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다수의 시간지연을 갖는 구간 시간지연 시스템을 위한 간단한 안정 판별식

(Simple Stability Criterion for Interval Time-Delay Systems with Multiple Delays)

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요 약

본 단편 논문에서는, 다수의 시간지연을 갖는 구간시간지연 시스템의 점근 안정성을 조사한다. 주어진 시스템의 시간지연에 독립적인 점근 안정 조건 식과 감쇄비를 보장하는 점근 안정 조건 식을 제시한다. 구해진 조건 식은 시스템 행렬들의 조합으로 구성된 행렬의 스펙트럼 반경으로 표시된다. 수치 예를 통하여 결과의 우수성을 보인다.

Abstract

In this note, the asymptotic stability of interval time-delay systems with multiple delays is investigated. Sufficient conditions for the stability independent of delay and decaying rate for the system are derived in terms of the spectral radius. Numerical computations are performed to illustrate the result.

I. Introduction

It is well-known that interval matrices, which are caused by unavoidable system parametric variations, changes in operating conditions, aging, etc, are real matrices in which all elements are known only to belong to a specified closed interval. In the past, a number of reports have been proposed for the stability analysis of interval systems[1-5]. In general, the stability analysis for interval systems becomes more complicated when these systems posses time-delays. The time-delay is frequently a source of instability and are encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic, hydraulic, or rolling mill systems. This is why the study of dynamic systems with time-delays has received considerable attention[8-13]. For that reason, recently, the stability analysis for interval time-delays systems has been studied by some researchers[6-7], and several sufficient conditions expressed in terms of matrix norms and measures are presented.

In this note, we are interested in the following linear time-delay systems with multiple delays described by :

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$$\dot{x}(t) = A_I x(t) + \sum_{k=1}^{n} B_{I,k} x(t-h_k)$$
$$x(t) = \phi(t), \quad t \in [-H, 0] \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, A_I and $B_{I,k} \in \Re^{n \times n}$ are matrices whose elements vary in prescribed ranges, e.g. A_I and $B_{I,k}$ are such that

$$\begin{aligned} A_I &= \{[a^I_{ij}]: \underline{a}_{ij} \le a^I_{ij} \le \overline{a}_{ij}, \quad i,j=1,2,\cdots,n\} \\ B_{I,k} &= \{[b^I_{k,ij}]: \underline{b}_{k,ij} \le b^I_{k,ij} \le \overline{b}_{k,ij}, \quad i,j=1,2,\cdots,n\}, \end{aligned} \tag{2}$$

the h_k's are the time-delays with $0 \le h_k \le H$, and $\phi(\cdot)$ is the given continuously differentiable function on $[-H, 0]$.

In this note, we present sufficient conditions for the stability independent of delay and decaying rate for the system(1). The derived sufficient conditions are expressed in terms of the spectral radius of the matrix which is the combination of the modulus matrices.

The rest of this note is organized as follows. In Section II, we state notations, definitions, and well-known lemmas about matrix properties. In Section III, two criteria for asymptotic stability of the systems, in terms of spectral radius, are presented. To show the effectiveness of the proposed criterion, numerical examples are given in Section IV. Finally, a conclusion is given in Section V.

II. Preliminaries

Before we develop our main result, we state some notations, definitions, and lemmas. Let $\rho[R]$ denote the largest modulus of the eigenvalues of the matrix R, which is known as the spectral radius of R. $|R|$ denotes a matrix formed by taking the absolute value of every element of R, and it is called the modulus matrix of R. I_n denotes the identity matrix of order n. The relation $R \le T$ represents that all the elements of matrices, R and T, satisfy $r_{ij} \le t_{ij}$ for all i and j. Also, $\|R\|$ and $\mu(R)$ denote matrix norm and corresponding matrix measure of R, respectively.

Now, consider a nominal time-delay systems described by

$$\dot{x}(t) = A_0 x(t) + B_0 x(t-h) \tag{3}$$

where A_0 and $B_0 \in \Re^{n \times n}$ are constant matrices.

Then, we have following definitions and lemmas.

Definition 1. The response of system (3) is said to be asymptotically stable if, for any initial state x_0, the response due to x_0 approaches zero eventually.

Definition 2. System (3) is said to have a decaying rate $\beta > 0$ if there exists a positive number k (depending on initial conditions) such that any solution $x(\cdot)$ of (3) satisfies

$$\|x(t_2)\| \le k \|x(t_1)\| e^{-\beta(t_2 - t_1)} \tag{4}$$

for all

$$t_1, t_2 \in \Re^+ \text{ and } t_2 \ge t_1.$$

Lemma 1. [8] System (3) is asymptotically stable if and only if the solutions of its characteristic equation

$$det(sI_n - A_0 - B_0 e^{-sh}) = 0 \tag{5}$$

are in the open left-half complex plane.

Lemma 1 can be rewritten in another form as :

Lemma 2. The response of system (3) is asymptotically stable if and only if

$$|det(sI_n - A_0 - B_0 e^{-sh})| > 0 \quad \text{for} \quad \Re s \ge 0.$$

Lemma 3. [14] For any $n \times n$ matrices R, T, and V, if $|R| \le V$, then

a) $|RT| \le |R||T| \le V|T|$
b) $|R+T| \le |R| + |T| \le V + |T|$
c) $\rho[R] \le \rho[|R|] \le \rho[V]$
d) $\rho[RT] \le \rho[|R||T|] \le \rho[V|T|]$
e) $\rho[R+T] \le \rho[|R+T|] \le \rho[|R|+|T|] \le \rho[V+|T|]$.

Lemma 4. [15] For an n x n matrix R, if $\rho[R] < 1$, then $|\det(I_n \pm R)| > 0$

Lemma 5. [9] If

$$z(t)=e^{\beta t}x(t) \tag{6}$$

where $x(\cdot)$ is the solution of (3) and $z(\cdot)=0$ is asymptotically stable, then the system (3) has a decaying rate β.

III. Stability Criteria

In this section, we derive a sufficient condition for the asymptotic stability of the system given in (1).

Denote

$$\begin{aligned} A_1 &= (\underline{a}_{ij}), \quad A_2=(\overline{a}_{ij}) \\ B_{k,1} &= (\underline{b}_{k,ij}), \quad B_{k,2}=(\overline{b}_{k,ij}) \\ &\qquad \forall\ k=1,2,\cdots,n \end{aligned} \tag{7}$$

and let

$$\begin{aligned} A &= (a_{ij})=\frac{1}{2}(\underline{a}_{ij}+\overline{a}_{ij}) \\ &= \frac{1}{2}(A_1+A_2) \\ B_k &= (b_{k,ij})=\frac{1}{2}(\underline{b}_{k,ij}+\overline{b}_{k,ij}) \\ &= \frac{1}{2}(B_{k,1}+B_{k,2})\ \forall\ k=1,2,\cdots,n \end{aligned} \tag{8}$$

where A and B_k are the average matrices between A_1 and A_2, and $B_{k,1}$ and $B_{k,2}$, respectively. Furthermore

$$\begin{aligned} \Delta A &= (a^I_{ij}-a_{ij})=A_I-A \\ \Delta B_k &= (b^I_{k,ij}-b_{k,ij})=B_{I,k}-B_k \\ &\qquad \forall\ k=1,2,\cdots,n \end{aligned} \tag{9}$$

where ΔA and ΔB_k are the bias matrices between A_I and A, and $B_{I,k}$ and B_k, respectively. Also,

$$\begin{aligned} A_M &= (\overline{a}_{ij}-a_{ij})=A_2-A \\ B_{k,M} &= (\overline{b}_{k,ij}-b_{k,ij}) \\ &= B_{k,2}-B_k\ \forall\ k=1,2,\cdots,n \end{aligned} \tag{10}$$

where A_M and $B_{k,M}$ are the maximal bias matrices between A_2 and A, and $B_{k,2}$ and B_k, respectively. Note that

$$|\Delta A|\le A_M \text{ and } |\Delta B_k|\le B_{k,M}\ \forall\ k=1,2,\cdots,n. \tag{11}$$

Let $F(s)=(sI-A)^{-1}$, and F_M be the matrix formed by taking the maximum magnitude of each element of $F(s)$ for $\Re s\ge 0$. Then, we have the following theorem.

Theorem 1. Assume that the matrix A is a Hurwitz. The interval time-delay systems given in (1) is asymptotically stable, if the following inequality is satisfied :

$$\rho\left[F_M\cdot\left(A_M+\sum_{k=1}^{n}(|B_k|+B_{k,M})\right)\right]<1. \tag{12}$$

Proof. The characteristic equation of the system given in (1) is

$$\lambda(s)=det\left[sI_n-A_I-\sum_{k=1}^{n}B_{k,I}e^{-h_k s}\right]=0. \tag{13}$$

By the identity

$$det[RT]=det[R]det[T] \tag{14}$$

for any two $n\times n$ matrices R and T, we have

$$\begin{aligned} & det\left[sI_n-A_I-\sum_{k=1}^{n}B_{k,I}e^{-h_k s}\right] \\ &= det\left[sI_n-(A+\Delta A)-\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\right] \\ &= det[sI_n-A]\cdot det \\ &\quad \left[I_n-(sI_n-A)^{-1}\left(\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\right)\right]. \end{aligned} \tag{15}$$

Since A is a Hurwitz matrix, it is obvious that

$$|det[sI_n-A]|>0 \quad \text{for } \Re s\ge 0. \tag{16}$$

Thus, to show that $\left|det\left[sI_n-\left(A_I+\sum_{k=1}^{n}B_{k,I}e^{-h_k s}\right)\right]\right|>0$ for $\Re s\ge 0$, we only need to show that

$$\begin{aligned} & det\Big[I_n-(sI_n-A)^{-1} \\ &\quad \left(\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\right)\Big]\ne 0 \end{aligned} \quad \text{for } \Re s\ge 0 \tag{17}$$

To verify stability of the system (1) from Lemma 2 and 4, we need to show that

$$\rho\left[(sI_n-A)^{-1}\left(\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\right)\right]<1 \quad \text{for } \Re s\ge 0. \tag{18}$$

Using Lemma 3, we have

$$
\begin{aligned}
&\rho\Big[(sI_n-A)^{-1}\Big(\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\Big)\Big]\\
&=\rho\Big[F(s)\Big(\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\Big)\Big]\\
&\le\rho\Big[|F(s)|\cdot\Big|\Delta A+\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\Big|\Big]\\
&\le\rho\Big[|F(s)|\cdot\Big(|\Delta A|+\Big|\sum_{k=1}^{n}(B_k+\Delta B_k)e^{-h_k s}\Big|\Big)\Big]\\
&\le\rho\Big[|F(s)|\cdot\Big(|\Delta A|+\Big|\sum_{k=1}^{n}(B_k+\Delta B_k)\Big|\Big)\Big]\\
&\le\rho\Big[|F(s)|\cdot\Big(|\Delta A|+\sum_{k=1}^{n}|B_k+\Delta B_k|\Big)\Big]\\
&\le\rho\Big[F_M\cdot\Big(A_M+\sum_{k=1}^{n}(|B_k|+B_{k,M})\Big)\Big]\\
&<1 \quad \text{for } \Re s\ge 0.
\end{aligned}
\tag{19}
$$

Hence, if condition (12) is satisfied,

$$\Big|det\Big[sI_n-\Big(A_I+\sum_{k=1}^{n}B_Ie^{-h_k s}\Big)\Big]\Big|\neq 0 \quad \text{for } \Re s\ge 0,$$

which guarantees the asymptotic stability of the systems (1). This completes the proof.

Remark 1. Since the matrix A is a Hurwitz, the matrix F_M always exists and it can obtained for some s on imaginary axis by the maximum modulus principle.

Remark 2. The sufficient conditions for stability of interval time-delay system with a single delay, h, by Tissir and Hmamed[6] are as follows:

$$
\begin{aligned}
&\mu(A+zB)+\|A_M\|+\|B_M\|<0,\\
&\qquad\forall\ z \text{ with } |z|=1,\\
&\mu(A)+\mu(zB)+\|A_M\|+\|B_M\|<0,\\
&\qquad\forall\ z \text{ with } |z|=1,\\
&\mu(A)+\|B\|+\|A_M\|+\|B_M\|<0
\end{aligned}
\tag{20}
$$

where $z=e^{jw}$ with $w\in[0,2\pi]$ and $j^2=-1$.

To apply the above sufficient conditions, it needs the requirement that $\mu(A+zB)<0$ or $\mu(A)<0$ while Theorem 1 allows more relaxed requirement that A is a Hurwitz matrix. Furthermore, the stability criterion of Theorem 1 is expressed in terms of the spectral radius of the matrices which is the combination of the modulus matrices. Therefore, there is better possibility that the proposed criterion is less conservative than those in (20), which use the matrix norms and matrix measure.

With the Definition 2 and Lemma 5, now we are ready to present Theorem 2.

Theorem 2. If the condition

$$\rho\Big[F_{SM}\Big(A_M+e^{\beta H}\sum_{k=1}^{n}(|B_k|+B_{k,M})\Big)\Big]<1 \tag{21}$$

is satisfied, then system (1) is stable with a decaying rate β where $\pm F_{SM}$ is a matrix formed by taking the maximum magnitude of each element of

$$F_S(s)=(sI_n-(A+\beta I_n))^{-1} \quad \text{for} \quad \Re s\ge 0.$$

Proof. Utilize (6) to transform (1) into

$$\dot z(t)=(A_I+\beta I_n)z(t)+\sum_{k=1}^{n}e^{\beta h_k}B_{k,I}\,z(t-h_k). \tag{22}$$

By same procedure as proof of Theorem 1, we can obtain

$$
\begin{aligned}
&\rho[(sI_n-A-\beta I_n)^{-1}\\
&\quad\Big\{\Delta A+\sum_{k=1}^{n}e^{\beta h_k}(B_k+\Delta B_k)e^{-h_k s}\Big\}\Big]\\
&=\rho\Big[F_S(s)\Big\{\Delta A+\sum_{k=1}^{n}e^{\beta h_k}(B_k+\Delta B_k)e^{-h_k s}\Big\}\Big]\\
&\le\rho\Big[|F_S(s)|\cdot\Big|\Delta A+\sum_{k=1}^{n}e^{\beta h_k}(B_k+\Delta B_k)e^{-h_k s}\Big|\Big]\\
&\le\rho\Big[F_{SM}\Big(A_M+e^{\beta H}\sum_{k=1}^{n}(|B_k|+B_{k,M})\Big)\Big]\\
&<1 \quad \text{for } \Re s\ge 0.
\end{aligned}
\tag{23}
$$

Therefore, by Lemma 2, 4, and 5, the system (1) is asymptotically stable with a decaying rate β.

Ⅳ. Numerical Examples

To demonstrate the application of the result, we give the following examples.

Example 1. Consider the interval time-delay system described by (1) where

$$
\begin{aligned}
A_1 &= \begin{bmatrix}-5 & -4\\ 1 & 0\end{bmatrix},\quad A_2=\begin{bmatrix}-3 & -2\\ 1 & 0\end{bmatrix}\\
B_{1,1} &= \begin{bmatrix}0.1 & 0\\ 0 & -0.1\end{bmatrix},\quad B_{1,2}=\begin{bmatrix}0.4 & 0\\ 0 & 0.1\end{bmatrix}\\
B_{2,1} &= \begin{bmatrix}0 & 0\\ -0.1 & 0\end{bmatrix},\quad B_{2,2}=\begin{bmatrix}0 & 0.2\\ 0.1 & 0\end{bmatrix}.
\end{aligned}
$$

From (8) and (10), the average matrices are

$$A=\begin{bmatrix}-4 & -3\\ 1 & 0\end{bmatrix},\quad B_1=\begin{bmatrix}0.25 & 0\\ 0 & 0\end{bmatrix},\quad B_2=\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix}$$

and the matrices A_M and B_M are

$$A_M=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix},\quad B_{1,M}=\begin{bmatrix}0.15 & 0\\ 0 & 0.1\end{bmatrix},\quad B_{2,M}=\begin{bmatrix}0 & 0.1\\ 0.1 & 0\end{bmatrix}.$$

Since $\mu(A+zB_1)=0.25>0$ and $\mu(A)=0.2361>0$, the criteria (20) in[6] are not applicable even when $B_{2,1}=B_{2,2}=0$. Hence one cannot state the stability from the result of Tissir and Hmamed[6]. However, the matrix A is a Hurwitz, therefore Theorem 1 can be applied. The rational function matrix $F(s)$ and F_M are easily computed as

$$F(s)=\frac{1}{s^2+4s+3}\begin{bmatrix}s & -3\\ 1 & s+4\end{bmatrix},\quad F_M=\begin{bmatrix}1/4 & 1\\ 1/3 & 4/3\end{bmatrix}.$$

Then, by checking the criterion (12) of Theorem 1, we obtain

$$\rho\Big[F_M\cdot\Big(A_M+\sum_{k=1}^{2}(|B_k|+B_{k,M})\Big)\Big]=0.9833<1.$$

This gives the asymptotic stability of the system.

Example 2. Consider the following system described by (1) where

$$A_1=\begin{bmatrix}-7 & 0.5\\ 0 & -5\end{bmatrix},\quad A_2=\begin{bmatrix}-5 & 1.5\\ 0 & -3\end{bmatrix}$$
$$B_1=\begin{bmatrix}-0.4 & 0.2\\ 0 & -0.3\end{bmatrix},\quad B_2=\begin{bmatrix}-0.2 & \nu\\ 0.2 & 0.3\end{bmatrix}$$

where ν is a parameter scalar for which we shall find the upper bound that guarantee the stability of the systems.

From (8) and (10), we have

$$A=\begin{bmatrix}-6 & 1\\ 0 & -4\end{bmatrix},\quad B=\begin{bmatrix}-0.3 & 0.5(\nu+0.2)\\ 0.1 & 0\end{bmatrix}$$
$$A_M=\begin{bmatrix}1 & 0.5\\ 0 & 1\end{bmatrix},\quad B_M=\begin{bmatrix}0.1 & 0.5\nu-0.1\\ 0.1 & 0.3\end{bmatrix}.$$

The rational function matrix $F(s)$ and F_M are computed as

$$F(s)=\begin{bmatrix}1/(s+6) & 1/((s+4)(s+6))\\ 0 & 1/(s+4)\end{bmatrix},\quad F_M=\begin{bmatrix}1/6 & 1/24\\ 0 & 1/4\end{bmatrix}.$$

Then, by simple computation of the inequality (12), the bound of ν for guaranteeing the asymptotic stability of the system is

$$0.2<\nu<60.6.$$

However, by applying the criterion in[6], the stability bound is $0.2<\nu<3.96$. In the example, we can see that our criterion is less conservative than others.

V. Conclusions

In this note, we have investigated the stability problems of interval time-delay systems with multiple delays. Using characteristic equation approach and the properties of spectral radius, sufficient conditions for delay-independent stability and decaying rate β of the systems have been developed. Finally, the applications of the obtained result have been illustrated by numerical examples.

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