

Robust Stability Analysis of Fuzzy Feedback Linearization Control Systems

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Abstract

In this paper, we have studied a numerical stability analysis method for the robust fuzzy feedback linearization regulator using Takagi-Sugeno fuzzy model. To analyze the robust stability, we assume that uncertainty is included in the model structure with known bounds. For these structured uncertainty, the robust stability of the closed system is analyzed by applying Linear Matrix Inequalities theory following a transformation of the closed loop systems into Lur'e systems.

Key words : Takagi-Sugeno fuzzy model, feedback linearization, linear matrix inequalities, robust control, fuzzy control

I. Introduction

Fuzzy feedback linearization is a feedback linearization method which uses a fuzzy model as a nonlinear system model. In [3], the fuzzy feedback linearization concept was introduced using Takagi-Sugeno fuzzy model. However, robustness issue which is significant in practical applications was not considered in this work. In some previous researches, adaptive techniques were applied [4]-[7]. While adaptive fuzzy feedback linearization guarantees Lyapunov stability in the presence of uncertainty, it has some practical limitations because of its complex structures.

LMI theory is the new and fast growing field and an valuable alternative to the analytical method [10], [11]. A variety of problems arising in system and control theory can be reduced to a few standard convex or quasiconvex optimization problems involving LMI. Since these resulting optimization problems can be easily solved by numerical computation, LMI techniques are very efficient and practical tools for the complex control problems. Specifically, a class of fuzzy control problems which is difficult to solve analytically, LMI techniques can afford the practical solutions. In the recent papers [12]-[16], the applicability of LMI techniques were showed excellently to fuzzy control systems.

To apply LMI techniques to our stability analysis problems, the closed system should be transformed into the standard form which has available LMI solution. In this paper, a numerical robust stability analysis for the fuzzy feedback linearization regulator is presented using Linear Matrix Inequalities (LMI) Theory. Well-known Takagi-Sugeno fuzzy model is used as the nonlinear plant model. Uncertainty is assumed to be included in the model structure with known bounds. For these structured uncertainty, the closed system can

be cast into Lur'e system by simple transformation. From the LMI stability condition for Lur'e system [17], we can derive the robust stability condition for the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model.

II. The fuzzy feedback linearization regulator based on T-S fuzzy model

2.1 Problem formulation

Consider the regulation problem of the following n -th order nonlinear SISO system

$$\dot{x}^{(n)} = f(x) + g(x)u \quad (1)$$

where f and g are unknown or uncertain, but bounded continuous nonlinear functions. Let $x = [x, \dot{x}, \dots, x^{(n-1)}]^T \in R^n$ be the state vector of the system which is assumed to be available.

In this paper, well-known Takagi-Sugeno fuzzy model is used to identify the unknown nonlinear system (1). Takagi-Sugeno fuzzy model is available in IF-THEN form (2) or Input-Output form (3).

IF-THEN form

plant rule i :

$$\text{IF } x \text{ is } M_{i1} \text{ and } \dot{x} \text{ is } M_{i2} \text{ and } \dots \text{ and } x^{(n-1)} \text{ is } M_{in}$$

$$\text{THEN } \dot{x}^{(n)} = (a_i + \Delta a_i(t))^T \cdot x + (b_i + \Delta b_i(t))u, \quad (2)$$

$$i = 1, 2, \dots, r$$

where $x = [x, \dot{x}, \dots, x^{(n-1)}]^T$,

$$a_i, \Delta a_i(t) \in R^{1 \times n}, b_i, \Delta b_i(t) \in R$$

In (2), M_{ij} is the fuzzy set and r is the number of rules. Also, $\Delta a_i(t)$ and $\Delta b_i(t)$ denotes the norm-bounded time-

varying modeling uncertainty.

• **Input-Output form**

$$x^{(n)} = \frac{\sum_{i=1}^r w_i(\mathbf{x}) \{ (\mathbf{a}_i + \Delta \mathbf{a}_i(t))^T \cdot \mathbf{x} + (b_i + \Delta b_i(t))u \}}{\sum_{i=1}^r w_i(\mathbf{x})}$$

$$= \sum_{i=1}^r h_i(\mathbf{x}) \{ (\mathbf{a}_i + \Delta \mathbf{a}_i(t))^T \cdot \mathbf{x} + (b_i + \Delta b_i(t))u \} \quad (3)$$

where

$$w_i(\mathbf{x}) = \prod_{j=1}^n M_{ij}(x^{(j-1)}), \quad h_i(\mathbf{x}) = \frac{w_i(\mathbf{x})}{\sum_{i=1}^r w_i(\mathbf{x})}$$

$M_{ij}(x^{(j-1)})$ is the grade of membership of $x^{(j-1)}$ in M_{ij} . It is assumed in this paper that

$$w_i(\mathbf{x}) \geq 0, \quad i=1,2,\dots,r, \quad \sum_{i=1}^r w_i(\mathbf{x}) > 0$$

Therefore,

$$h_i(\mathbf{x}) \geq 0, \quad i=1,2,\dots,r, \quad \sum_{i=1}^r h_i(\mathbf{x}) = 1$$

For (3) to be controllable, $\sum_{i=1}^r h_i(\mathbf{x}) b_i \neq 0$ for \mathbf{x} in certain controllability region $U_c \subset R^n$ is required. If this controllability requirement is satisfied, the following fuzzy feedback linearization regulator (4) can cancel the nonlinearity of (3) and achieve perfect linearization (5).

$$u = \frac{\widehat{\mathbf{a}}^T \cdot \mathbf{x} - \sum_{i=1}^r h_i(\mathbf{x}) \mathbf{a}_i^T \cdot \mathbf{x}}{\sum_{i=1}^r h_i(\mathbf{x}) b_i}$$

$$= \frac{\sum_{i=1}^r h_i(\mathbf{x}) (\widehat{\mathbf{a}}^T - \mathbf{a}_i^T) \cdot \mathbf{x}}{\sum_{i=1}^r h_i(\mathbf{x}) b_i} \quad (4)$$

where we use the same \mathbf{a}_i , b_i and $h_i(\mathbf{x})$ with the fuzzy model (3) for all i and $\widehat{\mathbf{a}} \in R^{1 \times n}$ is the linear state feedback gain vector. The perfectly linearized system can be written as (5).

$$x^{(n)} = \widehat{\mathbf{a}}^T \cdot \mathbf{x} \quad (5)$$

However, due to the inevitable uncertainty, perfect linearization can not be achieved in practical application. By substituting (4) into (3), the imperfectly linearized system can be written as (6). From the bounds of $\Delta \mathbf{a}_i(t)$ and $\Delta b_i(t)$, the bound of $\mathbf{a}_N(t)$ can be derived as in Appendix A. Thus, the closed system (6) can be treated as the linear system with the sector bounded nonlinearities. In the next section, the numerical robust stability analysis via LMI for the closed system (6) will be presented.

$$x^{(n)} = \widehat{\mathbf{a}}^T \cdot \mathbf{x} + \sum_{i=1}^r h_i(\mathbf{x}) \Delta \mathbf{a}_i(t)^T \cdot \mathbf{x}$$

$$+ \frac{\sum_{i=1}^r h_i(\mathbf{x}) \Delta b_i(t)}{\sum_{i=1}^r h_i(\mathbf{x}) b_i} \{ \sum_{i=1}^r h_i(\mathbf{x}) (\widehat{\mathbf{a}} - \mathbf{a}_i)^T \cdot \mathbf{x} \}$$

$$= \widehat{\mathbf{a}}^T \cdot \mathbf{x} + \mathbf{a}_N(t)^T \cdot \mathbf{x} \quad (6)$$

where

$$\mathbf{a}_N(t) = \sum_{i=1}^r h_i(\mathbf{x}) \Delta \mathbf{a}_i(t)^T \cdot \mathbf{x}$$

$$+ \frac{\sum_{i=1}^r h_i(\mathbf{x}) \Delta b_i(t)}{\sum_{i=1}^r h_i(\mathbf{x}) b_i} \{ \sum_{i=1}^r h_i(\mathbf{x}) (\widehat{\mathbf{a}} - \mathbf{a}_i)^T \cdot \mathbf{x} \}$$

2.2 Implementation

To implement the fuzzy feedback linearization regulator (4) in the fuzzy rule-based form, we utilize the control structure shown in Figure 1, which is the same one used in [8]. In this structure, the fuzzy feedback linearization regulator is divided into two blocks, a fuzzy rule-based controller block and a simple nonlinear function block. The fuzzy controller block shares the same fuzzy sets M_{ij} and the parameters \mathbf{a}_i and b_i with the fuzzy model in the premise parts for all i and j . Therefore, $\widehat{\mathbf{a}}$ is the only design parameter of the fuzzy controller block. The nonlinear function block simply divides u_1 by u_2 to produce the same u as in Eqn. (4). i -th rule of the fuzzy controller block can be represented by (7).

$$\text{IF } x \text{ is } M_{i1} \text{ and } \dot{x} \text{ is } M_{i2} \text{ and } \dots \text{ and } x^{(n-1)} \text{ is } M_{in}$$

$$\text{THEN } \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} (\widehat{\mathbf{a}} - \mathbf{a}_i)^T \cdot \mathbf{x} \\ b_i \end{bmatrix}, \quad i=1,2,\dots,r \quad (7)$$

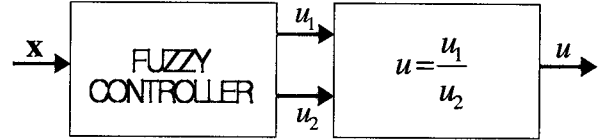


Fig. 1. Control structure of fuzzy feedback linearization regulator

III. LMI-based robust stability analysis

Consider the following Lur'e system (8)

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{p}$$

$$\mathbf{p}_i(t) = \phi_i(x_i(t)), \quad i = 1, \dots, n_p \quad (8)$$

where $\mathbf{p}(t) \in R^{n_p}$, and the functions ϕ_i satisfy the [0, 1] sector conditions

$$0 \leq \sigma \phi_i(\sigma) \leq \sigma^2 \quad \text{for all } \sigma \in R \quad (9)$$

or equivalently,

$$\phi_i(\sigma)(\phi_i(\sigma) - \sigma) \leq 0 \quad \text{for all } \sigma \in R$$

The linear system with the sector bounded nonlinearities can be cast into Lur'e system. Therefore, the closed system (6)

can be cast into Lur'e system. In Theorem 1, Lyapunov stability condition for Lur'e system is derived using LMI Theory. In the proof of Theorem 1, S-procedure in LMI techniques [10] is used.

Theorem 1 : Lur'e system (8) is stable in the sense of Lyapunov if there exist $P > 0$,

$$A = \text{diag} (\lambda_1, \dots, \lambda_{n_p}) \geq 0 \text{ and} \\ T = \text{diag} (\tau_1, \dots, \tau_{n_p}) \geq 0 \text{ which satisfy LMI (10).}$$

$$\begin{bmatrix} A^T P + P A & P B + A^T \Lambda + T \\ B^T P + \Lambda A + T & \Lambda B + B^T \Lambda - 2 T \end{bmatrix} < 0 \quad (10)$$

Proof : Let us choose a Lyapunov function

$$V(x) = x^T P x + 2 \sum_{i=1}^{n_p} \lambda_i \int_0^{x_i} \phi_i(\sigma) d\sigma \quad (11)$$

Thus the data describing the Lyapunov function are the matrix P and the scalars $\lambda_i, i = 1, \dots, n_p$. For $V(x)$ to be positive for nonzero x , we require

$$P > 0 \text{ and } A = \text{diag} (\lambda_1, \dots, \lambda_{n_p}) \geq 0.$$

The time derivative of $V(x)$ is

$$\frac{dV(x)}{dt} = 2 (x^T P + \sum_{i=1}^{n_p} \lambda_i p_i I_i) (Ax + Bp) \quad (12)$$

where I_i denotes the i -th row of $n \times n$ identity matrix.

Lyapunov stability condition $\frac{dV(x)}{dt} < 0$ holds for all nonzero x if and only if

$$(x^T P + \sum_{i=1}^{n_p} \lambda_i p_i I_i) (Ax + Bp) < 0 \quad (13)$$

for all nonzero x .

The S-procedure in LMI techniques then yields the following LMI condition

$$\begin{bmatrix} A^T P + P A & P B + A^T \Lambda + T \\ B^T P + \Lambda A + T & \Lambda B + B^T \Lambda - 2 T \end{bmatrix} < 0$$

where

$$T = \text{diag} (\tau_1, \dots, \tau_{n_p}) \geq 0, \\ A = \text{diag} (\lambda_1, \dots, \lambda_{n_p}) \geq 0.$$

Therefore, Lur'e system (8) is stable in the sense of Lyapunov if there exist $P > 0, A \geq 0$ and $T \geq 0$ which satisfy LMI (10).

Remark : When we set $A = 0$, we obtain the LMI

$$\begin{bmatrix} A^T P + P A & P B + T \\ B^T P + T & -2 T \end{bmatrix} < 0$$

which can be interpreted as a condition for the existence of a quadratic Lyapunov function $V(x) = x^T P x$ for Lur'e system.

To apply Theorem 1, the closed system (6) should be transformed into Lur'e system (8).

First, we divide (6) into the linear and the nonlinear part as (14)

$$x^{(n)} - \hat{a}^T \cdot x = a_N(t)^T \cdot x \quad (14)$$

Then, the differential equation (14) can be represented by the state-space equation of (15).

$$\dot{x} = \bar{A} x + \bar{B} p \quad (15)$$

$$\bar{p}_i(t) = \bar{\phi}_i(x_i(t)), \quad i = 1, \dots, n$$

where,

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{a}_1 & \hat{a}_2 & \hat{a}_3 & \dots & \hat{a}_n \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

and

$$\bar{\phi}_i(x_i(t)) = a_{Ni}(t)x_i(t), \quad i = 1, \dots, n$$

The sector condition of $\bar{\phi}_i$ is

$$\alpha_i \sigma^2 \leq \sigma \bar{\phi}_i(\sigma) \leq \beta_i \sigma^2, \quad i = 1, \dots, n \quad (16)$$

where $\alpha_i = \min_t (a_{Ni}(t))$ and

$$\beta_i = \max_t (a_{Ni}(t)).$$

The sector bounds α_i and β_i can be obtained by the method in Appendix.

By substituting the equations (17) into (15) and (16), the general $[\alpha_i, \beta_i]$ sector condition for $\bar{\phi}_i$ (16) can be cast into the $[0, 1]$ sector condition (9) for ϕ_i . This substitution procedure is called 'loop transformation'.

$$\bar{p}_i = \bar{\phi}_i(x_i(t)) = (\beta_i - \alpha_i) p_i + \alpha_i x_i \quad (17)$$

$$p_i(t) = \phi_i(x_i(t)), \quad i = 1, \dots, n$$

or in the matrix form.

$$\bar{p} = M p + N q$$

where $M = \text{diag} (\alpha_1, \dots, \alpha_n)$ and

$$N = \text{diag} (\beta_1 - \alpha_1, \dots, \beta_n - \alpha_n)$$

And the resulting Lur'e system of loop transformation can be expressed as (18)

$$\dot{x} = Ax + Bp \quad (18)$$

$$p_i(t) = \phi_i(x_i(t)), \quad i = 1, \dots, n_p$$

where $A = \bar{A} + \bar{B} N$ and $B = \bar{B} M$

$$\phi_i(x_i(t)) = \frac{a_{Ni}(t) - \alpha_i}{\beta_i - \alpha_i} x_i(t), \quad i = 1, \dots, n$$

Applying Theorem 1 to the transformed Lur'e system (18), we can derive the robust stability condition for the closed system (6), as in Theorem 2.

Theorem 2 : The closed system (6) is robust stable in the sense of Lyapunov if the corresponding Lur'e system (18) of the closed system (6) satisfies Theorem 1.

Proof : The closed system (6) can be transformed into the corresponding Lur'e system (18) by the above-mentioned loop transformation. Therefore, the stability of Lur'e system (18) implies the robust stability of the closed system (6). Thus, if the corresponding Lur'e system (18) of the closed system (6) satisfies Theorem 1, then we can conclude that the closed system (6) is robust stable in the sense of Lyapunov.

IV. Conclusion

In this paper, we have presented the LMI-based robust stability condition which can be solved numerically for the fuzzy feedback linearization regulator via Takagi-Sugeno fuzzy model. We implemented the fuzzy feedback linearization regulator based on Takagi-Sugeno fuzzy model and proposed the LMI-based robust stability condition.

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Appendix

In the followings, we need the basic assumption that

$$\sum_{i=1}^r h_i(\mathbf{x}(t)) = 1 \quad \text{and} \quad \max_{\mathbf{x}} h_i(\mathbf{x}(t)) = 1.$$

The maximum and minimum sector bounds of $a_{Nf}(t)$ can be computed from (25) and (26)

$$= \max_t \left\{ \frac{\sum_{i=1}^r h_i(\mathbf{x}(t)) \Delta a_{ij}(t)}{\sum_{i=1}^r h_i(\mathbf{x}(t)) b_i} \cdot \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij} \right\} \quad (25)$$

$$\min_t (a_{Nf}(t)) = \min_t \left\{ \frac{\sum_{i=1}^r h_i(\mathbf{x}(t)) \Delta a_{ij}(t)}{\sum_{i=1}^r h_i(\mathbf{x}(t)) b_i} \cdot \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij} \right\} \quad (26)$$

where $e_{ij} = \hat{a}_j - a_{ij}$

The second terms of (25) and (26) can be computed using the following property.

$$\min_i (\Delta a_{ij}(t)) \leq \sum_{i=1}^r h_i(\mathbf{x}(t)) \Delta a_{ij}(t) \leq \max_i (\Delta a_{ij}(t)) \quad (27)$$

The third terms of (25) and (26) can be computed from (28) and (29)

$$\max_t \left\{ \frac{\sum_{i=1}^r h_i(\mathbf{x}(t)) \Delta b_i(t)}{\sum_{i=1}^r h_i(\mathbf{x}(t)) b_i} \cdot \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij} \right\}$$

$$= \max \left\{ \frac{\overline{\Delta b^{nn}}}{b^p} \cdot \overline{e_j^{nn}}, \frac{\Delta b^n}{b^p} \cdot \underline{e_j^n}, \frac{\Delta b^n}{b^n} \cdot \overline{e_j^{nn}}, \frac{\Delta b^{nn}}{b^n} \cdot \underline{e_j^n}, \frac{\Delta b^{nn}}{b^p} \cdot \overline{e_j^n}, \frac{\Delta b^n}{b^p} \cdot \underline{e_j^{nn}}, \frac{\overline{\Delta b^n}}{b^n} \cdot \overline{e_j^n}, \frac{\Delta b^{nn}}{b^n} \cdot \underline{e_j^{nn}} \right\} \quad (28)$$

$$\min_t \left\{ \frac{\sum_{i=1}^r h_i(\mathbf{x}(t)) \Delta b_i(t)}{\sum_{i=1}^r h_i(\mathbf{x}(t)) b_i} \cdot \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij} \right\}$$

$$= \min \left\{ \frac{\Delta b^{nn}}{b^p} \cdot \underline{e_i^{nn}}, \frac{\Delta b^n}{b^p} \cdot \overline{e_j^n}, \frac{\overline{\Delta b^n}}{b^n} \cdot \underline{e_i^{nn}}, \frac{\Delta b^{nn}}{b^n} \cdot \overline{e_j^n}, \frac{\Delta b^{nn}}{b^n} \cdot \underline{e_j^n}, \frac{\overline{\Delta b^n}}{b^p} \cdot \underline{e_i^{nn}}, \frac{\Delta b^n}{b^n} \cdot \overline{e_j^{nn}}, \frac{\Delta b^n}{b^n} \cdot \underline{e_j^n} \right\} \quad (29)$$

where, $b_i^p = \{ b_i \mid b_i > 0 \}$, $b_i^n = \{ b_i \mid b_i < 0 \}$

$e_{ij}^{nn} = \{ e_{ij} \mid e_{ij} \geq 0 \}$, $e_{ij}^n = \{ e_{ij} \mid e_{ij} < 0 \}$

$\underline{b^p} \leq \sum_{i=1}^r h_i(\mathbf{x}(t)) b_i^p \leq \overline{b^p}$,

$\underline{b^n} \leq \sum_{i=1}^r h_i(\mathbf{x}(t)) b_i^n \leq \overline{b^n}$

$\underline{e_i^{nn}} \leq \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij}^{nn} \leq \overline{e_i^{nn}}$,

$\underline{e_j^n} \leq \sum_{i=1}^r h_i(\mathbf{x}(t)) e_{ij}^n \leq \overline{e_j^n}$

$\overline{\Delta b^{nn}} = \max_{i,t} \{ (\Delta b_i(t) \mid \Delta b_i(t) \geq 0) \}$,

$\underline{\Delta b^n} = \max_{i,t} \{ (\Delta b_i(t) \mid \Delta b_i(t) < 0) \}$

$\overline{\Delta b^{nn}} = \min_{i,t} \{ (\Delta b_i(t) \mid \Delta b_i(t) \geq 0) \}$,

$\underline{\Delta b^{nn}} = \min_{i,t} \{ (\Delta b_i(t) \mid \Delta b_i(t) < 0) \}$



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