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ON FUZZIFYING TOPOLOGICAL SPACES

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Abstract

The main aim of this paper is to study the concept of fuzzifying proximity and fuzzifying uniformity in the framework of fuzzifying topology. Some fundamental properties of them are esablished.

Key words : Fuzzifying topologies, Fuzzifying proximity and Fuzzifying uniformity

1. Introduction

Since the generaliation of the notion of an ordinary set into a fuzzy set by a Zadeh [15], many authors work for construction new branches of fuzzy mathematics. Proximities and uniformites have been studies in detalis [1, 2, 3, 4, 5, 6]. Samanta et al. [9, 10] give a new definition of fuzzy topology by introducing a concept of openness of fuzzy subsets. He introduced a new definition of fuzzy proximity [11]. In (1991) M. Ying [12] defined a fuzzifying topology on a set X as a mapping from 2^X, the family of all subsets to the closed unit interval I, satisfying the natural axioms. He also developed the fuzzifying topology in [13, 14] by using fuzzy logic and established.

2. Preliminaries.

For the sake of fixing notation, we recall some basic definitions. We shall let X be a nonempty and I be the closed unit interval and we let $I_0 = I - 0 = (0.1]$, $I_1 = I - 1 = [0.1)$. We denote the characteristic function of a subset A of 2^X by 1_A.

Definition 2.1([12]). A function $\tau : 2^X \to I$ is called a fuzzifying topology on X if it satisfies the following conditions:

(01) $\tau(X) = \tau(\emptyset) = 1$.

(02) $\tau(A \cap B) \geq \tau(A) \wedge \tau(B)$ for each $A, B \in 2^X$.

(03) $\tau(\bigcup_{i\in\Gamma} A_i) \geq \bigwedge_{i\in\Gamma} \tau(A_i)$ for any $\{A_i\}_{i\in\Gamma} \subset 2^X$.

The pair (X, τ) is called a fuzzifying topological space.

Let τ_1 and τ_2 be fuzzifying topologies on X. We say τ_1 is finer than τ_2(or τ_2 is coarser than τ_1) iff $\tau_2(A) \leq \tau_1(A)$ for all $A \in 2^X$.

Let (X, τ_1) and (Y, τ_2) be fuzifying topological spaces. A function f : $(X, \tau_1) \to (Y, \tau_2)$ is called a fuzzifying continuous map if $\tau_2(A) \leq \tau(f^{-1}(A))$ for all $A \in 2^Y$.

Let (X, τ) be a fuzzifying topoloical space. The mapping $\mathfrak{I}_\tau : 2^X \to I$ is called a fuzzifying cotopology satisfying the following properties:

(01)' $\mathfrak{I}(\emptyset) = \mathfrak{I}(X) = 1$.

(02)' $\mathfrak{I}(A \cup B) \geq \mathfrak{I}(A) \wedge \mathfrak{I}(B)$ for all $A, B \in 2^X$.

(03)' $\mathfrak{I}(\bigcap_{j\in J} A_j) \geq \bigwedge_{j\in J} \mathfrak{I}(A_j)$ for all $A_j \in 2^X$.

3. On fuzzifying proximities.

Definition 3.1. A function $\delta : 2^X \times 2^X \to I$ is called a fuzzifying proximity on X, if it satisfies the following axioms:

(FP1) $\delta(X, \emptyset) = 0$,

(FP2) $\delta(A, B) = \delta(B, A)$

(FP3) if $\delta(A, B) \neq 1$, *then* $A \subseteq B^C$,

(FP4) $\delta(A \cup B, C) = \delta(A, C) \vee \delta(B, C)$,

(FP5) For any $A, B \subseteq X$, there exists $C \subseteq X$ such that

$$\delta(A, B) \geq \inf_C \{\delta(A, C) \vee \delta(C^c, B)\}.$$

The pair (X, δ) is a fuzzifying proximity space.

Theorem 3.2. Let δ be a fuzzifying proximity on X. The mapping $C_\delta : 2^X \times I_1 \to 2^X$, is defined by

$$C_\delta(A, r) = \bigcap\{B^C \in 2^X \mid \delta(A, B) < 1 - r\}.$$

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Then it has the following properties:

(1) $C_\delta(\emptyset, r) = \emptyset$

(2) $A \subseteq C_\delta(A, r)$.

(3) If $A_1 \subseteq A_2$, then $C_\delta(A_1, r) \subseteq C_\delta(A_2, r)$.

(4) $C_\delta(A_1 \bigcup A_2, r) = C_\delta(A_1, r) \bigcup C_\delta(A_2, r)$.

(5) If $r_1 \leq r_2$, then $C_\delta(A, r_1) \subseteq C_A(\delta, r_2)$.

(6) $C_\delta(C_\delta(A, r), r) = C_\delta(A, r)$.

Proof. (1), (2), (3) and (5) are easily proved.

(4) From (3), we have

$$C_\delta(A_1 \bigcup A_2, r) \supseteq C_\delta(A_1, r) \bigcup C_\delta(A_2, r).$$

Conversely, suppose there exist $A_1, A_2 \in 2^X$ and $r \in I$ such that

$$C_\delta(A_1 \bigvee A_2, r) \subseteq C_\delta(A_1, r) \bigcup C_\delta(A_2, r).$$

There exist $x \in X$ and $t \in I_1$ such that

$$C_\delta(1_{A_1} \bigvee 1_{A_2}, r)(x) > t > C_\delta(1_{A_1}, r)(x) \bigvee C_\delta(1_{A_2}, r)(x).$$

Since $C_\delta(1_{A_i}, r)(x) < t$, for each $i \in \{1, 2\}$, there exist $1_{B_i} \in 2^X$ with $\delta(1_{B_i}, 1_{A_i}) < 1 - r$ such that

$$C_\delta(1_{A_i}, r)(x) \leq (1_{B_i^c})(x) < t.$$

On the other hand, since

$$\begin{aligned} &\delta(1_{B_1} \bigvee 1_{B_2}, 1_{A_1} \bigwedge 1_{A_2}) \\ &\leq \delta(1_{B_1} \bigvee 1_{B_2}, 1_{A_1}) \bigvee \delta(1_{B_1} \bigvee 1_{B_2}, 1_{A_2}) \\ &\leq \delta(1_{B_1}, 1_{A_1}) \bigvee \delta(1_{B_2}, 1_{A_2}) \\ &< 1 - r \end{aligned}$$

It implies

$$\begin{aligned} C_\delta(1_{A_1} \bigvee 1_{A_2}, r)(x) &\leq (1_{(B_1 \bigwedge B_2)'})(x) \\ &< t. \end{aligned}$$

It is a contradiction.

(6) Let $\delta(B, A) < 1 - r$. Then $B^c \supseteq C_\delta(A, r)$.

BY (FP5) of Definition 3.1, there exists δ such that $\delta \geq \delta \circ \delta$, where $\delta \circ \delta(A, B) = \inf_C \{\delta(A, C) \bigvee \delta(C^c, B)\}$. It follows

$$\delta(B, A) \geq \delta \circ \delta(B, A).$$

Since $\delta \circ \delta(B, A) < 1 - r$, there exists $C \in 2^X$ such that

$$1 - r > \delta \circ \delta(B, A) \geq \delta(B, C) \bigwedge \delta(C^c, A).$$

Hence

$$B^c \supseteq C_\delta(C^c, r), C^c \supseteq C_\delta(A, r).$$

Thus

$$B^c \supseteq C_\delta(C^c(A, r), r).$$

$$C_\delta(C_\delta(A, r), r) \subseteq C_\delta(A, r).$$

Theorem 3.3. Let(X, δ) be a fuzzifying proximity space. Define a map $\tau_\delta : 2^X \to I$ by

$$\tau_\delta(A) = \sup\{r \in I_1 \mid C_\delta(A^c, r) = A^c\}.$$

Then τ_δ is a fuzzifying topology on X induced by δ.

Proof. (O1) Since $C_\delta(\emptyset, r) = \emptyset$ and $C_\delta(X, r) = X$, for all $r \in I_1$, $\tau_\delta(\emptyset) = \tau_\delta(X) = 1$

(O2) Suppose there exist $A_1, A_2 \in 2^X$ and $t \in (0,1)$ such that

$$\tau_\delta(A_1 \bigwedge A_2) < t < \tau_\delta(A_1) \bigwedge \tau_\delta(A_2).$$

Since $\tau_\delta(A_1) > t$ and $\tau_\delta(A_2) > t$, there exist $r_1, r_2 > t$ such that

$$A_i^c = C_\delta(A_i^c, r_i), i = 1, 2$$

Put $r = r_1 \bigwedge r_2$. We have

$$C_\delta((A_1 \bigcap A_2)^c, r) = (A_1 \bigcap A_2)^c.$$

Consequently, $\tau_\delta(A_1 \bigwedge A_2) \geq r > t$. Hence

$$\tau_\delta(A_1 \bigcap A_2) \geq \tau_\delta(A_1) \bigwedge \tau_\delta(A_2).$$

(03) Suppose there exists a family $\{A_j \in 2^X \mid j \in \Gamma\}$ and $t \in (0, 1)$ such that

$$\tau_\delta\left(\bigcup_{j \in \Gamma} A_j\right) < t < \bigwedge_{j \in \Gamma} \tau_\delta(A_j).$$

Since $\bigwedge_{j \in \Gamma} \tau_\delta(A_j) < t$, for each $j \in \Gamma$, there exists $r_j > t$ such that

$$A_j^c = C_\delta(A_j^c, r_j).$$

Put $r = \bigwedge_{j \in \Gamma} r_j$. We have

$$C_\delta((\bigcup_{j \in \Gamma} A_j)^c, r) = (\bigcup_{j \in \Gamma} A_j)^c.$$

Consequently, $\tau_\delta(\bigvee_{j \in \Gamma} A_j) \geq r > t$. Hence

$$\tau_\delta\left(\bigvee_{j \in \Gamma} A_j\right) \geq \bigvee_{j \in \Gamma} \tau_\delta(A_j).$$

Theorem 3.4. Let (X, δ) be fuzzifying proximity space. A mapping $\mathfrak{J}_\delta : 2^X \to I$ defined by

$$\mathfrak{J}_\delta(A) = \inf_{x \in A^c} (1 - \delta(A, x))$$

is a fuzzifying cotopology on X.

Proof. (01)′ Clear.

(02)′

$$\begin{aligned}\Im_\delta(A\cup B) &= \inf_{x\in A^c\cap B^c}(1-\delta(A\cup B,x))\\ &= \inf_{x\in A^c\cap B^c}(1-\delta(A,x)\vee\delta(B,x))\\ &\geq \inf_{x\in A^c}(1-\delta(A,x))\wedge\inf_{x\in B^c}(1-\delta(B,x))\\ &= \Im_\delta(A)\wedge\Im_\delta(B)\end{aligned}$$

(03)′

$$\begin{aligned}\Im_\delta(\bigcap_j A_j) &= \inf_{x\in\bigcup A_j^c}(1-\delta\bigcap A_j,x))\\ &\geq \inf_{x\in\bigcup A_j^c}(1-\delta(A_j,x)\\ &= \wedge_j\inf_x(1-\delta(A_j,x))\\ &= \wedge_j\Im_\delta(A_j)\end{aligned}$$

4. Fuzzifying uniform spaces.

Definition 4.1. A nonzero function F : $2^{X\times X}\to I$ is caled a fuzzifying filter on $X\times X$ if it satisfying the foolwong condition:

SF1) if $A\not\in 1_\Delta$, then $F(A)=0$,

SF2) $F(A\wedge B)=F(A)\wedge F(B)$.

SF3) $F(X\times X)=1$,

Definition 4.2. A function $U:2^{X\times X}\to I$ is called a fuzzifyin uniformity on X if it satisfying for μ, $\omega\in 2^{X\times X}$, the following condition:

(FU1) U is a fuzzifying filter on $X\times X$,

(FU2) $U(u)\leq U(u^{-1})$, where $u^{-1}(x,y)=u(y,x)$

(FU3) $U(u)\leq\sup\{U(\omega)\mid\omega\circ\omega\subseteq u\}$.

The pair (X,U) is said to be a fuzzifying uniform space.

Let U_1 and U_2 be fuzzifying uniformities on X. We say U_1 is finer than U_2 (or U_2 is coarser than U_1) iff $U_2(u)\leq U_1(u)$ for all $u\in 2^{X\times X}$.

Theorem 4.3 Let(X,U) be a fuzzifying uniform space. For each $\alpha\in I_1$, let $U^\alpha=\{u\in 2^{X\times X}\mid U(u)>\alpha\}$. Then U^α is a uniformity on X.

Lemma 4.4. Let(X,U) be a fuzzifying uniform space. For each $u,u_1,u_2\in 2^{X\times X}$ and $A,A_1,A_2\in 2^X$, we have

(1) $A\subseteq u[A]$, for each $U(u)>0$,

(2) $u\subseteq u\circ u$, for each $U(u)>0$,

(3) $(v\circ u)[A]=v[u[A]]$,

(4) $(u_1\cap u_2)[A_1\cup A_2]\subseteq u_1[A_1]\cap U_2[A_2]$,

(5) if $f: X\to Y$ is a function, for each $\nu\in 2^{Y\times Y}$, we have

$$f^{-1}(\nu[f(A)]) = (f\times f)^{-1}(\nu)[A]$$

Theorem 4.5. Let (X,U) be a fuzzifying uniform spaces. The mapping $C_u: 2^X\times I_1\to 2^X$, is defined by

$$C_u(A,r)=\bigcap\{u[A]\mid U(u)>r\}.$$

For each $A,A_1,A_2\in 2^X$ and $r,r_1,r_2\in I_1$, we have the following properties:

(1) $C_U(\emptyset,r)=\emptyset$

(2) $A\subseteq C_U(A,r)$,

(3) if $A_1\subseteq A_2$, then $C_U(A_1,r)\subseteq C_U(A_2,r)$,

(4) $C_U(A_1\cup A_2,r)=C_U(A_1,r)\cup C_U(A_2,r)$,

(5) if $r_1\leq r_2$, then $C_U(A,r_1)\subseteq C_U(A,r_2)$,

(6) $C_U(C_U(A,r),r)=C_U(A,r)$.

Poof. (1) Since $u[\emptyset]=\emptyset$, $C_U(\emptyset,r)=\emptyset$

(2) For $U(u)>0$, by Lemma 4.4(1), $A\subseteq u[A]$ implies $A\subseteq C_U(A,r)$.

(3) and (5) are easily proed.

(4) From (3), we have

$$C_U(A_1\cup A_2,r)\subseteq C_U(A_1,r)\cup C_U(A_2,r).$$

Conversely, suppose there exist $A_1,A_2\in 2^X$ and $r\in I$ such that

$$C_U(A_1\cup A_2,r)\not\subseteq C_U(A_1,r)\vee C_U(A_2,r).$$

There exist $x\in X$ and $t\in I_1$ such that

$$C_U(1_{A1}\vee 1_{A2},r)(x)>C_U(1_{A1},r)(x)\vee C_U(1_{A2},r)(x).$$

Since $C_U(1_{A_i},r)(x)<t$, for each $i\in\{1,2\}$, there exist $u_i\in 2^{X\times X}$ with $U(u_i)>r$ such that

$$C_U(A_i,r)(x)\leq u_i[A_i](x)<t.$$

On the other hand, since $U(u_1\cap u_2)>r$ and from Lemma 4.4(4),

$$(u_1\cap u_2)[A_1\cup A_2]\subseteq u_1[A_i]\cap u_2[A_2],$$

we have

$$\begin{aligned}C_U(A_1\vee A_2,r)(x) &\leq (u_1\wedge u_2)[A_1\vee A_2](x)\\ &\leq u_1[A_1](x)\wedge u_2[A_2](x)\\ &< t.\end{aligned}$$

It is a contradiction.

(6) Suppose there exist $A\in 2^X$ and $r\in I_1$ such that

$$C_U(C_U(A,r),r)\not\subseteq C_U(A,r).$$

There exist $x\in X$ and $t\in I$ such that

$$C_U(C_U(1_A, r), r)(x) > t > C_U(1_A, r)(x).$$

Since $C_U(1_A, r) < t$, there exists $u \in 2^{X \times X}$ with $U(u) > r$ such that

$$C_U(1_A, r)(x) \le u[A](x) < t.$$

On the other hand, since $U(u) > r$, by (FU3), there exists $u_1 \in 2^{X \times X}$ such that

$$u_1 \circ u_1 \subseteq u, U(u_1) > r.$$

Since $C_u(A, r) \subseteq u_1[A]$, we have

$$\begin{aligned} C_u(C_u(A, r), r) &\subseteq C_u(u_1[A], r) \\ &\subseteq u_1[u_1[A]] \\ &= (u_1 \circ u_1)[A] \quad \text{(by Lemma 4.4(3))} \\ &\subseteq u[A]. \end{aligned}$$

Thus, $C_u(C_u(A, r), r)(x) \le u[A](x) < t$. It is a contradiction.

Theorem 4.6. Let (X, U) be a fuzzifying uniform space. Define a map $\tau u : 2^X \to I$ by

$$\tau_u(A) = \sup\{r \in I_1 \mid C_u(A^c, r) = A^c\}.$$

Then τ_u is fzifying topology on X induced by U.

Proof. (01) Since $C_u(\emptyset, r) = \emptyset$ and $C_u(X, r) = X$, for all $r \in I_1$, $\tau_u(X) = \tau_u(\emptyset) = 1$.

(02) Suppose there exist $A_1, A_2 \in 2^X$ and $t \in (0,1)$ such that

$$\tau_u(A_1 \wedge A_2) < t < \tau_u(A_1) \wedge \tau_u(A_2).$$

Since $\tau_u(A_1) > t$ and $\tau_u(A_2) > t$, there exist $r_1, r_2 > t$ such that

$$A_i^c = C_u(A_i^c, r_i), i = 1, 2.$$

Put $r = r_1 \wedge r_2$. We have

$$C_u((A_1 \wedge A_2)^c, r) = (A_1 \wedge A_2)^c$$

Consequently, $\tau_u(A_1 \wedge A_2) \ge r > t$. Hence

$$\tau_u(A_1 \wedge A_2) \ge \tau_u(A_1) \wedge \tau_u(A_2).$$

(03) Suppose here exists a family $\{A_j \in 2^X \mid j \in \Gamma\}$ and $t \in (0,1)$ such that

$$\tau_u\Big(\bigvee_{j \in \Gamma} A_j\Big) < t < \bigwedge_{j \in \Gamma} \tau_u(A_j).$$

Since $\bigwedge_{j \in \Gamma} \tau_u(A_j) > t$, for each $j \in \Gamma$, there existes $r_j > t$ such that .

$$A_j^c = C_u(A_j^c, r_j).$$

Put $r = \bigwedge_{j \in \Gamma} r_j$. We have

$$C_U\Big(\Big(\bigvee_{j \in \Gamma} A_j\Big)^c, r\Big) = \Big(\bigvee_{j \in \Gamma} A_j\Big)^C$$

Consequently, $\tau_u(\bigvee_{j \in \Gamma}) \ge r > t$. Hence

$$\tau_u\Big(\bigvee_{j \in \Gamma} A_j\Big) \ge \bigwedge_{j \in \Gamma} \tau_u(A_j).$$

Definition 4.7. Let (X, U) and (Y, V) be fuzzifing uniform spaces. A function $f : (X, U) \to (Y, V)$ is said to be fuzzifying uniform continuous if

$$V(v) \le U((f \times f)^{-1}(v)), \forall v \in 2^{Y \times Y}.$$

Theorem 4.8. Let $(X, U), (Y, V)$ and (Z, W) be fuzzifing spaces. If $f : (X, U) \to (Y, V)$ and $g : (Y, V) \to (Z, W)$ are fuzzifying uniform continuous, then $g \circ f : (X, U) \to (Z, W)$ is fuzzifying uniform continuous.

Proof. It follows that, for each $\omega \in 2^{Z \times Z}$,

$$\begin{aligned} W((g \circ f) \times (g \circ f))^{-1}(\omega)) &= W(((g \times g) \circ (f \times f))^{-1}(\omega)) \\ &= W((f \times f)^{-1}((g \times g)^{-1}(\omega))) \\ &\ge V((f \times f)^{-1}(\omega)) \\ &\ge U(\omega). \end{aligned}$$

Theorem 4.9. Let (X, U) and (Y, V) be fuzzifying uniform spaces. Let $f : (X, U) \to (Y, V)$ be fuzzifying uniform continuous. Then:

(1) $f(C_u(A, r)) \le C_v(f(A), r)$,

(2) $C_u(f^{-1}(B), r) \le f^{-1}(C_v(B, r))$,

(3) $f : (X, \tau_u) \to (Y, \tau_v)$ is fuzzifying continuous.

Proof. (1) Suppose tere exist $1_A \in 2^X$ and $r \in I_1$ such that

$$f(C_U(1_A, r)) \subseteq C_v(f(1_A), r).$$

There exist $y \in Y$ and $t \in I_0$ such that

$$f(C_U(1_A, r))(y) > t > C_v(f(1_A), r)(y).$$

Since $f^{-1}(\{y\}) = \emptyset$, provides a contradiction that $f(C_U(1_A, r))(y) = 0$, $f^{-1}(\{y\}) = \emptyset$, and there existis $x \in f^{-1}(\{y\})$ such that

$$f(C_U(1_A, r))(y) \ge C_U(1_A, r)(x) > t > C_v(f(1_A, r)(f(x)).$$

Since $C_v(f(1_A), r)(f(x)) < t$, there exists $v \in 2^{Y \times Y}$ with $V(v) > r$ such that

$$C_v(f(1_A), r)(f(x)) \le v[f(1_A)](f(x)) < t.$$

On the other hand, since f is fuzzifying uniform continuous,

$$u((f \times f)^{-1}(\nu)) \ge v(\nu) > r$$

It implies

$$\nu[f(1_A)](f(x)) = (f\times f)^{-}(\nu)[1_A](x) \text{ (by Lemma 4.4(5))}$$
$$=\sup_{z\in X}\{1_A(z)\wedge(f\times f)^{-1}(\nu)(z,x)\}$$
$$\geq Cu(1_A, r)(x)$$

Thus, $Cu(1_A, r)(x) < t$. It is a contradiction.

(2) For each $B\in 2^Y$ and $r\in I_1$, put $A=f^{-1}(B)$. From (1),

$$f(Cu(f^{-1}(B), r))\subseteq C\nu(f(f^{-1}(B)), r)\subseteq C\nu(B, r),$$

It implies

$$Cu(f^{-1}(B), r)\subseteq f^{-1}(f(Cu(f^{-1})))\subseteq f^{-1}(C\nu(B, r)).$$

(3) From (2), $C\mu(B, r)=B$ implies $Cu(f^{-1}(B), r)= f^{-1}(B)$. It is easily proved.

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