(9) as the relative deviation in the expected total cost for the optimal solution non-considering salvage value, T^*_{nc} vs the optimal solution considering salvage value, T^*_c. The relative deviation is a decreasing function of the declining rate p and converges to zero, which suggests that this model is more sensitive for the lower p.

$$\delta(T^*_{nc}, T^*_c) = ETC(T^*_c) - ETCS(T^*_{nc}) \quad (9)$$

4. Conclusion

We considered the periodic replacement policy considering the salvage value in random time horizon. The total expected cost is obtained. It is shown the optimal interval is obtained by numerical method and if the declining rate increases to infinity, the optimal interval converges to the optimal interval in the non-considering salvage case(Yun and Choi, 2000). This model improves the practicality of the assumption about the salvage value, which has been assumed to be zero. From a simple investigation of the total expected cost function and the example, we studied the relationship between the optimal value and parameters.

In this paper, exponential distributed time horizon and Weibul failure distribution is assumed and the periodic replacement has meaning. For the further studies, we can consider many variants of distributions.

References

1. Ansell, J., Bendell, A. and Humble, S., "Age Replacement under Alternative Cost Criteria", *Management Science*, Vol.30, No.3, pp.358~367, 1984.
2. Barlow, R. E. and Hunter, L. C., "Optimum Preventive Maintenance Policies", *Operations Research*, Vol.8, No.1, pp.90~100, 1960.
3. Barlow, R. E. and Proschan, F., *Mathematical theory of reliability*, Wiley, New York, 1965.
4. Beichelt, F., "A Unifying Treatment of Replacement Policies with Minimal Repair", *Naval Research Logistics*, Vol.40, No.1, pp.51~67, 1993.
5. Choi, C. H., "Maintenance and Warranty Policies with Uncertain Life Cycle", Ph. D. Thesis, *Pusan National University*, Korea, 2001.
6. Derman, C., Lieberman, G. J. and Ross, S. M., "On the Use of Replacements to Extend System Life", *Operations Research*, Vol.32, No.3, pp.616~627, 1984.
7. Hopp, W. J. and Nair, S. K., "Timing Replacement Decisions under Discontinuous Technological Change", *Naval Research Logistics*, Vol. 38, No.2, pp.203~220, 1991.
8. Jack, N., "Repair Replacement Modelling over Finite Time Horizons", *Journal of the*

Operational Research Society, Vol.42, No.9, pp.759~766, 1991.

9. Legát, V., aludová, A. H., Èervenka, V. and Jurèa, V., "Contribution to Optimization of Preventive Replacement", *Reliability Engineering and System Safety*, Vol.51, No.3, pp.259~266, 1996.
10. Lam, C. T. and Yeh, R. H., "Optimal Maintenance-Policies for Deteriorating Systems under Various Maintenance Strategies", *IEEE Transactions on Reliability*, Vol.43, No.3, pp.423~430, 1994.
11. Martino, J., *Technological Forecasting for Decision Making*, North Holland, New York, 1983.
12. Pierskalla, W. P. and Voelker, J. A., "A Survey of Maintenance Models : The Control and Surveillance of Deteriorating Systems", *Naval Research Logistics Quarterly*, Vol.23, No.3, pp.353~388, 1976.
13. Simmons, D. M., *Nonlinear programming for operations research*, Prentice-Hall, 1975.
14. Valdez-Flores, C. and Feldman, R. M., "A Survey of Preventive Maintenance Models for Stochastically Deteriorating Single-Unit Systems", *Naval Research Logistics Quarterly*, Vol.36, No.4, pp.419~446, 1989.
15. Wells, C. E., "Replacement vs Repair of Failed Components for a System with a Random Lifetime" , *IEEE Transactions on Reliability*, Vol.37, No.3, pp.280~286, 1988.
16. Yun, W. Y. and Choi, C. H., "Optimum Replacement Intervals with Random Time Horizon", *Journal of Quality in Maintenance Engineering*, Vol.6, No.4, pp.269~274, 2000.