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# Optimal Design for A Heteropolar Magnetic Bearing Considering Nonlinearities

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## ABSTRACT

Although the design of magnetic bearing needs a systematic optimization due to several design variables, constraints, geometric limitations, nonlinearities, and so on, the present designs for magnetic bearings have been conducted in the linear region of the characteristics for magnetic materials by trial and error considering design constraints. This article, therefore, provides the possibility of a genetic algorithm(GA) based optimization with two dimensional nonlinear finite element magnetic field analysis for the design of a radial heteropolar magnetic bearing. The magnetic bearing design by GA based optimization makes good agreements with that by a commercial optimization software DOT using the sensitivity analysis.

**Key Words** : Optimization, Genetic algorithm, Magnetic bearing

## 1. Introduction

A magnetic bearing is a electro-magnet which levitates a rotor and controls its location by changing forces generated by magnetic field. Since the non-linearity of magnetic material is included and the various performance limits and geometric constraints should be satisfied in designing the magnetic bearing, the design procedure of magnetic bearing is very complicated.

The high speed rotor system with magnetic bearings has been the subject of much research in recent years due to the potential for active vibration control(AVC), extension of life through no physical contact, no need of lubrication, wide range of temperature, auto- matic balancing even though the magnetic bearing is relatively expensive comparing to other bearings. [1]~[4]

However, since the present designs for magnetic bearings have been conducted through one-dimensional magnetic circuit analysis in the range assumed to be linear of the characteristics for magnetic materials, by trial and error considering the limits and constraints, it is therefore difficult to obtain the maximum magnetic force. In addition, although modeling errors due to linearization can be compensated in designing controllers, the complicated control algorithm may be required, if the errors are excessive.

Therefore, a systematic optimal design is needed to obtain the maximum magnetic force while satisfying the limits and constraints including the exact nonlinear magnetic force calculation. Since the typical 1st, 2nd order optimization methods using sensitivity have problems such as mathematical difficulties and local maximization,[7][8] a new optimization tech- nique is needed.

This article, therefore, provides an optimal design procedure for a heteropolar radial magnetic bearing using a genetic algorithm of zero-order overall searching method in connection with a nonlinear finite element magnetic field analysis for more accurate magnetic force calculation. Finally the magnetic bearing design by the GA based optimization are compared with that by a commercial optimization software DOT using the sensitivity analysis.

## 2. Theory

### 2.1 Genetic Algorithm

A genetic algorithm(GA) is a powerful stochastic searching method applicable to a board range of problems. The genetic algorithm is based on principles from evolution theory to achieve optimization.[5][9][10] In order to apply a GA for the constrained optimization problem, fitness is defined to include the force and the penalty values so that the constrained problem is transformed to a unconstrained problem.

### 2.2 Governing equations for an octagonal radial magnetic bearing design

A typical octagonal ($N-S-S-N-N-S-S-N$ arrangement) heteropolar magnetic bearing shown in Fig. 1 is equipped to the shaft of radius, $r_s$.

Design variables, which are the width of pole leg, $W_1$, the height of pole leg, $H_1$, and the axial length of pole leg, $B_1$ are defined as the numbers randomly generated within reasonable limits in applying GA.

The number of coil turns wrapped around each pole, $N$ is defined as

$$N = e_f \times nh_1 \times nd_1 \quad (1)$$

where the packing factor, $e_f$ is applied to be 0.7. $nh_1$ and $nd_1$, which represent the integer numbers of coils along the height of the bobbin and along the width of the bobbin, respectively, are calculated as;

$$nh_1 = \frac{H_1 - 2 \cdot t_{bb}}{2 \cdot r_w} \quad (2)$$

$$nd_1 = \frac{1}{2 \cdot r_w}\left\{(r_s + W_1 + c)\tan(\frac{\alpha}{2}) - t_{bb} - \frac{W_1}{2}\right\} \quad (3)$$

where the angle between two consecutive poles, $\alpha$ is $45^\circ$. $r_w$ and $t_{bb}$ represent the radius of wire with insulation and the thickness of the bobbin, respectively. Then, the out diameter of magnetic bearing, $OD$ is defined as

$$OD = 2\times\{r_s + (2 \cdot W_1) + c + H_1\} \quad (4)$$

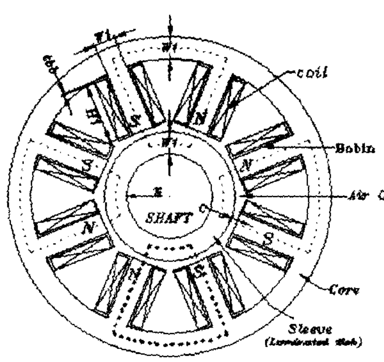



Fig. 1 A Schematics of a heteropolar magnetic bearing

The upper and lower bias current, $I_{bT}$ and $I_{bB}$, are calculated from the magnetic flux densities at the $B_T$ point and $B_B$ point in the region assumed to be linear as shown in Fig. 2.

$$I_{bT} = \frac{C_f c B_T}{\mu_0 N}, \quad I_{bB} = \frac{C_f c B_B}{\mu_0 N} \quad (5)$$

where $c$ and $C_f$ represents the nominal radial clearance and the correction factor(=1.2), respec- tively, and $\mu_o$ is the magnetic permeability of free space.

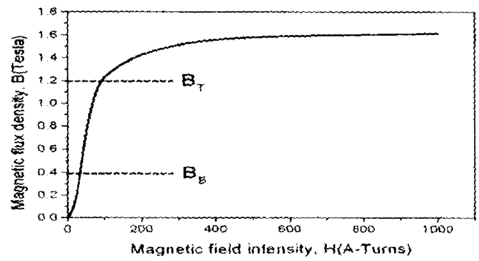



Fig. 2 The B-H curve for silicon steel

Total rotor power loss in magnetic bearing suggested by Kasarda[14], $P_T$ is occurred due to both windage effects and iron power losses which are expressed as

$$P_T = P_e + P_{ha} + P_{hr} + P_W \quad (6)$$

where $P_e$ is eddy current power loss, $P_{ha}$ is alternating hysteresis power loss per cycle for one rotor lamination stack, $P_{hr}$ is rotational hysteresis loss, and $P_w$ is the windage losses due to air-friction.

The heat generated in the coil is same to that by convection in coil surface.[15] Therefore the maximum temperature in the coil resistance, $T_s$ is

$$T_s = \frac{(I_{bT}+I_{bB})^2 R}{2hA_{coil}} + T_\infty \quad (7)$$

where $h$ is convection factor of air. $A_{coil}$ is the coil cross-section area, $R$ is the coil resistance, and $T_\infty$ is the temperature, in the atmosphere.

Since the magnetic permeability of the silicon iron, which is material of core and sleeve of magnetic bearing, is much higher than that of air, most of the reluctance of core and sleeve in electromagnet is assumed to be zero in one dimensional magnetic circuit analysis. Although modeling errors due to linearization have been compensated in designing controllers, the complicated control algorithm may be required to calculate the exact nonlinear magnetic force because it is actually nonlinear even if the characteristic of magnetic material are shown to be linear in the selected region of the B-H curve.

### 2.3 Magneitc field analysis

The solution of the magnetostatic boundary value problem(BVP) is the magnetic vector potential, $\overrightarrow{A_Z}$, which minimize the corresponding functional, $I$, by a variational approach.

$$I(A_z) = \int_\Omega \frac{1}{2\mu(B)} |\nabla A_z|^2 d\Omega - \int_\Omega A_z J_s d\Omega \quad (8)$$

where $\mu(B)$ is a nonlinear permeability of magnet, and $J_S$ is the source current density.

The nonlinear magnetic field analysis is per- formed using Plane13 (4 node) in a commercial software, ANSYS.

The magnetic force is obtained by the local Jacobian analysis method during postprocessing step from the virtual work generated by applying the virtual displacements for the nodes at sleeve around air gap.

## 3. Optimal design for the magnetic bearing using a genetic algorithm

### 3.1 Optimization problem establishment

The optimization problem established in this investigation is to find the set of design variables, which makes the magnetic force, $F_{M.F}$, calculated by the nonlinear electro- magnetic field analysis maximize while satisfying the performance limits and the geometric constraints.

The constraint conditions are to (a) restrict the outer diameter of magnetic bearing to $0.110$ m considering the total outer diameter of high-speed machine tools, (b) restrict the energy loss to 750 W at maximum rotating speed, 60,000 rpm, (c) limit the maximum temperature in the coil to 85℃ to avoid damaging the insulation film of coil, (d) wrap the wire over 5 floors at bottom of the bobbin so that one pole do not interfere the adjacent pole and the number of coil turns is assured sufficiently.

The optimization problem established in this investigation is summarized as ;

$$\begin{gathered} Maximize \quad F_{M.F.} \\ OD \le 0.110\,(m)\,, \qquad P_T \le 750\,(W) \quad (9) \\ T_S \le 85\,(℃)\,, \qquad n d_1 \ge 5 \end{gathered}$$

### 3.2 Design variables

Three design variables are selected with geometric variables, which are made up of the axial length of the magnetic bearing, $B$, the height of the pole leg, $H$, and the width of the pole leg, $W_1$ . In optimal design by GA, each design variable is given by a set of selectable values and then each value is decoded to a binary code. It is, therefore, necessary to determine the selectable values in the range of design for each variable. The axial length of pole may be determined about 35~40.5 mm due to the restriction of installing space considering the axial lengths of both shaft and built-in motor. The height of pole may be about 15~21 mm considering the limit of the outside diameter of bearing. The width of pole may be given to be around 8~14 mm because of the geometric constraint for bearing.

The design variables in this investigation are summarized as;

$$B_1\ (m) : 0.035 \le B_1 \le 0.0405$$

$$H_1\ (m) : 0.015 \le H_1 \le 0.021 \qquad (10)$$

$$W_1\ (m) : 0.008 \le W_1 \le 0.014$$

Since the limits of design variables is determined, the chromosome length having information for design variables should be determined as a number of bits. The chromosome lengths of three design variables, i.e, $B_1$, $H_1$, and $W_1$ are assigned to be 7 , 7 , 7 bits, respectively. So the total length of a chromosome is 21 bit.

### 3.3 Fitness evaluation

The optimization problem established in this article as Eq. (9) is a typical constrained optimization problem. Since genetic operators used to manipulate the chromosomes often yield infeasible offspring, the major concern for applying GA to the constrained optimization is how to handle constraints. The penalty technique is often used to handle infeasible solutions. In essence, this technique transforms the constrained problem into an unconstrained problem by penalizing infeasible solutions with constructing the evaluation function called the fitness, which is the objective function added to a penalty term for any violation of the constraints.

Therefore the optimization problem is trans- formed to an unconstrained optimization problem with defining the fitness described as;

$$Fitness = F_{M.F} - P_1 - P_2 - P_3 - P_4 \qquad (11)$$

$$P_1 = \left(\frac{OD - 0.11}{0.11}\right) \times pratio \quad \text{if} \quad OD > 0.11 \qquad (12)$$

$$P_2 = \left(\frac{P_T - 750}{750}\right) \times pratio \quad \text{if} \quad P_T > 750 \qquad (13)$$

$$P_3 = \left(\frac{T_S - 85}{85}\right) \times pratio \quad \text{if} \quad T_S > 85 \qquad (14)$$

$$P_4 = \left(\frac{5 - nd_1}{5}\right) \times pratio \quad \text{if} \quad nd_1 < 5 \qquad (15)$$

where $F_{M.F}$ is the object function and $P_i$ is the penalty term for $i$th constraint.

Pratio is a non-dimensional weighting factor added to handle infeasible solutions. In this article, pratios in all of constraints are identical and variable pratios are applied as generations proceed: 20% for 1 through 15 generation, 50% for 16 through 30 generation, 70% for 31 through 45 generation, and 80% over 46 generation.

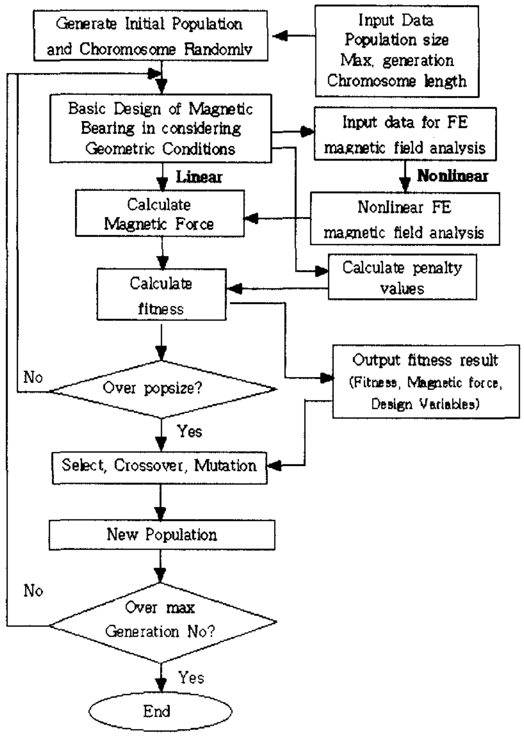



Fig. 3 Flow chart for optimal design of the heteropolar magnetic bearing with G.A.

### 3.4 Optimal design using GA

The optimal design procedure of the magnetic bearing with GA is shown in the Fig. 3. Optimal values are obtained when the fitness values are saturated, i.e. converged through iteration with zero penalty value.

## 4. RESULT AND REVIEW

### 4.1 Optimization using GA

Fig. 4 shows the optimization results using GA, which represent the biggest of fitnesses during the population loop in each generation. As the number of generation is increased, the fitness is increased to the direction of evolution. Optimal fitness by nonlinear analysis, 401.517 is obtained about 6.21% less than that by linear analysis, 428.136. The design variables searched globally in the infeasible area as well as feasible area. Fig. 4 shows convergence to magnetic force in the feasible area with zero penalty values.

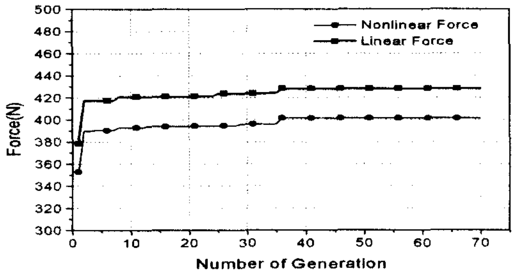



Fig. 4 Maximum fitness values in each generation

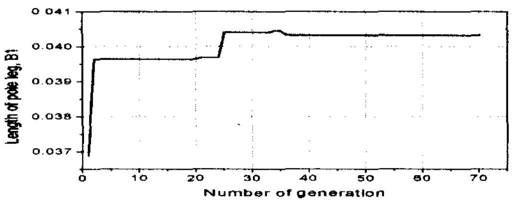
## 4.2 Optimal configuration for magnetic bearing system

As the generation increases, the design values are changed to increase the fitness. Fig. 5 shows that the given three design variables have good convergences for both linear and nonlinear optimization.

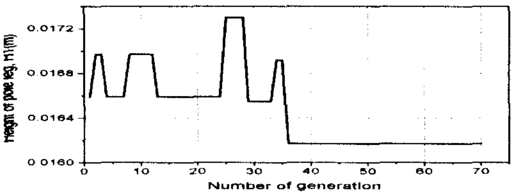
In Fig. 5, as the fitness is converged, axial length of the pole leg, $B_1$, is converged to the maximum value in its design limits because of enlarging the area of the pole for obtaining more magnetic force. The height of the pole leg, $H_1$, has a tendency to converge to the minimum value in design limit for satisfying the constraint condition of outer diameter. The width of the pole leg, $W_1$, is converged to the middle value of the design interval due to the restrictions of outer diameter and the width of bobbin.

Table 1 shows the values of the optimal design variables at the maximum fitness which can be recognized as the optimal design.

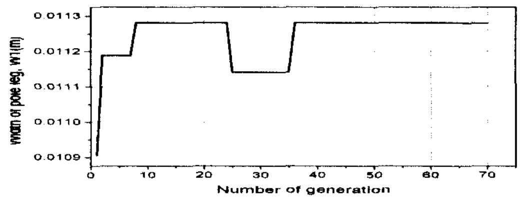
When the fitness converges to the maximum, i.e., the optimization is completed, the temperature in the coil rises to 79.5℃ the power loss for the magnetic bearing is 741W.




(a) Changing of pole length




(b) Changing of pole height




(c) Changing of pole width

Fig. 5 Optimal design value in each generation

Table 1. Optimal design values for design variables

| Design Variables | Optimal value(m) | Variables concerned Design | Final Values |
|---|---|---|---|
| Axial length of the pole leg | 0.04032813 | Number of turns | 112 |
| Height of the pole leg | 0.01617188 | Bias current for the top magnets(A) | 4.093 |
| Width of the pole leg | 0.01128125 | Bias current for the bottom magnets(A) | 1.364 |
| Final Fitness **(Maximum magnetic force, N)** | | Nonlinear analysis result : 401.517 N Linear analysis result : 428.136 N | |

## 4.3 Verification for the optimal design

A commercial optimization program DOT by Vanderplant Research & Development, Inc, (1995) using sensitivity analysis is used to compare with the optimization code using the genetic algorithm made in this article. The equation for magnetic force in DOT is derived by one dimensional linear magnetic circuit analysis. The optimizations by DOT produce the different results for various initial conditions, i.e, the phenomena of typical local minimum as shown in Table 2 and Fig. 6. Since the results for case 1 by DOT is similar to those by GA optimization, which indirectly verifies the optimization code by GA.

Table 2 Several initial values for DOT

| | B1 (m) | H1 (m) | W1 (m) |
|---|---|---|---|
| case 1 | 0.036 | 0.017 | 0.01 |
| case 2 | 0.035 | 0.015 | 0.008 |
| case 3 | 0.0405 | 0.021 | 0.014 |
| case 4 | 0.03775 | 0.018 | 0.011 |

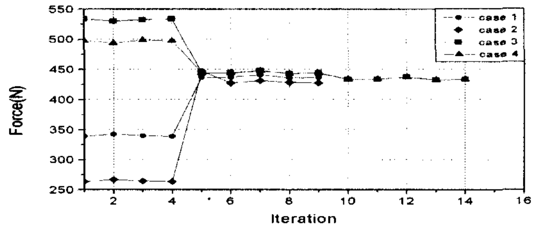



Fig. 6 DOT Optimal value at each case

## 5. Conclusion

(1) A systematic optimal design for a general eight pole heteropolar radial magnetic bearing was performed by a new optimization technique using a genetic algorithm in connection with nonlinear magnetic field analysis and was verified by comparing with the sensitivity analysis using a commercial optimal design program, DOT.

(2) Local minimum problem in optimization design with sensitivity analysis was overcome using zero-order overall searching method, GA.

(3) The design considering nonlinearity may be expected to lessen the errors in lineariza- tion.

## Acknowledgement

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## References

1. Bleuler, H., "Survey of Magnetic Levitation and Magnetic Bearins Type," J. of JSME III, Vol. 35, No. 3, pp. 335-342, 1992.
2. Ha Youngho, and Lee Jongwon, "Intelligent type Magnetic bearing," J. of KSME Vol. 34, No. 10, pp. 742-755. 1994.
3. Brunet, M., "Practical Applications of the Active Magnetic Bearings to the Industrial World," Proc. of the 1st Int. Symp. on Magnetic Bearings, pp. 225 ~ 244, 1988.
4. Dussaux, M., 1990, "Industrial Applications of the Active Magnetic Bearings Technology," Proc. of the 2nd Int. Symp. on Magnetic Bearings, pp. 33 ~ 38, 1990.
5. Bukyia Gengmung, "Genetic Algorithm, Daechung Information System, 1996.
6. Kim, C., "Magnetic Bearing Eddy Current Effects on Rotordynamic System Response," Ph.D. Thesis, Mechanical Engineering, Taxas A&M University, 1995.
7. Imlach, J., Allaire, P.E., Humphris, R.R., and Barett, L.E., "Magnetic Bearing Design Optimization," IMechE, pp. 53 ~ 59, 1988.
8. Pang, D., Kirk, J. A. An, and, D.K. and Huang, C., "Design Optimization for Magnetic Bearing," Proc. of the 26th IECEC, USA, Vol. 35, No. 4, pp. 186-191, 1991.
9. David E. Goldberg, "Genetic Algorithm in Search, Optimization, and Machine Learning," Addison-Wesley Publishing Company, Inc., 1989.
10. Zbigniew Michalewicz, "Genetic Algorithm," Books Publishing Green, 1996.
11. Hwang Sangmun, "A study on the Optimal Design of Dynamic system Using a Genetic Algorithm," Thesis, Mechanical Engineering, Changwon University, 1997.
12. Vanderplaats Research & Development, Inc., "DOT Users Manual version 4.20," 1995.
13. ANSYS, "Magnetics," User's Guide for Revision 5.0A, Swanson Analysis System, Inc., Vol. I, DN-S231, Houston, PA 15342, 1994.
14. Mary E.F. Kasarda, "The Measurement and Characterization of Power Losses in High Speed Magnetic Bearings," Ph.D. Thesis, Mechanical and

Aerospace Engineering, Virginia University, pp. 21~26, 1997.

15. Shankar Jagannathan, "Analysis and Design of Magnetic Bearings," Master Thesis, Mechanical Engineering, Virginia University, pp. 27-30, 1991.