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Optimal Structural Design for Flexible Space Structure with Control System Based on LMI

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A simultaneous optimal design problem of structural and control systems is discussed by taking a 3-D truss structure as an object. We use descriptor forms for a controlled object and a generalized plant because the structural parameters appear naturally in these forms. We consider a minimum weight design problem for structural system and disturbance suppression problem for the control system. The structural objective function is the structural weight and the control objective function is H_∞ norm from the disturbance input to the controlled output in the closed-loop system. The design variables are cross sectional areas of the truss members. The conditions for the existence of controller are expressed in terms of linear matrix inequalities (LMI). By minimizing the linear sum of the normalized structural objective function and control objective function, it is possible to make optimal design by which the balance of the structural weight and the control performance is taken. We showed in this paper the validity of simultaneous optimal design of structural and control systems.

Key Words : Optimal Structural Design, Simultaneous Optimal Design, H_∞ Controller, Descriptor System Forms, 3-D Structure, LMI Approach

1. Introduction

In the field of designing flexible structures such as large space structures, they are required to have lighter weight in consideration of transportation cost. However, when the structures are made lighter, their stiffness decreases thus even a little disturbance causes serious vibration problems. Besides, generally, the internal damping of space structures is so small that once vibrations are caused, it takes long time for the amplitude to come to rest. Hence, the active controller has been considered for the vibration suppression. Therefore, to satisfy the design specifications of the structural system and control system simultaneously, it is necessary to design the structural system and the control system to be the combined system, when there is the close relationship between the systems designs like flexible space structure.

Recently, the study of validity on simultaneous optimal design of structural and control systems has attracted great attention. Iwatsubo *et al.* minimized linear sum of a quadratic evaluation function of structural weight and linear regulator for the beam model by which a continuous body is imitated (Iwatsubo et al., 1993). Salama *et al.* (1988) minimized linear sum of quadratic evaluation function of structural weight and linear regulator for 3 D.O.F beam model under the restriction of the natural frequency of closed-loop system (Salama et al., 1988). Onoda *et al.* (1987) minimized linear sum of structural weight and the control energy under the restriction of the vibration energy (Onoda et al., 1987). Rao *et al.* (1990) accomplished the combined design of

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structural weight and linear regulator by using game theory for the truss structure (Rao et al., 1990). Khot *et al.* (1988) minimized the structural weight under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure (Khot et al., 1988). Grandhi(1989) minimized the structural weight or frobenius norm under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure (Grandhi, 1989). Kajiwara *et al.* (1993) minimized the quadratic evaluation functions of structural weight and linear regulator for 3 D.O.F system model (Kajiwara et al., 1993). Moreover, Tada *et al.* (1997) minimized the objective function of the control system that shows the effect of the vibration by initial external force for 3-D truss structure (Tada et al., 1997).

Conventionally, these problems have been dealt with as an optimization problem based on state space equation form for plant description. In this paper, we use descriptor form for a controlled object and a generalized plant. The descriptor form is a natural representation of linear dynamical systems, and makes it possible to analyze a larger class of systems than state equations do(Youn, 2000 ; Sung et al., 2001 ; Obinata et al., 1996). The design method by linear matrix inequality (LMI) is useful in the control system design problem recently(Ohara and Masu, 1997). An advantage of using LMI is "LMI is solvable". That is, the variables with which LMI is satisfied can be efficiently obtained by a numerical method based on convex programming etc. Therefore, if it is possible to reduce to the LMI problem, it is becoming of an efficient analysis and the design possible as for the analysis and the design problem. It seems that the necessity for thinking about the formulation of the problem in the direction of LMI approach is high in the optimal design problem.

In this paper, we deal with the simultaneous optimal design of structural and control systems for 3-D truss structure modeled by the finite element method as design object. The structural objective function is the structural weight and the control objective function is H_∞ norm. The design variables are cross sectional areas of the truss members. The objective function of simultaneous optimal design problem is the linear sum of the normalized structural objective function and control objective function. By minimizing this objective function, it is possible to make optimal design by which the balance of structural weight and control performance is taken. We consider in this paper the validity of simultaneous optimal design of structural and control systems. A numerical example is shown in chapter 4.

2. System Formulation and H_∞ Control Problem

In this paper, a material and structural arrangement make three-dimensional truss structure that consists of n truss members not changed a design object. Generally, flexible structures modeled by the finite element method are expressed by the following equation of motion.

$$M(a)\ddot{q}+D(a)\dot{q}+K(a)q=L_1w+L_2u \quad (1)$$

Where $M(a)$, $D(a)$, $K(a)$ are the mass, the damping, and the stiffness matrices, respectively. q, w, u are the displacement, the disturbance input and the control input, and L_1, L_2 are the disturbance and control input matrices. $a=[a_1, \cdots, a_n]^T$ is a vector which consists of cross sectional areas $a_i(i=1,\cdots,n)$ of truss members which are design variables. The descriptor system form of Eq. (1) becomes

$$E\dot{x}=Ax+B_1w+B_2u \quad (2)$$

$$z=C_1x+D_{12}u \quad (3)$$

$$y=C_2x+D_{21}w \quad (4)$$

$$E\begin{bmatrix} I & 0 \\ 0 & M(a) \end{bmatrix}, A=\begin{bmatrix} 0 & I \\ -K(a) & -D(a) \end{bmatrix}$$

$$B_1=\begin{bmatrix} 0 \\ L_1 \end{bmatrix}, B_2=\begin{bmatrix} 0 \\ L_2 \end{bmatrix}, x=\begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

where x, z, y are the descriptor variables, the controlled output and the measured output. C_1, C_2 are the matrices that depend on arrangement of sensors and actuators, and $D_{12}=D_{21}=0$. In this paper, we use H_∞ control theory based on LMI, so it is not necessary to satisfy the standard H_∞ control constraints, $D_{12}^T C_1=0$, $D_{12}^T D_{12}=I$ and B_1

$D_{12}^T=0$, $D_{21}D_{21}^T=I$. Because the descriptor equation can preserve physical variables and a physical structure of the control object system, it can be said that it will be an expression as natural as the system that exists. In descriptor system form, coefficient matrices of the motion equation are represented linearly, so it is possible to say that descriptor form is more excellent than state equations for system modeling. The state equation representation of Eq. (1) becomes the following.

$$\dot{x}=A_s x+B_{1s}w+B_{2s}u$$

$$A_s=\begin{bmatrix} 0 & I \\ -M(a)^{-1}K(a) & -M(a)^{-1}D(a) \end{bmatrix}$$

$$B_{1s}=\begin{bmatrix} 0 \\ M(a)^{-1}L_1 \end{bmatrix},\ B_{2s}=\begin{bmatrix} 0 \\ M(a)^{-1}L_2 \end{bmatrix}$$

In this case, mass matrix $M(a)$ appears in the form of inverse matrix $M(a)^{-1}$. In general, the change in the parameter of the structure system appears complexly in the change of the procession of the coefficient matrices of the state equation because all elements of $M(a)^{-1}$ change even when only one of elements of $M(a)$ changes. For this case, adopting the descriptor equation form more than the case of the state equation expression can shorten the calculation processing time.

In this paper, the control system (Yim and Park, 2001) is designed with the H_∞ control to suppress the effect of the disturbance. In Fig. 1, H_∞ control problem is to find a controller K(s) such that the closed-loop system is internally stable and the following H_∞ norm condition is satisfied

$$N=\| T_{zw}(s)\|_\infty<\gamma,$$
$$\| T_{zw}(s)\|_\infty=\sup_w \sigma_{\max}(T_{zw}(j\omega)) \qquad (5)$$

where $T_{zw}(s)$ is the transfer function from the

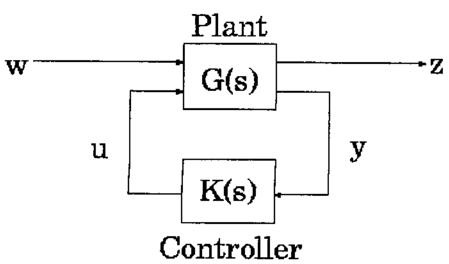


Fig. 1 H_∞ control system

disturbance input w to the controlled output z in the closed-loop system, γ is a prescribed positive number, and $\sigma_{\max}(T_{zw})$ is the maximum singular value of T_{zw}. Equation (5) which is expressed in the frequency domain is equivalent to the following equation in the time domain.

$$\int_0^\infty z^T(t)z(t)\,dt<\gamma^2\int_0^\infty w^T(t)w(t)\,dt \qquad (6)$$

It is considered from Eqs. (5) and (6) that H_∞ norm, N, denotes the degree of disturbance suppression because the right-hand side of Eq. (6) denotes the effect of disturbance.

The necessary and sufficient conditions for the existence of H_∞ controller are that there exist X and Y which satisfy the followings

$$\begin{bmatrix} E & 0 \\ 0 & E^T \end{bmatrix}\begin{bmatrix} X & I \\ I & Y \end{bmatrix}=\begin{bmatrix} X^T & I \\ I & Y^T \end{bmatrix}\begin{bmatrix} E^T & 0 \\ 0 & E \end{bmatrix}\geq 0 \quad (7)$$

$$\begin{bmatrix} B_{\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} AX+X^TA^T & B_1 & X^TC_1^T \\ B_1^T & -\gamma I & 0 \\ C_1X & 0 & -\gamma I \end{bmatrix}\begin{bmatrix} B_{\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix}<0 \quad (8)$$

$$\begin{bmatrix} C_{\frac{1}{2}}^T & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} Y^TA+A^TY & YB_1 & C_1^T \\ B_1^TY & -\gamma I & 0 \\ C_1 & 0 & -\gamma I \end{bmatrix}\begin{bmatrix} C_{\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix} \quad (9)$$

where $B_{\frac{1}{2}}$ and $C_{\frac{1}{2}}$ are

$$B_{\frac{1}{2}}=\begin{bmatrix} B_2 & B_{\frac{1}{2}}^T \end{bmatrix}=[0 \quad I],\ C_{\frac{1}{2}}^T[C_2^T \quad C_{\frac{1}{2}}]=[0 \quad I].$$

3. Simultaneous Optimal Design Problem

In this paper, we consider a minimum weight design problem for structural system and suppression problem of the effect of disturbances for control system as the purpose of the design. Taking a 3-D truss structure as a design object, the mass, damping, and stiffness matrices of the system can be modeled as the function of the cross sectional areas of the truss members by using the finite element method (Mori, 1989). The structural objective function is the structural weight W and the control objective function is N which means H_∞ norm of the transfer function from the disturbance input to the controlled output in closed-loop system as the left hand side of Eq. (5). The design variables are cross sectional areas of the truss members. The objective function in this approach is the linear sum of the normalized structural objective function and control objective function as follows

$$J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0} \tag{10}$$

where w_W and w_N are the weighting factors for structural weight and H_∞ norm, and W_0 and N_0 are the initial value of the structural weight and of H_∞ norm for normalization of objective functions. By minimizing the objective function J, it is possible to make optimum design (Yoo et al., 2000 ; Han, 2000) by which the balance of structural weight and control performance is taken.

We formulate a simultaneous optimal design problem as follows :

$$\min_a J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0} \; (w_W + w_N = 1)$$

$$\text{subject to} \begin{cases} \text{Equation. (7) is satisfied.} \\ \text{There exist } X \text{ in Eq. (8).} \\ \text{There exist } Y \text{ in Eq. (9).} \end{cases}$$

where a is the set of cross sectional areas of the truss members. The simplex method (Box et al., 1972) is used to solve the optimization problem above. We take into consideration other side constraints for design variables of structural systems (Kim et al., 2000) as

$$a^{\min} \leq a \leq a^{\max} \tag{11}$$

where $a^{\min}$ and $a^{\max}$ are lower and upper limits of the cross sectional area.

The design method using the LMI approach in this paper is described shortly as follows. First, the control system variables (X, Y) are fixed, and optimal design problem only concerning the structural variables (a) are solved by linear approximation. In the design problem above, the control system variables are not taken into consideration. Then, it is necessary to change the control system variables in a new structure. Whether the constraints of Eqs. (7) ~ (9) are examined for a new structure. The design problem newly linearized is solved when consisting. However, when the value of the objective function increases more than the value before, the move limit in successive linearization is one by one reduced. The LMI problem by which the condition of $J' < J$ is added to the constraints of Eqs. (7) ~ (9) is solved for the structure newly obtained, and the control system variables X, Y are changed when not consisting. J is a value of the objective function in the iterating calculation of the previous state. If the solution is not obtained, the move limit is reduced. And, the solution when the move limit became small enough will be considered to be the optimal solution.

4. Numerical Example

We take a 3-D truss structure shown in Fig. 2 as an object of numerical example. 1, ⋯, 10 are nodes and [1], ⋯, [12] are truss members. Considering non-dimensional form, the length of long members is 10, short members $2\sqrt{2}$, density 1.0, and Young's modulus 10^4. The nodes 5, 6, 7, 8, 9, and 10 are fixed. The sensors and actuators are located at the node 1 in x, y and z directions, then the disturbance input L_1 and the control input L_2 are decided by positioning of sensors and actuators. The sensors and actuators are located in the same directions and at the same positions, the controlled output matrix C_1 and measured output matrix C_2 are $C_1 = C_2 = [L_2^T \; 0]$. The disturbances are added continuously at the nodes 2, 3, and 4 in y direction. The damping matrix is assumed by

$$D(a) = \alpha M(a) + \beta M(a) \tag{12}$$

where $\alpha = \beta = 0.001$. Structural objective function W is calculated by

$$W = \sum_{i=1}^{12} \rho_i l_i a_i \tag{13}$$

which ρ_i, l_i and a_i are density, length and cross sectional area of truss member. Control objective function N is H_∞ norm of the transfer function from the disturbance input to the controlled output in closed-loop system, and the influence of the vibration from external disturbances can be suppressed by this value small. The solution of H_∞ optimal control problem is obtained by iterating and solving H_∞ quasi-optimal control

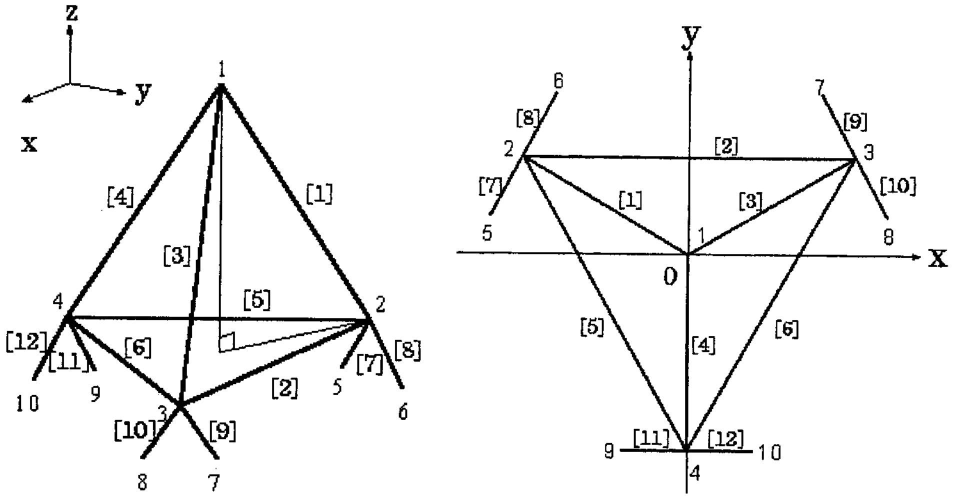


Fig. 2 3-D truss structure

problem. That is, if the controller that stabilizes the transfer function from the disturbance input to the controlled output exists, H_∞ quasi-optimal control problem has the solution enough for big γ. The solution does not exist when γ is reduced enough. Because the solution of H_∞ optimal control problem is given by that boundary, and is obtained by the method of bisection which is the numerical analysis. This is called γ-iteration (Mita, 1994). H_∞ optimal control problem is solved by composing the object system which is the function of the cross sectional area of the truss member, and using γ-iteration in the following interval,

$$0<\gamma\leq 10 \tag{14}$$

and N of control objective function is minimum value of the γ. Our purpose of the design is to find cross sectional areas of members for the minimization of these two objective functions of structural and control systems W and N.

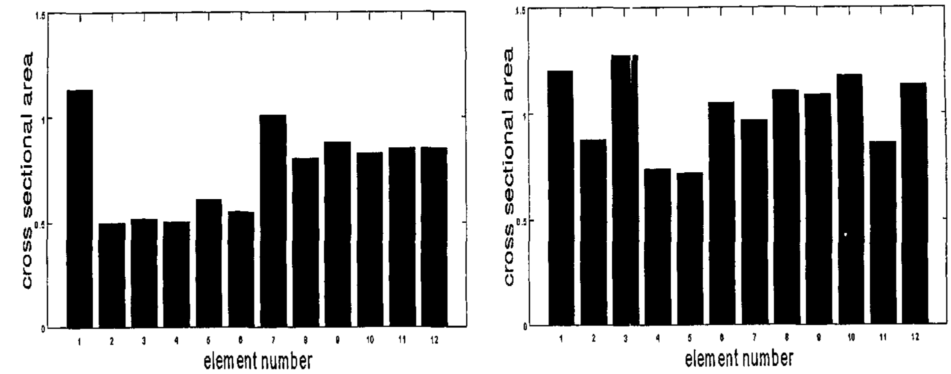
We adopt the initial structure in which all members have uniform cross sectional areas ($a_i=1$: $i=1, \cdots, 12$). In this case, structural weight W is 76.97 and the value of H_∞ norm N is 0.49. We use these structural weight and N_∞ norm as the initial values of objective functions W_0 and N_0 in Eq. (10).

First, we perform the combined optimal design in case of the set of weighting factor for the structural and control objective functions, $(w_W, w_N)=(0.5, 0.5)$, to find design variables under following condition.

$$0.5\leq a_i\leq 1.5\,(i=1, \cdots, 12) \tag{15}$$

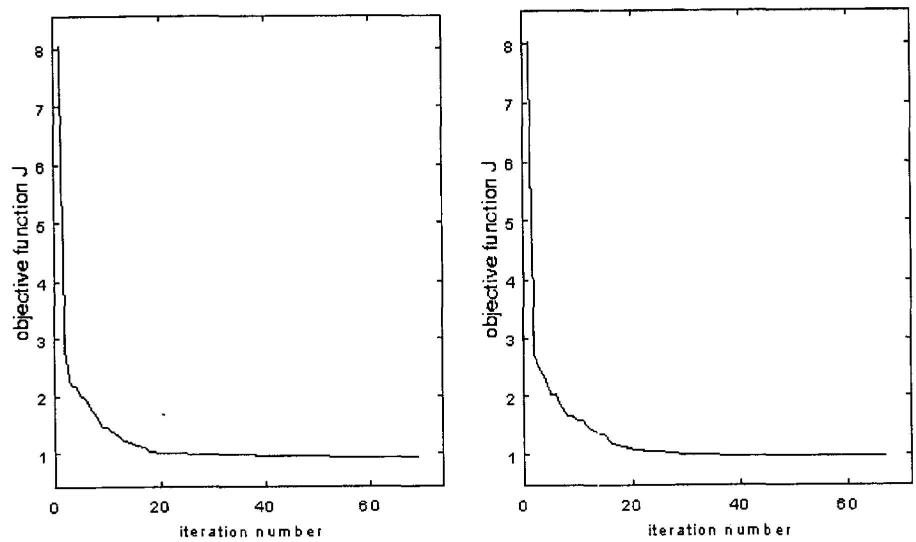
In this case, from the result by the minimization of J in Equation. (10), structural weight W is 52. 98 and H_∞ norm N is 0.54. We get lighter structural weight W than the initial weight W_0. But the value of H_∞ norm N is increased than N_0, that is, the suppression problem of the effect of disturbances gets worse than initial structure. Figure 3 (a) shows the distribution of the optimum cross sectional areas for this case (case 1).

Next, in the case of the set of weighting factor for, $(w_W, w_N)=(0.4, 0.6)$, we perform the combined optimization under the same side condition for the cross sectional area. In this case, structural weight is 76.61 and value of H_∞ norm is 0.45. W is increased than case 1. But we get both lighter structural weight W and smaller H_∞ norm N than the initial structure. We consider that this design takes the balance of structural weight and control performance. And this result also shows the effect of weighting factor for structural and control objective functions. Figure 3 (b) shows the distribution of the optimum cross sectional areas for this case (case 2). Figure 4 shows the behavior of the objective function to convergence



(a) Case 1 : $(w_W, w_N) = (0.5, 0.5)$ (b) Case 2 : $(w_W, w_N) = (0.4, 0.6)$

Fig. 3 Distribution of cross sectional area



(a) Case 1 : $(w_W, w_N) = (0.5, 0.5)$ (b) Case 2 : $(w_W, w_N) = (0.4, 0.6)$

Fig. 4 Behavior of the objective function to convergence

Next, time responses of the displacement of the direction of y of node 1 when assuming that the disturbance of y direction at nodes 2, 3, and 4 is the one like Fig. 5 are shown. Figure 6 (a) shows the time response for the initial structure, and Fig. 6 (b) shows the time response for the structure obtained when the combined optimal design is performed for weighting factor $(w_W, w_N) = (0.4, 0.6)$. When comparing time responses before and behind the combined optimal design at point to which the disturbance is input, it is understood that the influence on the displacement of the direction of at y node 1 is a little in the case of simultaneous optimal design.

Finally, we show the results of solving the same benchmark problem in Fig. 7, which compare the simultaneous optimal design result of reference by Tada and Park (2000) based on the Riccati equation that is the equality restriction condition, with the simultaneous optimal design result of this paper based on LMI that is the inequality restriction condition. In Fig. 7, Riccati means optimal design result of reference by Tada and Park (2000), and LMI means optimal design result of this paper. As for case 2 x mark in LMI, case 1 o and case 2 o mark in reference by Tada and Park (2000), it is understood that the value of structure weight W and H_∞ norm N has become small more

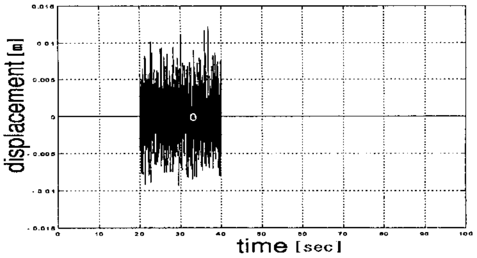
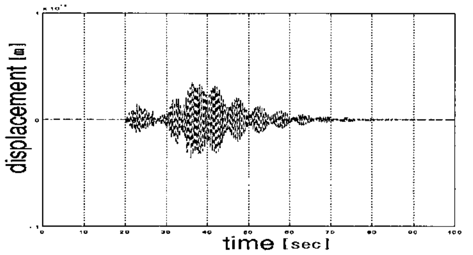
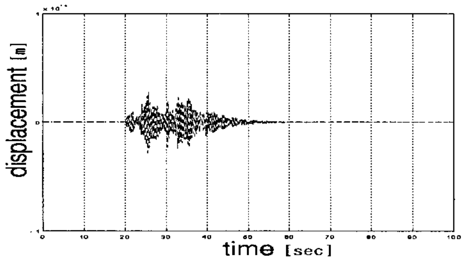


Fig. 5 Disturbance of y direction at node 2, 3, 4



(a) Uniform cross sectional areas (initial structure)



(b) Optimum cross sectional areas (case 2)

Fig. 6 Response of y direction displacement at node 1

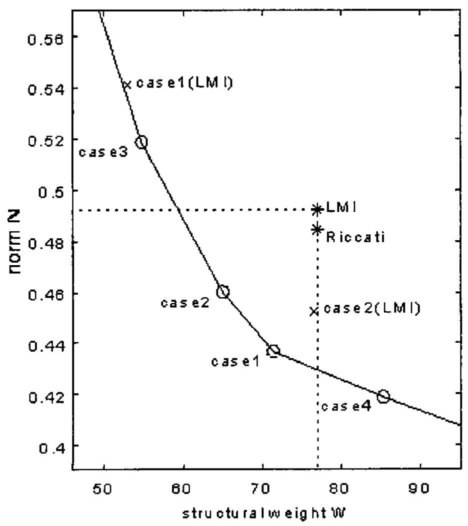


Fig. 7 Relation of W and N for Riccati and LMI

than the result of single control system design (*LMI and *Riccati: $(a_i=1 : i=1, \cdots, 12)$) in both methods.

5. Conclusion

In this paper, we formulated combined optimal design problem of structural and control systems represented by descriptor system forms and suggested a design method for three-dimensional truss structure as the design object. By our simultaneous design method, we obtained the reduction of the cost for structural system design and the improvement of the suppression for the effect of disturbances for control system compared with the design that considers only the control system. We recognized the relation of competition between two objective functions and effect of weighting factors for two objective functions. And it was possible to reduce the time of simulation by use of descriptor system forms, because inverse matrices of mass matrices are not calculated in optimization process, which contains a lot of iterating calculation processes.

In this research, the conditions of constraint described by Linear Matrix Inequality (LMI) were used in the optimal design method. This was based on the tendency to which recognition with thought of obtaining the solution in the range that the value of the objective function was specified (inequality constraint) effective had deepened recently in the use of a recent numerical analysis technique. However, it relatively takes the calculation time so much to calculate the inequality condition of constraint described by LMI compared with the equation condition of constraint described by the equation (case 1 o: 444sec ↔ case 2 x: 134939sec, case 3 o: 376sec ↔ case1 x: 127062sec in CPU time on Fig. 7). For this case, it is thought that the approach of using the descriptor equation expression by which the calculation processing time can be shortened more than the case of the state equation expression is effective.

Acknowledgements

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