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상관변수에 의한 공정관리 절차

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A Process Control Procedure Based on the Correlated Variable

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A process control procedure is suggested when screening inspection is performed with a surrogate variable correlated with the performance variable. Assuming bivariate normal distribution for the performance and screening variable, the procedure is designed on the basis of the time required for detecting process shift.

Keywords : process control, surrogate variable, screening, performance variable

1. Introduction

Recent advances in inspection systems have made 100 % inspection very popular at one or more stages of the manufacturing process. When the major quality characteristic (performance variable) is difficult or expensive to measure, the inspection is usually performed with a surrogate variable correlated with the performance variable. For example, suppose that the voltage at an internal point of an electronic device is the performance variable of interest. Measuring the voltage at the internal part will be costly, since it requires disassembling some part of the device. The voltage at an external point may be used for screening inspection. There have been a number of studies concerning the screening procedure. For detailed literature review, see Tang and Tang(1994). For more recent works, see Boys *et al.* (1996), Greenshtein and Rabinowitz (1997), Gong *et al.* (1997), Shaoxiang and Lambrecht (1997), and Hong *et al.* (1998)

The objective of screening procedures is basically to improve the outgoing quality from the current level to a prespecified higher level. This, however, may not be achieved if the process is unstable. For attainment of a state of statistical stability of a process, a wide variety of Shewhart control charts and their modifications have been developed for application to different situations. The control charts are usually based on samples taken over fixed or variable time intervals. Under 100 % inspection, however, the inspection data can be used and sampling is not necessary. For this situation, Bourke (1991) suggested a run-length control chart to detect a shift in fraction nonconforming. Hui (1991) studied a complete inspection plan with feedback control for continuous performance variable. These works considered the case where inspection is performed with the performance variable.

In this paper, we consider the case where screening inspection is performed with a correlated variable. The performance and screening variables are assumed to be jointly normally distributed with known parameters. An upper specification limit is assumed to be given on the performance variable. The screening limit is determined so that the prespecified outgoing quality can be attained after screening. A control procedure is designed based on the information of screening results.

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2. The Process Control Procedure

Let Y be the performance variable which requires destructive testing for measurement. Assume the upper specification U is given on Y. Suppose that a screening variable X is measured instead of Y for 100 % inspection. We assume that

$$X = a + bY + \varepsilon \qquad (1)$$

where a and $b(b>0)$ remain constants even when the process changes, and ε is a normal random variable with mean 0 and variance σ_ε^2. We further assume that the process mean may change but the variance remains constant over time.

The control procedure for the manufacturing process is :

i) Set $R := 0$
ii) $R := R+1$
iii) For each incoming item, take a measurement x of X. If $x \le \omega$, accept the item and go to ii); otherwise, reject it.
iv) Compare with a predetermined number R_L. If $R > R_L$, go to i); otherwise, continue to v).
v) Measure Y for the next n items incoming from the process and denote the measured values by $y_1, y_2, \cdots, y_n$. Calculate $\bar{y} = \sum_{i=1}^{n} y_i/n$.
vi) Compare $\bar{y}$ with a predetermined number $\overline{Y_U}$. If $\bar{y} \le \overline{Y_U}$, go to i); otherwise, stop the process and take a corrective action.

The procedure is illustrated with <Figure 1>. The meaning of the symbols may be summarized as follows :

R = the number of items inspected after the previous rejection. $R-1$ is the run length of accepted items.
R_L = the threshold value of R for direct measurement of Y. If $R \le R_L$, Y is directly measured to determine whether the process is out of control.
ω = the cutoff value of X determining whether accept or reject each item.
$\overline{Y_U}$ = the threshold value of $\bar{y}$, the average of the observed values of Y, whether to stop the process and take a corrective action.

For use of the procedure, ω, R_L, n, and $\overline{Y_U}$ must be determined. We assume here that the sample size n is predetermined considering the cost of testing.

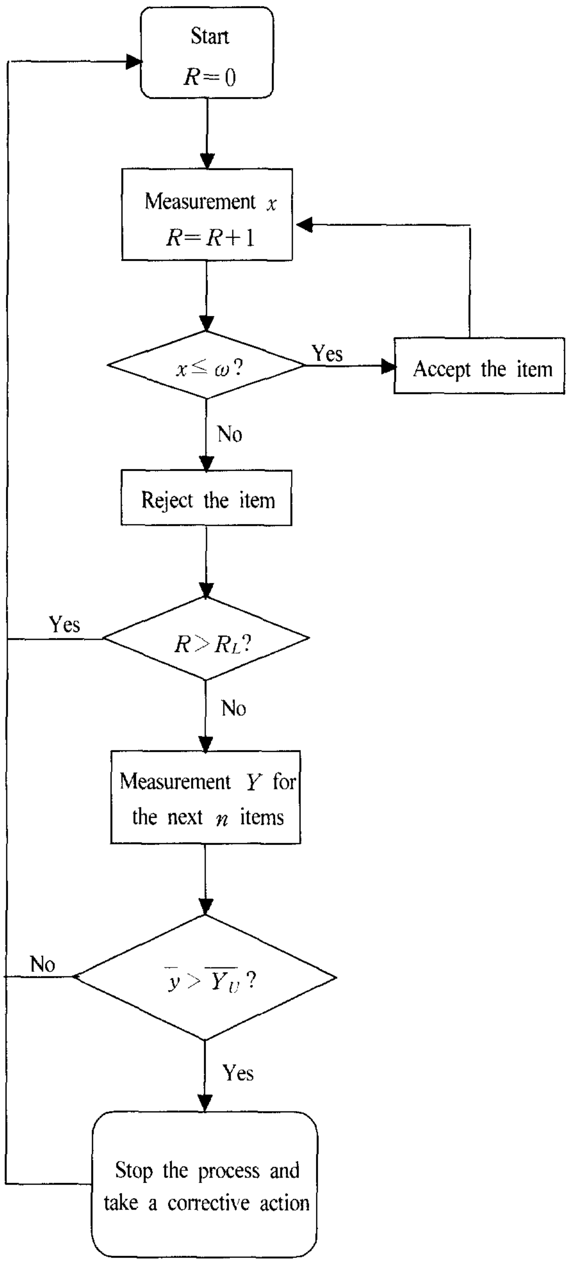

Figure 1. The Control Procedure.

3. The Design Criteria

The process control procedure illustrated in <Figure 1> will produce a series of cycles, each of which includes a start point, a stop point for correcting the process, K samples of size n for inspection with Y, $\sum_{i=1}^{K} M_i$ items rejected by screening, and $\sum_{i=1}^{K} (R_{ij} - 1)$ items accepted. Here, M_i is the number of the rejected items based on X between the $(i-1)^{th}$ and i^{th} direct inspections on Y, and R_{ij} is the number of items

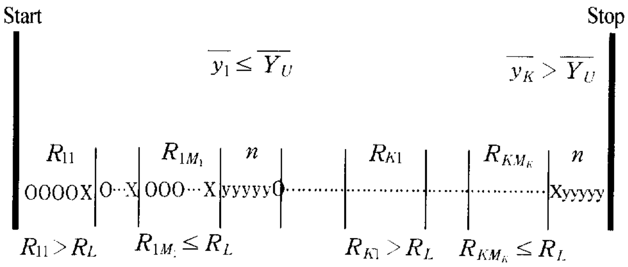

Figure 2. Cycle of the Process.

inspected with X between the $(j-1)^{th}$ and j^{th} rejected items (including the j^{th} item) after the $(i-1)^{th}$ direct inspection on Y. <Figure 2> depicts a cycle generated by the procedure.

Let T be the time elapsed from the start point to the stop point of the procedure, where the unit time is the time needed for producing and inspecting an item. Then

$$T = \sum_{i=1}^{K}\left(\sum_{j=1}^{M_i} R_{ij} + n\right) \tag{2}$$

which is a random variable since K, M_i, $i=1,2,\cdots,K$, and R_{ij}, $j=1,2,\cdots,M_i$, are random variables.

Let μ_x, μ_y and σ_x^2, σ_Y^2 be the means and variances of X and Y, respectively, and ρ be their correlation coefficient. Let μ_{x0} and μ_{y0} are the target means of X and Y, respectively, when the process is in control. Suppose that the process mean shifts from $\mu_y = \mu_{y0}$ to $\mu_y = \mu_{y0} + d\sigma_y$ when the process is out of control. For given values T_0 and $T_1\,(T_0 \gg T_1)$, we are going to determine R_L and $\overline{Y_U}$ so that

$$\begin{aligned} E[\,T \mid \mu_y = \mu_{y0}\,] &\ge T_0, \\ E[\,T \mid \mu_y = \mu_{y0} + d\sigma_y\,] &\le T_1 \end{aligned} \tag{3}$$

Note that T_0 and T_1 are the target values of T when the process is in control and out of control, respectively. If we set T_0 larger, we will have smaller α risk. If we set T_1 smaller, we will have smaller β risk.

4. The Values of the Design Parameters

4.1 The Screening Cutoff Value

The objective of screening is to assure the customer of high quality and most screening procedures are designed so that a specified quality level can be attained. Suppose that a prespecified outgoing quality δ is to be attained after screening. Then, the screening cutoff value ω should be determined so that

$$\Pr[\,Y \le U \mid X \le \omega\,] = \delta \tag{4}$$

Since the process may change over time, the distribution parameters of (X, Y) will change. Thus, there is no ω which make Equation (4) always hold. When the manufacturing process is in control, however, it should hold. We determine ω so that Equation (4) hold when the process is in control. Equation (4) is reduced to

$$\frac{\Psi(h, g; \rho)}{\Phi(h)} = \delta \tag{5}$$

where $\Psi(.,.;.)$ is the standard bivariate normal distribution function with correlation coefficient ρ, $\Phi(.)$ is the standard normal distribution function, $g = (U - \mu_{y0})/\sigma_y$, and $h = (\omega - \mu_{x0})/\sigma_x$. Once h is determined, ω can be easily obtained by $\omega = \mu_{x0} + h\sigma_x$. The solution of Equation (5) can be obtained using IMSL subroutines DNORDF and DBNRDF.

Let γ be the proportion of conforming items before screening when the process is in control. Then, since $\gamma = \Pr[\,Y \le U\,] = \Phi((U - \mu_{y0})/\sigma_y) = \Phi(g)$,

$$g = \Phi^{-1}(\gamma) \tag{6}$$

where $\Phi^{-1}(.)$ is the inverse function of the standard normal distribution function. If the values of γ, δ and ρ are given, h can be obtained using Equation (5) and (6). <Table 1> gives numerical values of h for various values of γ, δ and ρ. It shows that h takes larger value if ρ becomes larger. With larger ρ, the more information X gives on Y, which, through larger value of h, consequently relieves those items rejected due to lack of information on Y.

4.2 R_L and $\overline{Y_U}$

Suppose the process mean has shifted from $\mu_y = \mu_{y0}$ to $\mu_y = \mu_{y0} + d\sigma_y$. The screening procedure does not guarantee the outgoing quality δ any more. Since $b = \rho\,\sigma_x/\sigma_y$, the mean of X is $\mu_x = \mu_{x0} + d\rho\sigma_x$ and the actual outgoing quality is

$$\Pr[\,Y \le U \mid X \le \omega\,] = \frac{\Psi(h - d\rho, g; \rho)}{\Phi(h - d\rho)} \tag{7}$$

Table 1. Values of for given γ, δ, and ρ

		$\gamma = 0.6$	$\gamma = 0.7$	$\gamma = 0.8$
$\delta = 0.95$	$\rho = 0.90$	-0.09	0.27	0.73
	$\rho = 0.95$	0.11	0.45	0.88
$\delta = 0.975$	$\rho = 0.90$	-0.30	0.05	0.47
	$\rho = 0.95$	-0.04	0.28	0.68

Suppose that the manufacturer wants to assure the outgoing quality greater than or equal to δ_L even if the manufacturing process changes over time, that is, δ_L is the conservative level of the outgoing quality to be guaranteed. Then, any change in the process that makes the outgoing quality smaller than δ_L (out-of-control state) must be detected and corrected. We first determine the amount of change d that must be detected using the following equation :

$$\delta_L = \frac{\Psi(h - d\rho, g; \rho)}{\Phi(h - d\rho)} \quad (8)$$

Next, we must find E(T) which is a function of R_L and $\overline{Y_U}$. Let $\Pr[X \le \omega] = \pi$ and $\Pr[\overline{Y} \ge \overline{Y_U}] = \theta$. Then

$$\Pr[R_{ij} = r] = (1-\pi)^{r-1}\pi, \quad r = 1, 2, \cdots \quad (9)$$

for $i = 1, 2, \cdots, K$, $j = 1, 2, \cdots, M_i$. Since $\{K = k\} \equiv \{\overline{Y_1} \le \overline{Y_U}, \cdots, \overline{Y}_{K-1} \le \overline{Y_U}, \overline{Y_K} > \overline{Y_U}\}$, the probability function of K can be obtained by

$$\Pr[K = k] = (1-\theta)^{k-1}\theta, \quad k = 1, 2, \cdots \quad (10)$$

$E(T)$ can be obtained by taking expectation of the conditional expectation $E[T \mid K]$. But

$$E[T \mid K = k] = E\left[\sum_{i=1}^{K}\left(\sum_{j=1}^{M_i} R_{ij} + n\right) \middle| K = k\right] = kE\left[\sum_{j=1}^{M_i} R_{ij} + n\right] \quad (11)$$

and R_{ij} does not depend on K for all i and j. Since $E(K) = 1/\theta$,

$$E(T) = \frac{1}{\theta} E\left[\sum_{j=1}^{M_i} R_{ij} + n\right] \quad (12)$$

In the Appendix, we show that

$$E\left[\sum_{j=1}^{M} R_{ij} + n\right] = n + \frac{1}{\pi[1 - (1-\pi)^{R_L}]} \quad (13)$$

And we finally obtain

$$E[T] = \frac{1}{\theta}\left\{n + \frac{1}{\pi[1 - (1-\pi)^{R_L}]}\right\} \quad (14)$$

Finally, we determine R_L and $\overline{Y_U}$ so that the out-of-control state can be can be detected before a specified length of time elapses. Denote $\overline{Y_U}$ as $\overline{Y_U} = \mu_{y0} + l\sigma_y/\sqrt{n}$. Then, the design problem is equivalent to determine R_L and l such that

$$ET0 = E[T \mid \mu_y = \mu_{y0}] = \frac{1}{\theta_0}\left\{n + \frac{1}{\pi[1 - (1-\pi_0)^{R_L}]}\right\} \ge T_0,$$

$$ET1 = E[T \mid \mu_y = \mu_{y0} + d\sigma_y] = \frac{1}{\theta_1}\left\{n + \frac{1}{\pi[1 - (1-\pi_1)^{R_L}]}\right\} \le T_1 \quad (15)$$

where

$$\theta_0 = P[\overline{Y} \ge \overline{Y_U} \mid \mu = \mu_{y0}] = 1 - \Phi(l),$$

$$\theta_1 = P[\overline{Y} \ge \overline{Y_U} \mid \mu = \mu_{y0} + d\sigma_y] = 1 - \Phi(l - d\sqrt{n}),$$

$$\pi_0 = P[X \le \omega \mid \mu_x = \mu_{x0}] = \Phi(h), \text{ and}$$

$$\pi_1 = P[X \le \omega \mid \mu_x = \mu_{x0} + d\rho\sigma_x] = 1 - \Phi(h - d\rho).$$

Example. Suppose the current and target proportions of acceptable items are 0.8 and 0.95, respectively, and the time requirements are $T_0 = 600$ and $T_1 = 60$. Further, the manufacturer wishes the proportion of acceptable items to be at least 0.90 after screening even if the process mean shifts. The performance and screening variables are jointly normally distributed with correlation coefficient 0.9. Suppose that the major quality characteristic of $n = 4$ items are directly inspected and determine whether the process mean is shifted or not.

Using IMSL(1990) subroutines, we obtain $h = 0.73$, $d = 0.55$, $l = 2.37$, $R_L = 2$, and $ET0 = 604.7$, $ET1 = 59.0$. <Table 2> shows some solutions for $\rho = 0.9$, 0.95 and $\gamma = 0.6, 0.7, 0.8$. Note that, for given γ, we have smaller R_L when $\rho = 0.95$ than when $\rho = 0.90$, having less chance to measure Y directly. For the extreme case of $\rho = 1$, there is virtually no need to measure Y directly and all the inspection and process control can be perfectly performed with X.

Table 2. Solutions for and $\rho = 0.9, 0.95$ and $\gamma = 0.6, 0.7, 0.8$

		h	d	l	R_L	$ET0$	$ET1$
$\rho = 0.90$	$\gamma = 0.6$	-0.09	0.63	2.32	7	607.9	58.7
	$\gamma = 0.7$	0.27	0.61	2.35	3	613.2	56.6
	$\gamma = 0.8$	0.73	0.55	2.37	2	604.7	59.0
$\rho = 0.95$	$\gamma = 0.6$	0.11	0.73	2.31	2	605.4	57.8
	$\gamma = 0.7$	0.45	0.68	2.32	1	610.0	57.0
	$\gamma = 0.9$	0.88	0.57	2.36	1	604.3	56.7

5. Conclusion

A process control procedure is presented when 100 % inspection is performed with a surrogate variable instead of the performance variable. Assuming bivariate normal distribution, a control scheme is suggested on the basis of the time length between the start and stop points of a cycle of the procedure. Methods of finding the screening cutoff value and design parameters are provided and an example is given for illustration.

Appendix

Derivation of (13)

Note that

$$\Pr[R_{ij} = r \mid R_{ij} > R_L] = (1-\pi)^{r-R_L-1}\pi,$$
$$r = R_L+1,\ R_L+2, \cdots$$

and

$$\Pr[R_{ij} = r \mid R_{ij} \le R_L] = \{(1-\pi)^{r-1}\pi\}$$
$$/\{1-(1-\pi)^{R_L}\}.$$

Thus,

$$E[R_{ij} \mid R_{ij} > R_L] = R_L + 1/\pi,$$

$$E[R_{ij} \mid R_{ij} \le R_L] = 1/\pi - \{R_L(1-\pi)^{R_L}\}/$$
$$/\{1-(1-\pi)^{R_L}\}$$

Therefore,

$$E\left[\sum_{j=1}^{M_i} R_{ij} + n \mid M_i = m\right] = E\left[\sum_{j=1}^{m} R_{ij} + n \mid R_{i1} > R_L, \right.$$
$$\left. R_{im-1} > R_L,\ R_{im} \le R_L\right]$$
$$= n + \sum_{j=1}^{m-1} E[R_{ij} \mid R_{ij} > R_L] + E[R_{im} \mid R_{im} \le R_L]$$
$$= n + (m-1)\left(\frac{1}{\pi} + R_L\right)$$
$$+ \left\{\frac{1}{\pi} - \frac{(1-\pi)^{R_L}}{1-(1-\pi)^{R_L}} \cdot R_L\right\}$$

But

$$\Pr[M_i = m] = \{(1-\pi)^{R_L}\}^{m-1}\{1-(1-\pi)^{R_L}\},$$
$$m = 1, 2, \cdots$$

Thus,

$$E\left[\sum_{j=1}^{M_i} R_{ij} + n\right] = E\left[E\left[\sum_{j=1}^{M_i} R_{ij} + n \mid M_i\right]\right]$$
$$= \sum_{m=1}^{\infty} E\left[\sum_{j=1}^{M_i} R_{ij} + n \mid M_i = m\right] \Pr[M_i = m]$$
$$= n + \frac{1}{\pi\{1-(1-p)^{R_L}\}}$$

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