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Technology in Mathematics Education
-For Better or for Worse-

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The use of technology may be helpful for mathematics education. However, misuse of technology can result indisaster for mathematics education, especially at elementary school levels. With much concern, this article reports some instances of the latter. It then proposes some examples of proper uses of technology to teach mathematics in the secondary school level. It emphasizes that careful design and experiments of broad scope are necessary before introducing technology in real mathematics classrooms.

I

These days, calculators and computers are widely used in mathematics education around the world. There is no doubt that, if used correctly, they could be useful and powerful tools in mathe-matics education. We can immediately recall some clear examples that show their usefulness. In contrast, it is not easy to find a story about successful computer (or calculator) aided mathe-matics learning. In fact, quite often we have read sad stories about it (Gelernter, 2000; Golden, 2000; Harvin, 1998; Healy, 1998; Tien, 1998). This not only tells us that technology carelessly applied is not useful in mathematics learning, but that it can be harmful. "Technology in Mathematics Education" was one of the main issues at the International Round Table, which was held right after the opening ceremony of ICME9. The panelists mentioned such risks. We also refer readers to Koblitz (1996) and Dubinsky & Noss (1996).

However, many models of teaching mathematics with calculators or computers are still being introduced and tried in mathematics classrooms. Recently, the Alliance for Children of the U. S. A. reported that such a trend has been initiated by some related business corporations for whom financial profit appear to have a higher priority than mathematics education (Digital Chosun Daily News, 2000).

1) The author would like to thank Professor Jerry Becker for supplying many references for the topic of this article.

Here, we open the discussion by noting that there is no reliable evidencethat calculators and computers are necessary for mathematics education at the primary and the secondary school levels. It is also noteworthy that computers could even be harmful to children's physical health.

II

The foregoing remarks appear contradictory: Calculators and computers can be useful and powerful tools in mathematics education, but, it is not easy to find any stories about successful computer (or calculator) aided mathematics learning. First, we consider reasons for the latter.

(1) To keep up with social changes, mathematics education needs to be changed accordingly. This, in turn, results in changes in the curriculum of school mathematics. That's why some of the new topics and new teaching modelshave been suggested for inclusion in the curriculum. But, we have had to remember that such proposals have not always been successful. One of the main reasons for failures has been that we have acted with too much haste. The Russian approach on this issue deserves a serious consideration(Malaty, 1998).Normally if someone proposes a new topic or teaching model to be included in the curriculum, the proposer must provide good reasons for it. The first step is to study the proposal carefully for its validity. If the proposal proves to be valid and hence deserves to be considered more seriously, it must then be tested under real conditions. In other words, a new proposal has to be carefully checked to assure that it will be used successfully in real classrooms. If the new proposal also passes this second test, the teachers must be ready to competently use it. This is the final and the most important test. The teachers must clearly understand the basic philosophy and contents of the new proposal. They must also be certain about how they will implement the new proposal in their classrooms. If we recall that the teachers are the ones who play the key role in education(Campanile, 2001), we realize that the significance of that final step cannot be ignored orthought of too lightly. A new proposal is acceptable for inclusion in the curriculum only when all these procedures can be carried out successfully.

Let me remind the readers of a real story about the national curriculum of mathematics of a country A. In this story, another country B was involved. B was an advanced and influential country. Note that in this story the curriculum was the mathematics curriculum. Country B's curriculum ended as a failure in 1955. However, in 1955, country A officially announced her national curriculum. It was deeply influenced by country B's curriculum which had already failed.

It was obvious that A's new curriculum would end in a failure. If A had really wanted to follow B, A should have paid a lot of attention to what had been going with B's curriculum. But, unfortunately, A went ahead too fast without taking the time to follow the necessary testing process described above.

The New Math Movement in America had been influenced by the "The Process of Education" written by J. S. Bruner in 1961. But around 1970 that movement finished in a total failure. In 1971, even Bruner agreed that his ideas had been too "ideal" (Bruner, 1971). Nevertheless, country A in the above story officially announced a changed curriculum that was deeply influenced by the New Math. Country A repeated B's history of failure. This author feels so sad that A is again repeating that sorry history, this time with technology in mathematics education.

It naturally requires quite long time to prepare a new curriculum proposal for introduction into classrooms. It is very likely that even a good proposal will be doomed to failure if it is introduced into classrooms too quickly. We have a lot of experience with such situations. Mac Donald (1998) reported a problem with student-centered learning. From 2001, Korean schools are following the 7thnational curriculum. We are already suffering from many of its problems. One of the main reasons is that some topics have been taken out and others have been put in without in-depth studies. The teaching models using computers (or calculators) are typical examples of those that have been included in the curriculum without sufficient study.

I have met some junior high school teachers who reported that, if the curriculum changes too much for their ability to implement it, they either ignore it or just pretend to follow it. They don't have any other choice when they are not ready for the new curriculum. Although these teachers were convinced that technology is effective and necessary in mathematics education, when they were urged to use it, many of them just wanted to pretend to use it because they were not ready. This story reminds us of E. G. Begle. He warned us about serious risks involved in big changes in a mathematics curriculum after he witnessed the failure of the New Math proposal in which he had been deeply involved.

This author has a daughter in the 5th grade. Her teacher likes computer-aided instruction. Last semester, my daughter had a lot of trouble with mathematics. She is not a poor student, but she did a terrible job in mathematics. To make matters worse, she began to dislike mathematics. I found out that her teacher had used the computer quite often in mathematics class. My daughter understood almost nothing about the delicate mathematical concepts she should have learned. I had to teach her again at home. She knows something about mathematics now, and this should mostly be credited to her father, who taught her the mathematics without using a computer.

If we can't fully employ the testing procedures summarized in this paper, it would be better to have no change in the curriculum.

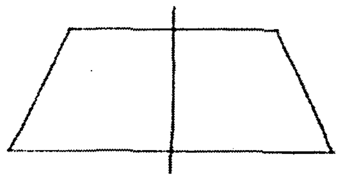
(2) Some educators have asserted that computers are useful in helping students through the difficult steps of doing mathematics. Here is one example that they have given. When students experience difficulty in proving the Pythagorean theorem, computers can help them to develop an ability to prove the theorem by using simulations. It is, of course, true that computers can help students to see what the theorem is about. Actually there are a lot of such examples using computers or graphic calculators. However we need to distinguish the steps. In teaching set theory, we often use Venn diagrams to help the students to understand some of the facts. But we need to note that the Venn diagrams cannot be the proofs. They merely help students understand what the claims of the theory are about.

The proofs of the claims are totally different. We cannot say that the diagrams will help students to develop the ability to prove. In fact, technology can be not only unhelpful, but also harmful, for learning the "proof" of the theorem. This is because students need to have an opportunity to overcome the difficult steps by themselves. They have to suffer through some cognitive conflicts. By means of the struggling, they can obtain the mathematical way of thinking, the mathematical "muscles." Such struggling is essential in order for them to internalize mathematicalconcepts. All that teachers have to do is to help their students carefully to pass through the obstacles successfully. When a learner falls down during a skiing lesson, all the instructor really does is to teach him how to get up and keep moving by himself. For the poor, the proverb says, it is better to teach them how to get bread than to give them bread. Depriving the students of the opportunity to struggle with a tough situation would be the misuse of technology, and it would contradict the basics of doing mathematics and of teaching how to do mathematics.

I remember an experience while I was teaching about the projection of certain solid bodies. To help the students understand the projection clearly, we used, as an example, a real quadrangular pyramid or tetrahedron and showed the students its shadow. It certainly seemed to be a good approach. The problem was that, whenever we talked about the projection, we used real solid bodies and their shadows. The teaching result was disastrous. When the students could not use the real solid bodies, they had big trouble with the projection.

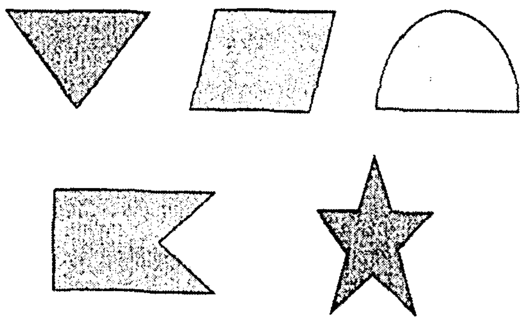
(3) Some educators have asserted that, if students are permitted to use calculators, then they will save much time that would otherwise be wasted on simple computations. We agree with this in the case of university students. But, as applied to the education of primary and secondary school students, of this is a quite dangerous idea.

The following is from a doctoral thesis on teaching some 5th graders the line symmetry using a geometry software. The teacher is supposed to ask the students to find the axis of symmetry using the software.



In the textbook, we have the following question:

선대칭도형을 모두 찾아보아라.



The author of the thesis claims that without computers, it is time-consuming to check the symmetrical property of the given figures. The following is the photocopy of the thesis:

에 제시된 도형들 가운데 선대칭도형을 찾아보는 활동을 하게 된다. 그림에서 제시된 활동을 할 때, 아동들은 도형을 눈으로 관찰하여 선대칭도형인지를 판단하게 된다. 이런 판단의 과정에서, 주어진 도형이 선대칭도형이 되는지를 추측해보는 활동은 가치로운 활동이다. 그러나 자신의 판단을 확인하기 위해서는 직접 접어보는 활동을 해 보아야 한다. 이런 경우 교과서에 제시된 도형을 오려서 확인해 볼 수는 없으므로, 직접 접어서 포개어보기 위해서는 모든 그림을 본떠서 잘라내어야만 한다. 이런 활동은 어렵고 번거로울 뿐 아니라 많은 시간이 소요된다.

In mathematics, logic and form are important. However, the mathematical habit is important, as well. How can the students obtain the habits required for mathematical thinking? Much computation is one of the good answers. We may say that doing mathematics is a kind of labor. This means that something like physical work is involved in doing mathematics.

Calculators cannot be a substitute for such valuable labor. To understand this philosophy, you might think about education in the apprentice system. It would also be helpful to remember how Sullivan taught Helen Keller.

When I was in the U.S.A. for my doctoral degree, I often met students who had to use calculators to subtract one two-digit number from another. They were typical examples of the results of technology-aided learning.

Recently, Golden (2000) reported on a classroom in America where the teacher was using calculators too often. I felt miserable about the situation.

At least at the levels of the first and second grades of primary school, computing skills and mental arithmetic should be emphasized. Technology should not be allowed to help such students do basic arithmetic.

(4) As remarked before in this presentation, while technology may have its uses, there is little room for the computers or calculators at the levels of primary and secondary school mathematics. It would be good to think about some concrete examples at this point.

The new technologies are perhaps more relevant for probability than for other parts of the mathematics curriculum which have fairly unified methods. We note that an attractive and useful function of computers is simulation. However, it is not good to simulate throwing a coin, or even a thumbtack. One may want to teach the concept of randomness or probability through such simulations. Remembering that computers are faithfully following the program, these simulations have nothing to do with the randomness. Furthermore, it'd be very probable for the students to have a serious misconception of computers.

As we mention later in this article, the Monte Carlo approach has been reported useful in teaching probability. In Monte Carlo method, the randomness (pseudo-random number) plays the key roles. In other words, we can benefit from the capability of pseudo-random generating of computers. However, the story should be different when we teach the concept of randomness. The value of experience with physical simulation as a prerequisite for a metaphorical understanding of computer generators should be emphasized (Kapadia & Borovcnik, 1991).

The Geometer's Sketchpad(GSP) that is used quite widely can measure a given angle roughly.

It is quite often that we have the following situation: GSP measures the three angles of a triangle as 30, 80, and 71. But GSP concludes that the sum of these three angles is 180. In other words, it claims that $30+80+71=180$. We can easily understand that this type of situations can't be avoided by the inherent property of computers. It is not impossible that a student be excited with the discovery of a triangle in which the sum of three angles is 181. At this moment we will be confused what we are doing.

III

We don't want to be misunderstood. We are not claiming that technology should be expelled from the primary and secondary school mathematics. In fact, we have presented some reasonable models for gifted or university students (Shin & Han, 2000). All weclaim is that at primary and secondary school levels, a new curriculum proposal should be well designed with great care. That is, the cognitive training of the students and the contents to be taught should be examined carefully and in detail. To repeat, we only want to say there is not a lot of room for technology in school mathematics on primary and secondary school levels.

However, we here propose some examples of teaching material using technology to show how calculators and computers can be useful inmathematics education. But, we want to emphasize that in these examples technology does a useful but simple job in the teaching of mathematics. We also note that our examples are mainly for teaching junior high school students on some special topics. Kombarov (2001) agrees with us at this point.

(1) Probability and statistics

In handling of some statistical data, calculators or computers are definitely useful. As an example, we are given the heights of seven basketball players. We want to compute the mean of the data. If calculators are not available, the data needs to be modified so that the sum can be divided by 7. But calculators are available, so we don't need to do that.

The concepts of randomness and probability are so delicate that it is not easy to formalize them mathematically. In fact, they are the sources of many paradoxes(Kapadia & Borovcnik, 1991; Paulos, 1988; Salmon, 1973). The randomness is also one of the important concepts in the foundations of mathematics. Chaitin(1975) explains this fact by observing: "Although randomness can be precisely defined and can even be measured, a given number cannot be proved to be random. This enigma establishes a limit to what is possible in mathematics." So, it is not surprising for us to have some difficulties with teaching probability. The computers can be used quite effectively in probability education (Kapadia & Borovcnik, 1991).

(2) Definition of the transcendental number e (Shin & Han, 2000)

Shin and Choi (2000) and Choi (2001) have proposed some teaching materials for gifted students through cognitive conflict. In those models, technology as well as internet and problem posing approaches can be incorporated quite efficiently.

(3) Factoring and primality (Shin & Han, 2000)

In this ageof information, we need a new paradigm of proof. This new type of proof may not be called "proof" in the classical sense of proof. Some thinkers call it "verification". Some models have been proposed for secondary schools (Shin, 2001). To understand themechanism of this scheme of verification, we need to understand the following fact: For computers, factoring and primality are totally different problems. In other words, a primality test is feasible even though factoring is infeasible. This fact is fundamental in modern cryptography. So, it is quite necessary for students to have experience with such a situation. In this case, it seems to be impossible to do it without computers. In applied mathematics at the university level, the technology can find ample room for itself. In particular, computers are indispensable for cryptography and coding theory.

(4) The Monty Hall dilemma (Shin & Han, 2000)

There are problems that can get even an outstanding mathematician like Paul Erdos into trouble. The Monty Hall dilemma is one of them.

You're on a game show, and you're given the choice of three doors. Behind one door is a car, behind each of the other two are goats. You choose, say, door 1, and the host, who knows where the car is, opens another door, behind which is a goat. He now gives you the choice of sticking with door 1 or switching to one of the other doors. What should you do? (Hoffman, 1998)

Even Paul Erdos couldn't be convinced by the correct answer to this problem. A computer simulation succeeded in convincing him of the answer. More detailed description of teaching model of this can be found in Choi (2001).

(5) Monte Carlo method

The concepts of randomness and probability are so delicate that it is not easy to formalize them mathematically. In fact, they are the sources of many paradoxes(Kapadia & Borovcnik, 1991; Paulos, 1988; Salmon, 1973). In all the verification schemes that have resulted in a hot debate, the probability is involved in some form.

The randomness is one of the important concepts in the foundations of mathematics. Chaitin(1975) explain this fact by observing: "Although randomness can be precisely defined and can even be measured, a given number cannot be proved to be random. This enigma establishes a limit to what is possible in mathematics." But we also need to note that the probability can play a powerful role in some types of verification(Shin, 2001). This fact impels us to introduce the concepts of randomness and probability in the secondary school mathematics in connection with the verification schemes. It would be good to introduce the Monte Carlo method. Brunner(1997), Geer(1999), and McClintock & Jiang(1997)have proposed some learning models through Monte Carlo method. In these cases the computers are definitely playing a very useful role.

(6) Creative mathematics through music composition

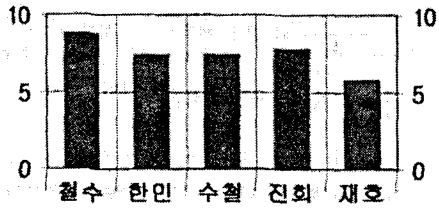
Upitis, Phillips and Higginson (1997) have reported a successful program of music composition for creative mathematics. In the project the computers are clearly indispensable. However, they have made it clear that "This was not a case of using the computer because it was in the classroom-it was an example of finding a computer because the computer, synthesiser, and software were the best tools available for the project we had undertaken." The example in Ⅱ(3) is an example in which the computers are used not because they are the best tools, but because they are in the classroom. Furthermore, Upitis (1995) says that "I am concerned that people not using technology will feel 'left out' or 'behind the times'. …. I am equally concerned that some who make liberal use of technology will want to use technology for certain things that are better accomplished with much older forms of non-computer-driven technology (like pen and paper)." The following is also one of the typical examples that Upitis et al. are worrying about:

표 3. 학생별 1학기 수학성적

	3월	4월	5월	6월	7월
경대	8	9	8	9	10
철수	10	10	9	8	7
한민	7	8	7	9	6
수철	6	5	7	9	10
진회	5	7	8	9	10
재호	5	6	6	5	7

문제5B 문제5A에 있는 표를 어느 한 선생님이 컴퓨터를 이용해서 학생의 평균 성적을 구하여 그래프로 나타내었다. 이를 이용하여 우수반 학생을 결정하시오.

경대	8.80
철수	8.80
한민	7.40
수철	7.40
진회	7.80
재호	5.80



(7) Fractal geometry

Newton's iteration method for a solution to f(x) = 0 was applied to complex functions by Cayley. Exploring iterations of functions of complex variable, Gaston Julia and Pierre Fatou noticed the fractal phenomenon in 1918. Their finding, however, did not receive much attention, mainly because graphics using technology were not available. Now, with the advent of the advanced technology, thefractal geometry has actively been introduced in the secondary school mathematics as well as the university mathematics (Peitgen, Jurgens, & Saupe, 1992). We note that the fractal is connected with many mathematical problems; Cantor set, countability, space-filling curve, the definition of dimension etc. This means that it is an interesting topic for the secondary school mathematics. In teaching and learning the fractal, the use of technology is indispensable. The computers or the graphic calculators are "the best tools available for the project." We want remark that the fractal geometry and the Euclidean geometry is totally different on the point of the use of technology.

(8) Mathematical modeling

Mathematical modeling is a form of the "Realistic Mathematics Education."It has proven its positive effects on mathematical beliefs and learning accomplishment of the students (Sung, 2000). It is quite obvious that technology can be useful in this case. In fact, many models has been proposed using computers (Swetz & Hartzler, 1991).

In Fujita (2000), the author has reported a mathematical modeling on a problem of pollution caused by oil spilled over the sea surface. The simulation in applied analysis using computer is important in the example.

IV

These days, cyber-schools are quite popular in Korea. They seem to be all right for certain areas other than mathematics, areas such as lifelong learning about health or handling money. Teaching mathematics with computers seems to be all right for adults who are already qualified and ready to use the technology. But a cyber-school is not good for mathematics education, at least,not at the primary and secondary school levels. To go one step further, any use of technology for mathematics education at primary and secondary school levels requires a lot of care. To be frank with you, I usually say the following: Do not use computers or calculators when you teach mathematics at primary and secondary school levels. If you believe it is impossible to teach some mathematical concept without a computer (or a calculator), then use it as little as possible.

Sharygin(2000) claims that "The Russian mathematical education is presently exposed to many dangers, but the most serious of them according to V. I. Arnold(and to myself) is 'Americanization' of the Russian mathematical education."On the issue of technology in mathematics education, Sharygin(2001) agrees with Koblitz(1996).

A new and good idea can be tried as soon as possible in many fields: a new fashion inart, a new product in a company's line of business, or even a new approach in mathematics itself. However, in education the approach must be different. There, a newly suggested idea should not be implemented quickly. It deserves quite a long time for prior study and experimentation.

To many of us, it sounds strange that somewant to use computers in teaching philosophy. The story is not so different when we talk about teaching mathematics. However, we can definitely get some advantages from computers in teaching and learning some topics in mathematics. But even in such promising situations, we need to check carefully many aspects of the proposed teaching models utilizing computers.

In Loveless & Diperna the following has been reported:

"Only 38 percent of education professors think calculators will hamper the learning of basic arithmetic. ⋯ . But 86 percent of the public rejects the use of calculators in early grades, and 73 percent of teachers want students to memorize the multiplication tables and learn pencil-and paper arithmetic before using calculators. … "The teachers of teachers need to think about a "butterfly effect" in mathematics education.

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