A study on Stress Singularities for V-notched Cracks in Anisotropic and/or Pseudo-isotropic Dissimilar Materials

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ABSTRACT

V-notched crack problems can be formulated as eigenvalue problems. The problem of a v-notched crack in anisotropic and/or pseudo-isotropic dissimilar materials was formulated as an eigenvalue problem to discuss stress singularities. The eigenvalue problem was solved by the commercial numerical program; MATHEMATICA. The specific data of eigenvalues possessing the stress singularity were obtained. Stress singularities for v-notched cracks in anisotropic and/or pseudo-isotropic dissimilar materials were discussed according to the relation between wedge angle and material property. It was shown that there are three cases of eigenvalues possessing the stress singularity; one real, two real and one complex.

Keywords : Anisotropic: Dissimilar materials: V-notched cracks: Eigenvalue problem: Stress singularities

1. Introduction

By increasing the use of bonded structures, more research on fracture mechanics of the interface is required. Structures bonded with dissimilar materials always have v-notched cracks. The v-notched crack has stress singularities. The interface crack occurs from the wedge due to stress singularities.

The stress components in the vicinity of a v-notched crack are represented as $\sigma_{ij} = Kr^{\lambda-1} f_{ij}(r, \theta)$ in linear fracture mechanics. It is well known that $\lambda - 1$ is the order of stress singularity, and K is the stress intensity factor. The order of stress singularity for a crack in a homogeneous isotropic or anisotropic material is constant at -0.5. Therefore, the stress intensity factors have had a great role in the fracture criterion. The order of stress singularity for a v-notched crack in a homogeneous material or dissimilar materials is not a constant, $-0.5 \leq Re(\lambda - 1) < 0$ and also is real or complex. A new general fracture criterion for v-notched cracks including cracks is needed for evaluation of the interface strength.

Bogy[1,2] commenced research on the stress singularity problem for a v-notched crack in isotropic dissimilar materials using the Mellin transform method. His work was followed by others: Hein and Erdogan[3], Dunders and Lee[4], Dempsey and Sinclair[5,6], Ting and Chou[7], Ting[8], etc. They used several available methods for singularity analysis. Bogy[1,2], Hein and Erdogan[3] used the Mellin transform method which is elegant but complicated. Dempsey and Sinclair[5,6] used the straightforward Airy stress function method. Ting and Chou[7], Ting[8] researched stress singularities for v-notched cracks in anisotropic dissimilar materials using the Williams eigenvalue expansion approach[9].

In this paper, two-term William's' type complex stress function method is used for the stress singularity analysis of a v-notched crack in anisotropic and/or pseudo -isotropic dissimilar materials. The problem of a v-notched crack in anisotropic and/or pseudo-isotropic dissimilar materials is formulated as an eigenvalue problem. The eigenvalue problem is solved by the commercial numerical program, MATHEMATICA. Stress singularities for v-notched cracks in anisotropic and/or pseudo-isotropic dissimilar materials are discussed according to the relation between wedge angle and material property.

2. Basic equatons

The equilibrium equation set as a governing equation in a linear two-dimensional elastic body is as follows:

$$\sigma_{ij,j} + b_i = 0, \quad (i, j = x, y) \tag{1}$$

where b_i is the body force component.

Neglecting the body forces, the equilibrium equations are satisfied if the stresses are related to the stress function, $U(x, y)$ by:

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 U}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{2}$$

Hooke's law of anisotropic materials for plane stress is[11]:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} \tag{3}$$

where

$$\begin{aligned} & a_{11} = 1/E_{xx}, \quad a_{22} = 1/E_{yy}, \\ & a_{12} = -\nu_{xy}/E_{xx} = -\nu_{yx}/E_{yy} \\ & a_{21} = a_{12}, \quad a_{66} = 1/G_{xy}, \\ & a_{16} = \eta_{xy,x}/E_{xx} = \eta_{x,xy}/G_{xy}, \quad a_{16} = a_{61}, \\ & a_{26} = \eta_{xy,y}/E_{yy} = \eta_{y,xy}/G_{xy}, \quad a_{26} = a_{62} \end{aligned} \tag{4}$$

in which a_{ij} denotes the coefficient of deformation, E_{xx}, E_{yy} are Young's moduli for x, y direction, G_{xy} is the shear modulus, ν_{xy} is Poisson's ratio, $\eta_{xy,x}$ and $\eta_{xy,y}$ are called the coefficients of mutual influence of the first kind, and $\eta_{x,xy}$ and $\eta_{y,xy}$ are called the coefficients of mutual influence of the second kind. For plane strain, a^*_{ij} is given by:

$$a^*_{ij} = a_{ij} - (a_{i3}a_{j3}/a_{33}) \tag{5}$$

If the principal axes 1, 2 of a material coincide with x, y axes as shown in Fig.1, the material is orthotropic and the coefficients of deformation a_{ij} are:

$$\begin{aligned} & a_{11} = 1/E_{11}, \quad a_{22} = 1/E_{22}, \\ & a_{12} = -\nu_{12}/E_{11} = -\nu_{21}/E_{22} \\ & a_{66} = 1/G_{12}, \quad a_{16} = a_{61} = a_{26} = a_{62} = 0. \end{aligned} \tag{6}$$

The compatibility equation is:

$$\frac{\partial^2 \varepsilon_{yy}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \tag{7}$$

Substituting equations (2) and (3) into (7), the governing equation is obtained as follows[11]:

$$a_{22}\frac{\partial^4 U}{\partial x^4} - 2a_{26}\frac{\partial^4 U}{\partial x^3 \partial y} + (2a_{12} + a_{66})\frac{\partial^4 U}{\partial x^2 \partial y^2} - 2a_{16}\frac{\partial^4 U}{\partial x \partial y^3} + a_{11}\frac{\partial^4 U}{\partial y^4} = 0 \tag{8}$$

3. Stress complex function for v-notched cracks in pseudo-isotropic and anisotropic dissimilar materials

As shown in Fig. 1, area 1 is composed of an anisotropic material and area 2 is composed of a pseudo-isotropic material for a two-dimensional linear elastic problem. The stress complex function for the

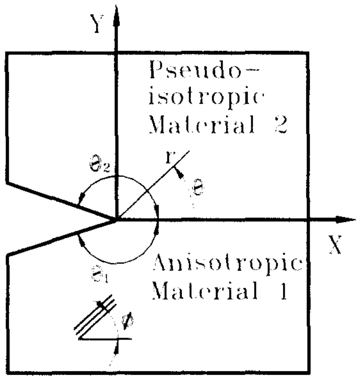

Fig.1 V-notched crack in pseudo-isotropic and anisotropic dissimilar materials

problem of v-notched cracks in the anisotropic material is assumed by two-term William's type stress function as[8]:

$$U(z) = Az^{\lambda+1} + Bz^{\bar{\lambda}+1} \quad (9)$$

where $z = x + \mu y$ and λ, μ, A and B are complex constants. The assumed root to equation (9) is substituted into differential equation (8) and the characteristic equation for μ is obtained as follows;

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0 \quad (10)$$

The characteristic equation (10) for an orthotropic material is given by equation (6) as follows;

$$a_{11}\mu^4 + (2a_{12} + a_{66})\mu^2 + a_{22} = 0 \quad (11)$$

The characteristic equations (10) and (11) can not have real roots, but have only the four following complex roots.

$$\begin{aligned} &\mu_1 = \alpha_1 + i\beta_1, \quad \mu_2 = \alpha_2 + i\beta_2, \\ &\mu_3 = \overline{\mu_1}, \quad\quad \mu_4 = \overline{\mu_2} \end{aligned} \quad (12)$$

where α_j and β_j are real, the imaginary part of μ_1 and μ_2 is positive. μ_3 and μ_4 are the conjugates of μ_1 and μ_2, respectively. Accordingly, stress complex function U(z) as the solution to differential equation (1) should be the following equation[8];

$$\begin{aligned} U(z) = 2Re\ [&A_1 z_1^{\lambda+1} + A_2 z_2^{\lambda+1} \\ &+ B_1 \bar{z}_1^{\bar{\lambda}+1} + B_2 \bar{z}_2^{\bar{\lambda}+1}] \end{aligned} \quad (13)$$

In addition, stress complex function of the pseudo-isotropic material for v-notched cracks in pseudo-isotropic and anisotropic dissimilar materials will be considered.

The root to characteristic equation (11) for a pseudo-isotropic material which is a special case of orthotropic materials should become $\mu_1 = \mu_2$ as a double root and a pure imaginary part. In order that the root of characteristic equation (11) becomes a double root, the modulus of shear elasticity of a pseudo-isotropic material is the following;

$$G_{12} = \frac{E_{11}}{2(\nu_{12} + \sqrt{E_{11}/E_{22}}\ )} \quad (14)$$

The materials become pseudo-isotropic ones if $i\beta_1 = i\beta_2$, and become isotropic ones if $\beta_1 = \beta_2 = 1$. Since the root of characteristic equation (11) is a double root if $i\beta_1 = i\beta_2$, the stress complex function of pseudo-isotropic materials must have a different form from the stress complex function of anisotropic materials. The form of stress complex function for pseudo-isotropic materials can be expressed as:

$$U(z) = 2Re\{F_1(z_1) + \bar{z}_1 F_2(z_1)\} \quad (15)$$

where

$$F_1(z_1) = A_1 z_1^{\lambda+1} + B_1 \bar{z}_1^{\bar{\lambda}+1} \quad (16)$$

$$F_2(z_1) = A_2 z_1^{\lambda} + B_2 \bar{z}_1^{\bar{\lambda}} \quad (17)$$

4. Stress and displacement fields for v-notched cracks in pseudo-isotropic and anisotropic materials.

Using equation (8), stress and displacement fields (8) of the anisotropic material for v-notched cracks in pseudo-isotropic and anisotropic materials can be obtained and is omitted in this paper. Using equation (15), the stress complex function is obtained as:

$$\sigma_{jxx} = 2Re\{\ \mu_{j1}^2 U_{j1}'' + \ \mu_{j2}^2 U_{j2}''\} \quad (18)$$

$$\sigma_{jyy} = 2Re\{\ U_{j1}'' + U_{j2}''\} \quad (19)$$

$$\sigma_{jxy} = -2Re\{\mu_{j1} U_{j1}'' + \mu_{j2} U_{j2}''\} \quad (20)$$

Substituting equations (18), (19) and (20), the strain can be obtained from the compatibility equation. Then, by integrating the strain, the displacement fields are obtained as:

$$u_{jx} = 2Re\{p_{j1} U_{j1}' + p_{j2} U_{j2}'\} - \omega_{j3} y + u_{jox} \quad (21)$$

$$u_{jy} = 2Re\{q_{j1} U_{j1}' + q_{j2} U_{j2}'\} + \omega_{j3} x + u_{joy} \quad (22)$$

where

$$p_{j1} = a_{j11}\mu_{j1}^2 + a_{j12} - a_{j16}\mu_{j1} \quad (23)$$

$$p_{j2} = a_{j11}\mu_{j2}^2 + a_{j12} - a_{j16}\mu_{j2} \quad (24)$$

$$q_{j1} = a_{j12}\mu_{j1} + \frac{a_{j22}}{\mu_{j1}} - a_{j26} \quad (25)$$

$$q_{j2} = a_{j12}\mu_{j2} + \frac{a_{j22}}{\mu_{j2}} - a_{j26} \quad (26)$$

where ' denotes differentiation and ω_{j3}, u_{jox} and u_{joy} are the arbitrary constants introducing rigid displacements of the body. This paper neglects rigid-body motion.

In the case of a v-notched crack problem, the stress and displacement components of the polar coordinate system are more convenient than the cartesian coordinate system, so that the following transformation equations are used:

$$u_i' = a_{ij}u_j \quad (27)$$

$$\sigma_{ij}' = a_{il}a_{jm}\sigma_{lm} \quad (28)$$

where a_{ij} denotes the two-dimensional transformation matrix which is given as:

$$a_{ij} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad (29)$$

5. Characteristic equation of λ for v-notched cracks in pseudo-isotropic and anisotropic materials.

The dissimilar material is composed of an anisotropic material in area 1 and a pseudo-isotropic material in area 2 as shown in Fig. 1. Boundary conditions for the dissimilar materials are as follows;

If $\theta = -\theta_1$

$$\sigma_{1\theta\theta} = 0, \quad \sigma_{1r\theta} = 0 \quad (30)$$

If $\theta = 0$

$$\sigma_{1\theta\theta} = \sigma_{2\theta\theta}, \quad \sigma_{1r\theta} = \sigma_{2r\theta} \quad (31)$$

$$u_{1r} = u_{2r}, \quad u_{1\theta} = u_{2\theta} \quad (32)$$

If $\theta = \theta_2$

$$\sigma_{2\theta\theta} = 0, \quad \sigma_{2r\theta} = 0 \quad (33)$$

Substituting stress and displacement fields for v-notched cracks in pseudo-isotropic and anisotropic materials into boundary conditions of equation (30) to (33), the following equation is given as;

$$[D(\lambda)]\{A\} = \{0\} \quad (34)$$

where

$$\{A\} = [A_{11}\ \overline{B_{11}}\ A_{12}\ \overline{B_{12}}\ A_{21}\ \overline{B_{21}}\ A_{22}\ \overline{B_{22}}]^T \quad (35)$$

$$\{0\} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T \quad (36)$$

$$[D(\lambda)] = \begin{bmatrix} s_{11} & 0 \\ s_{12} & s_{21} \\ u_{12} & u_{21} \\ 0 & s_{22} \end{bmatrix} \quad (37)$$

where

$$[s_{11}] = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \end{bmatrix}$$

$$d_{11} = \{\cos(-\theta_1)^2 + \mu_{11}^2 \sin(-\theta_1)^2 + 2\mu_{11}\sin(-\theta_1)\cos(-\theta_1)\} \times (\lambda+1)\lambda\{\cos(-\theta_1) + \mu_{11}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{12} = \{\cos(-\theta_1)^2 + \overline{\mu_{11}}^2 \sin(-\theta_1)^2 + 2\overline{\mu_{11}}\sin(-\theta_1)\cos(-\theta_1)\} \times (\lambda+1)\lambda\{\cos(-\theta_1) + \overline{\mu_{11}}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{13} = \{\cos(-\theta_1)^2 + \mu_{12}^2 \sin(-\theta_1)^2 + 2\mu_{12}\sin(-\theta_1)\cos(-\theta_1)\} \times (\lambda+1)\lambda\{\cos(-\theta_1) + \mu_{12}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{14} = \{\cos(-\theta_1)^2 + \overline{\mu_{12}}^2 \sin(-\theta_1)^2 + 2\overline{\mu_{12}}\sin(-\theta_1)\cos(-\theta_1)\} \times (\lambda+1)\lambda\{\cos(-\theta_1) + \overline{\mu_{12}}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{21} = [(1-\mu_{11}^2)\sin(-\theta_1)\cos(-\theta_1) - \mu_{11}\{\cos(-\theta_1)^2 - \sin(-\theta_1)^2\}] (\lambda+1)\lambda\{\cos(-\theta_1) + \mu_{11}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{22} = [(1-\overline{\mu_{11}}^2)\sin(-\theta_1)\cos(-\theta_1) - \overline{\mu_{11}}\{\cos(-\theta_1)^2 - \sin(-\theta_1)^2\}] (\lambda+1)\lambda\{\cos(-\theta_1) + \overline{\mu_{11}}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{23}=[(1-\mu_{12}^2)\sin(-\theta_1)\cos(-\theta_1) - \mu_{12}\{\cos(-\theta_1)^2-\sin(-\theta_1)^2\}] \times(\lambda+1)\lambda\{\cos(-\theta_1)+\mu_{12}\sin(-\theta_1)\}^{\lambda-1}$$

$$d_{24}=[(1-\overline{\mu_{12}}^2)\sin(-\theta_1)\cos(-\theta_1) - \overline{\mu_{12}}\{\cos(-\theta_1)^2-\sin(-\theta_1)^2\}] (\lambda+1)\lambda\{\cos(-\theta_1)+\overline{\mu_{12}}\sin(-\theta_1)\}^{\lambda-1}$$

$$[s_{12}] = \begin{bmatrix} (\lambda+1)\lambda & (\lambda+1)\lambda & & \\ (\lambda+1)\lambda\mu_{11} & (\lambda+1)\lambda\overline{\mu_{11}} & & \\ & & (\lambda+1)\lambda & (\lambda+1)\lambda \\ & & (\lambda+1)\lambda\mu_{12} & (\lambda+1)\lambda\overline{\mu_{12}} \end{bmatrix}$$

$$[s_{21}] = \begin{bmatrix} -(\lambda+1)\lambda & -(\lambda+1)\lambda & & \\ -(\lambda+1)\lambda\mu_{21} & -(\lambda+1)\lambda\overline{\mu_{21}} & & \\ & & -(\lambda+1)\lambda & -(\lambda+1)\lambda \\ & & -\lambda(\lambda-1)\mu_{21} & -\lambda(\lambda-1)\overline{\mu_{21}} \end{bmatrix}$$

$$[u_{12}] = \begin{bmatrix} (\lambda+1)p_{11} & (\lambda+1)\overline{p_{11}} & & \\ (\lambda+1)q_{11} & (\lambda+1)\overline{q_{11}} & & \\ & & (\lambda+1)p_{12} & (\lambda+1)\overline{p_{12}} \\ & & (\lambda+1)q_{12} & (\lambda+1)\overline{q_{12}} \end{bmatrix}$$

$$[u_{21}] = \begin{bmatrix} -(\lambda+1)p_{21} & -(\lambda+1)\overline{p_{21}} & & \\ -(\lambda+1)q_{21} & -(\lambda+1)\overline{q_{21}} & & \\ & & -(\lambda p_{21}+p_{22}) & -(\lambda\overline{p_{21}}+\overline{p_{22}}) \\ & & -(\lambda q_{21}+q_{22}) & -(\lambda\overline{q_{21}}+\overline{q_{22}}) \end{bmatrix}$$

$$[s_{22}] = \begin{bmatrix} d_{75} & d_{76} & d_{77} & d_{78} \\ d_{85} & d_{86} & d_{87} & d_{88} \end{bmatrix}$$

$$d_{75}=(\cos\theta_2{}^2+\mu_{21}{}^2\sin\theta_2{}^2+2\mu_{21}\sin\theta_2\cos\theta_2) \times(\lambda+1)\lambda(\cos\theta_2+\mu_{21}\sin\theta_2)^{\lambda-1}$$

$$d_{76}=(\cos\theta_2{}^2+\overline{\mu_{21}}^2\sin\theta_2{}^2+2\overline{\mu_{21}}\sin\theta_2\cos\theta_2) \times(\lambda+1)\lambda(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{\lambda-1}$$

$$\begin{aligned} d_{77}=[&\{2+(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &\times(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}\}\cos\theta_2{}^2 \\ &-\mu_{21}{}^2\{2-(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &\times(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}\}\sin\theta_2{}^2 \\ &+2\mu_{21}(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &\times(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}\cos\theta_2\sin\theta_2] \\ &\times\lambda(\cos\theta_2+\mu_{21}\sin\theta_2)^{\lambda-1} \end{aligned}$$

$$\begin{aligned} d_{78}=[&\{2+(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &\times(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}\}\cos\theta_2{}^2 \\ &-\overline{\mu_{21}}^2\{2-(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &\times(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}\}\sin\theta_2{}^2 \\ &+2\overline{\mu_{21}}(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}\cos\theta_2\sin\theta_2] \\ &\times\lambda(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{\lambda-1} \end{aligned}$$

$$d_{85}=\{(1-\mu_{21}{}^2)\sin\theta_2\cos\theta_2 - \mu_{21}(\cos\theta_2{}^2-\sin\theta_2{}^2)\} \times(\lambda+1)\lambda(\cos\theta_2+\mu_{21}\sin\theta_2)^{\lambda-1}$$

$$d_{86}=\{(1-\overline{\mu_{21}}^2)\sin\theta_2\cos\theta_2 - \overline{\mu_{21}}(\cos\theta_2{}^2-\sin\theta_2{}^2)\} \times(\lambda+1)\lambda(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{\lambda-1}$$

$$\begin{aligned} d_{87}=[&\{2+(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &\times(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}\}\cos\theta_2\sin\theta_2 \\ &+\mu_{21}{}^2\{2-(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}\}\cos\theta_2\sin\theta_2 \\ &-\mu_{21}(\lambda-1)(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2) \\ &\times(\cos\theta_2+\mu_{21}\sin\theta_2)^{-1}(\cos\theta_2{}^2-\sin\theta_2{}^2)] \\ &\times\lambda(\cos\theta_2+\mu_{21}\sin\theta_2)^{\lambda-1} \end{aligned}$$

$$\begin{aligned} d_{88}=[&\{2+(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &\times(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}\}\cos\theta_2\sin\theta_2 \\ &+\overline{\mu_{21}}^2\{2-(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &\times(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}\}\cos\theta_2\sin\theta_2 \\ &-\overline{\mu_{21}}(\lambda-1)(\cos\theta_2+\mu_{21}\sin\theta_2) \\ &\times(\cos\theta_2+\overline{\mu_{21}}\sin\theta_2)^{-1}(\cos\theta_2{}^2-\sin\theta_2{}^2)] \\ &\times\lambda(\cos\theta_2+\mu_{21}\sin\theta_2)^{\lambda-1} \end{aligned}$$

The condition to obtain an acceptable solution to equation (34) is $|D(\lambda)|=0$ where $|D(\lambda)|$ denotes the determinant of $|D(\lambda)|$ which is named characteristic equation and has a root to eigenvalue λ.

6. Numerical analysis and discussion

6.1 Stress singularities for v-notched cracks in pseudo-isotropic and anisotropic materials

Fig. 2 shows complex root μ of characteristic equation (11) with respect to the ratio of the elastic moduli of a pseudo-isotropic material satisfying equation (14) and the pseudo-isotropic material becomes an isotropic material in case that complex root, $\mu_1=\mu_2=i$. First, in order to demonstrate the validity of the numerical expressions for pseudo-isotropic materials, evaluated for the wedge angle are eigenvalues having stress singularity in plane stress state for v-notched cracks in an isotropic material which is a special case of pseudo-isotropic materials and seen in Table 1 and Fig. 3. It can be seen that the numerical analysis is equal to the result of Dunn et al[12]. Therefore, the vaidity of those numerical expressions can be verified.

In order to numerically analyze stress singularity in plane stress state for v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials, an orthotropic material for material 1 and a pseudo-isotropic material for material 2 in Fig. 1 are selected as;

$$\begin{aligned} &E_{111}=6.0GPa,\ E_{122}=2.5GPa,\\ &G_{112}=1.0GPa,\ \nu_{112}=0.29,\\ &E_{211}=n\times E_{111}\ E_{222}=n\times E_{122},\\ &G_{212}=E_{211}/\{2\times(\nu_{212}+\sqrt{E_{211}/E_{222}}\ )\},\\ &\nu_{212}=0.29,\qquad \textit{Plane stress.} \end{aligned} \tag{38}$$

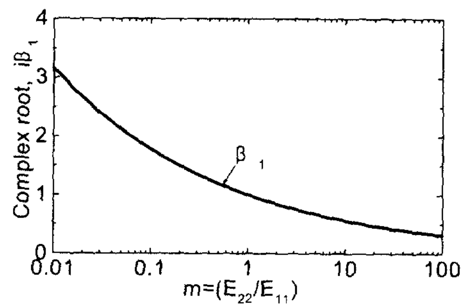

Fig. 2 Complex root for ratio of elastic moduli

Table 1 Eigenvalues for v-notched cracks in a isotropic material

Wedge angle θ 1=θ 2	λ 1	λ 2
90°	1.0	1.0
100°	0.818696	1.0
110°	0.697165	1.0
120°	0.615731	1.0
130°	0.562839	0.980475
140°	0.530396	0.84344
150°	0.512221	0.730901
160°	0.50349	0.638182
170°	0.500426	0.562007
180°	0.5	0.5

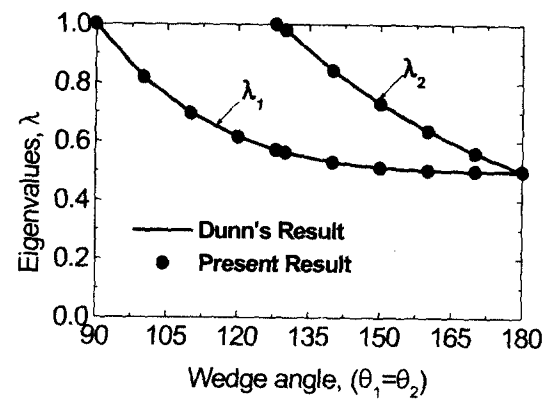

Fig. 3 Eigenvalues for v-notched cracks in an isotropic material

where n denotes the ratio of the elastic modulus of a pseudo-isotropic material(material 2) to that of an anisotropic material(material 1) and the first of subscripts expresses material 1 or 2. Since the elastic modulus of the orthotropic material is described as above, eigenvalues having stress singularity for v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials are to be analyzed with varying the elastic modulus of the anisotropic material, that is, n. Fig. 4 shows eigenvalues with stress singularity with respect to the variation of elastic modulus ratio, n, for several wedge angles between $\theta_1=\theta_2=90°$ and 115° . Real eigenvalues exist for the whole range of the elastic modulus ratio from $\theta_1=\theta_2=$ 90° to 107.835° and it is shown that stress singularity is more intense in case the elastic modulus is distant from 1 than in case that it is close to 1 while the opposite phenomenon occurs in case that the wedge angle, $\theta_1=\theta_2$ is bigger than 107.835° . As shown in Fig. 5, one real eigenvalue starts to generate two different real eigenvalues in case that the wedge angle varies from about 119° to 125° . As shown in Fig. 6, a complex eigenvalue for the wedge angle of 135° starts to be generated from two different real eigenvalues for $\theta_1=\theta_2$ more than the vicinity of 126.6° . As the wedge angle is increased gradually, the area of the two different real eigenvalues is reduced. However, it can be seen in Fig. 7 that the area of the complex eigenvalue is increased. As shown in Fig. 8, complex eigenvalues are seen at

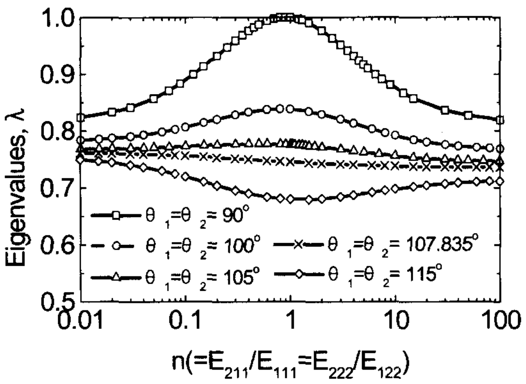

Fig. 4 Eigenvalues vs. n for a v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials

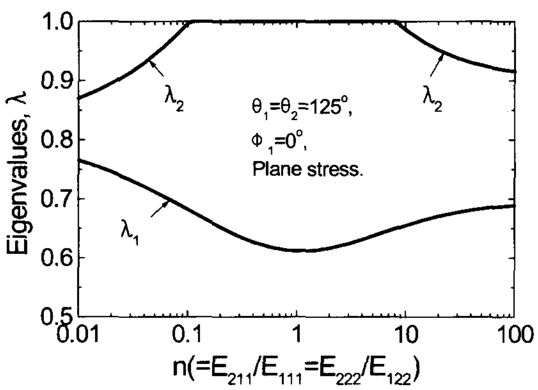

Fig. 5 Eigenvalues vs. n for a v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials

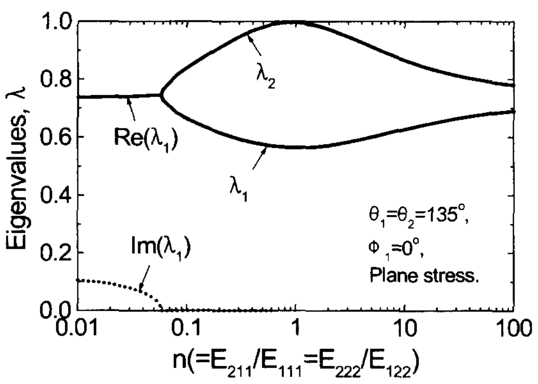

Fig. 6 Eigenvalues vs. n for a v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials

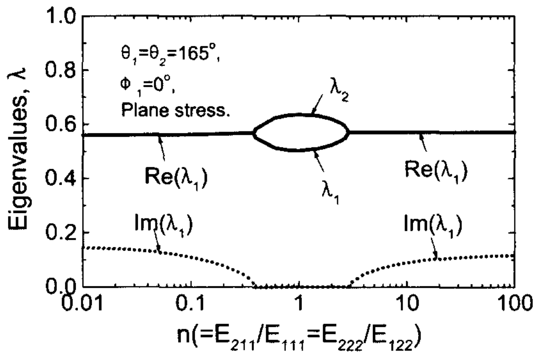

Fig. 7 Eigenvalues vs. n for a v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials

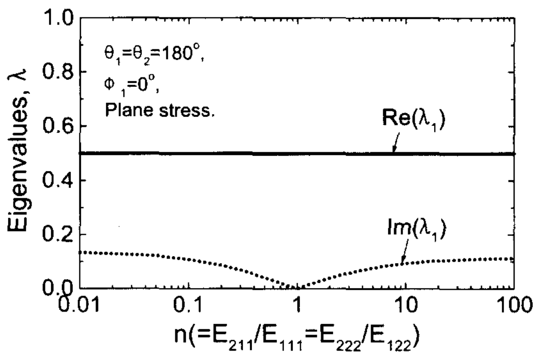

Fig. 8 Eigenvalues vs. n for a v-notched cracks in pseudo-isotropic and orthotropic dissimilar materials

180° of the wedge angle for the entire range except that n=1. Such feature tends to have a similar tendency with the analytic result[8] for v-notched cracks in anisotropic dissimilar materials.

6.2 Stress singularities for v-notched cracks in isotropic and anisotropic materials

Stress singularity for v-notched cracks in isotropic and anisotropic dissimilar materials which conform to a special case of pseudo-isotropic materials is to be analyzed. If principal axis 1 of an orthotropic material generates the rotation angle of Φ with x-coordinate as in Fig. 1, the material becomes anisotropic one according to the compatibility equation. Poisson's ratio ν_{212}=0.29 and elastic modulus E_2 is determined to be equal to elastic modulus E_{111} of an orthotropic material of equation (38) and then, the anisotropic material which is made from the orthotropic material rotated to ϕ is analyzed in plain stress state.

Fig 9. illustrates eigenvalues with stress singularity with respect to the variation of rotation angle ϕ when the wedge angle $\theta_1 = \theta_2$ is 90° , 95° , 105° , and 120° . It can be known that when the wedge is 90, eigenvalues are seen symmetric around 45° of the rotation angle, stress singularity is the most intense at 45° of the rotation angle, and stress singularity is extinguished at 0 ° and 90° of the rotation angle. When the wedge angle is increased to $\theta_1 = \theta_2 = 95$ ° , stress singularity is the most intense not at the rotation angle of 45° but of the vicinity of 43° . It is shown in Fig. 10, 11 and 12, that stress singularity gets more intense and then has two real eigenvalues as the wedge angle is increased. In the case of 170° of the wedge angle as in Fig. 13, the area having two real eigenvalues and the area having one complex area exist. For the interface crack at 180° of the wedge angle, one constant complex eigenvalue exists in the whole range as in Fig. 14.

When the difference between the elastic modulus of an anistropic material and that of an isotropic material is relatively big for the problem of v-notched cracks in isotropic and anisotropic dissimilar materials, the influences of the wedge angle and the rotation angle on stress singularity is to be examined. Fig. 15 and 16 display the analytical result of the orthotropic material of equation

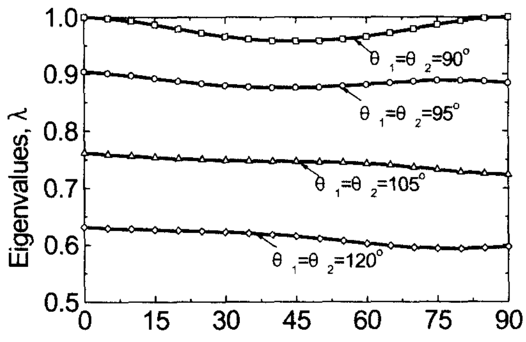

Fig. 9 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

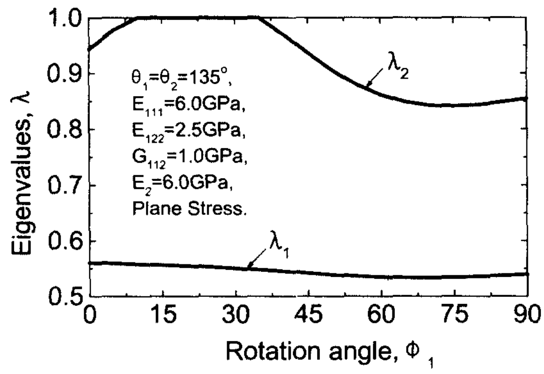

Fig. 10 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

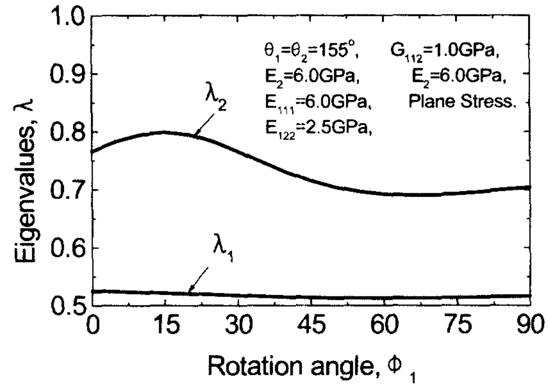

Fig. 11 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

(38) and the isotropic with E_2=6.0GPa, 0.6GPa, 0,06GPa and 0.006GPa, respectively and ν=o.29 for v-notched cracks in isotropic and anisotropic dissimilar materials.

Eigenvalues having stress singularity with respect to the variation of rotation angle ϕ at the wedge angle of $\theta_1 = \theta_2 = 90°$ are displayed in Fig. 15 and eigenvalues at the wedge angle of 115° are displayed in Fig. 16. It can be known that as the ratio of elastic moduli of the two materials in Fig. 15 and 16 becomes smaller, the width of variation of the eigenvalues with stress singularity with respect to the variation of rotation angle ϕ is increased gradually.

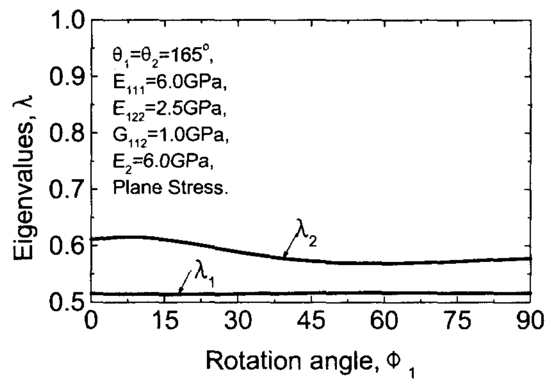

Fig. 12 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

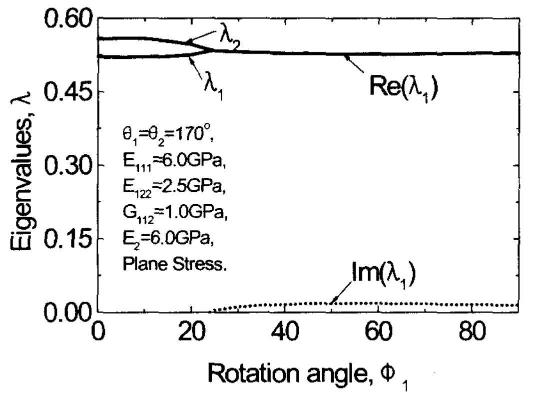

Fig. 13 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

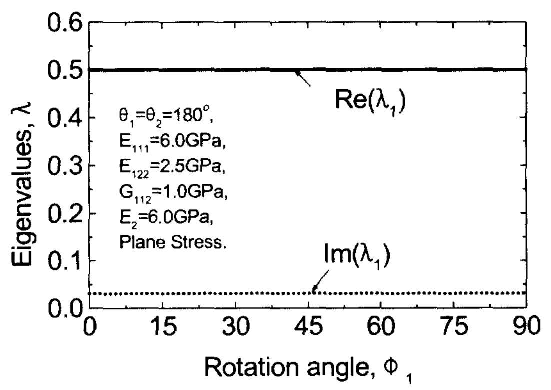

Fig. 14 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

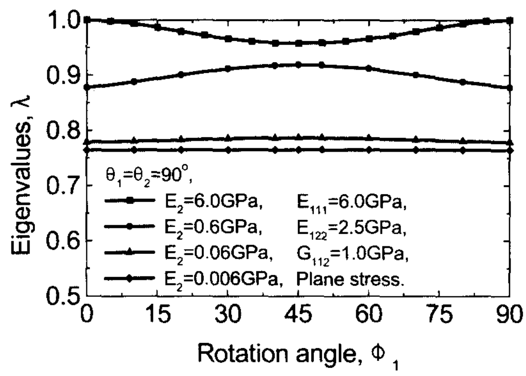

Fig. 15 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

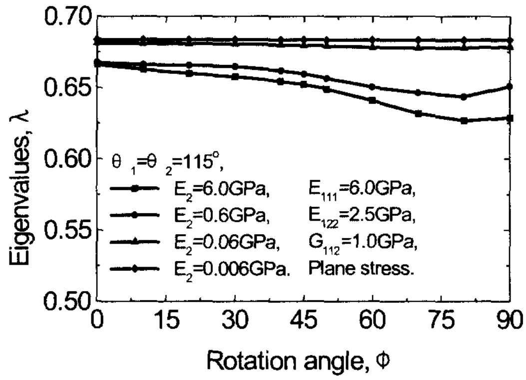

Fig. 16 Eigenvalues of v-notched cracks in isotropic and anisotropic dissimilar materials

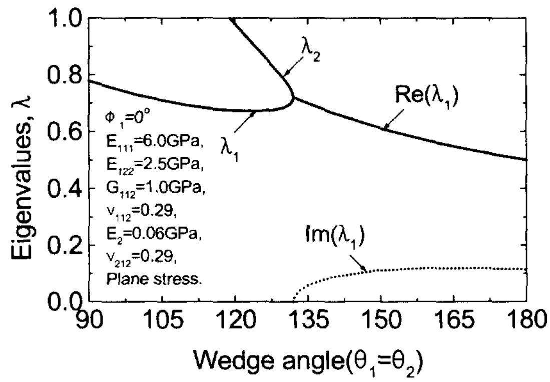

Fig. 17 Eigenvalues vs. n for a v-notched cracks in isotropic and orthotropic dissimilar materials

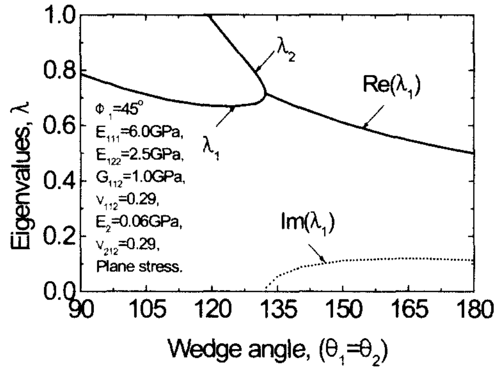

Fig. 18 Eigenvalues vs. n for a v-notched cracks in isotropic and orthotropic dissimilar materials

Fig. 17 displays the eigenvalues having stress singularity with respect to the variation of the wedge angle for the rotation angle of 0° , namely, an orthotropic material which corresponds to a special case of anisotropic ones with the anisotropic material of equation (38) and the isotropic material whose elastic modulus and Poisson's ratio are 0.06GPa and 0.29, respectively. And, Fig. 18 shows eigenvalues with stress singularity with respect to the variation of the wedge angle for the anisotropic material with the rotation angle of 45° . It can be seen from Fig. 15, 16, 17 and 18 that stress singularity is hardly affected by the rotation angle in the case of considerably big difference between elastic moduli of the two materials for the problem of v-notched cracks in isotropic and anisotropic dissimilar materials.

7. Conclusions

For the problem of v-notch in pseudo-isotropic and anisotropic dissimilar materials, the stress complex function of a pseudo-isotropic material was assumed and then, eigenvalues possessing stress singularity were numerically analyzed and examined. The conclusions are reached as follows;

1) According to the analytical result for pseudo-isotropic materials with the assumed stress complex function, it can be seen that the result is able to be acceptable for an isotropic material coinciding to a special case of pseudo-isotropic materials and equal to the traditional analytical result for v-notched cracks in isotropic materials.

2) For v-notched cracks in pseudo-isotropic and orthotropic, isotropic and anisotropic dissimilar materials, stress singularity has a similar tendency to the results for isotropic and isotropic, anisotropic and anisotropic dissimilar materials.

3) It can be seen that in case that difference between elastic moduli of the two materials for the problem of v-notched cracks in isotropic and anisotropic dissimilar materials, eigenvalues possessing stress singularity are affected by material properties and the wedge angle of the two materials regardless of the rotation angle of the anisotropic material.

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