■ 論 文 ■

A Variational Inequality Model of Traffic Assignment By Considering Directional Delays Without Network Expansion

네트웍의 확장없이 방향별 지체를 고려하는 통행배정모형의 개발

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Key Words : Variational Inequality, User Optimal Traffic Assignment Model, Directional Delays, Diagonalization Algorithm, Link-Based Vine Shortest path Algorithm

요 약

네트웍확장은 평형통행배정모형에서 교차로의 지체와 같이 방향별로 발생하는 교차로의 움직임을 고려하기위한 필수불가결한 방법으로 사용되어져왔다. 그러나 이 방법은 교차로에서 발생하는 가능한 방향별움직임을 가상링크를 추가하여 표현함으로 네트웍의 복잡성이 증가하고 계산노력이 많이 요구되어진다. 본 연구에서는 이러한 방향별지체와 이에 관련된 움직임을 네트웍의 구조를 확장함이 없이 이용가능한 사용자최적통행배정모형을 새로운 변동부등식를 통해 제안한다. 제안된 식에 회전지체함수가 직접적으로 내재되므로 네트웍의 어떤 변화도 요구되지 않으며 교차로의 방향별움직임에서 나타나는 상호연관성이 변동부등식의 특성을 통해 명쾌하게 반영된다. 제안된 변동부등식의 해법으로서 변형된 대각화알고리즘이 제안되며 이때 링크표지덩굴망알고리즘이 각 교차로에서 발생하는 방향별지체를 고려하여 최적경로를 발견하는데 응용된다. 제안된 모델을 통한 실험결과로서 자용자최적평형조건이 만족됨이 확인되었으며 교차로주변에서 회전금지와 함께 목격되는 유턴, 피턴과 같이 두번 이상 같은 교차로를 통과하는 통행행태가 운전자의 경로파악시 발생됨을 확인되었다. 본 연구에서 제안된 모델은 네트웍의 교차로의 움직임을 파악하기위해 구축하는데 요구되는 노력을 감축하고, 컴퓨터계산노력을 절감하며, 향후 첨단여행정보시스템을 구축하는데 기여할 것으로 기대된다.

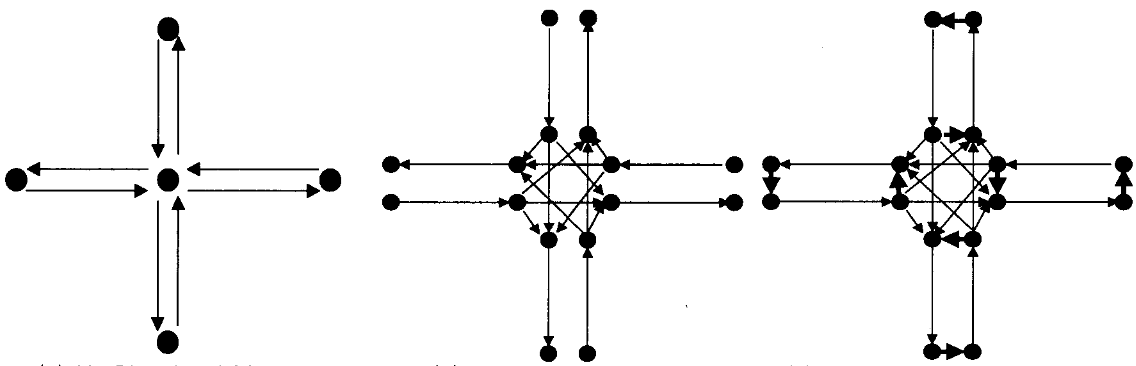
Ⅰ. Introduction

Route choice behavior of user-optimal(UO) traffic equilibrium assignment is determined in terms of route travel cost utilized by drivers. In real transportation networks, route travel cost is represented by link travel time as well as movement delays. As approaching toward more urbanized areas, intersection movement delays usually explain higher portion of route travel cost than rural areas. For this reason, treatment of intersection delays has been an important issue to imitate more reasonable route choice mechanism in UO traffic assignment models, especially in urban areas.

Network expansion has been an unavoidable way to treat intersection delays in the conventional UO traffic assignment models. To represent directional movements, it emulates all movements possibly shown in intersections like projected segments of approaching roads, which are treated as additional links in networks(Allsop and Charlesworth, 1977; Meneguzer, 1996), and therefore dramatic increase of number of links and nodes is not avoidable when junction delays are considered. As seen in ⟨Figures 1(a) and 1(b)⟩, a four-leg intersection, which is represented with 5 nodes and 8 links without consideration of any directional movement, needs to be expanded up a network with 24-links and 6-nodes size. From ⟨Figure 1(c)⟩, when considering both directional movements and U-Turn together, the network has to become even a larger one consisting of 32 links and 16 nodes. As a practical example, in ADVANCE project(Berka, etc., 1994), a sub-network of Chicago, which originally had 7850 links, was built to include 22918 links to take these directional movements of selected intersections into detail considerations. From these facts, it can be simply notified that network expansion does not only increase complexities for building network, but shrink computational performance by considerably increasing the size of networks.

This paper proposes a new formulation, which does not require any network modification for solving a UO traffic assignment model. The Variational Inequality(VI) theory is cast to reflect the interactive phenomena of directional flows into the formulation. The diagonalization algorithm is modified to test and prove the convergence of the proposed formulation. Model validation is demonstrated through three numerical case studies. The proposed formulation is expected to improve computational performance by reducing network-building complexities for treatment of interaction movements in urban transportation network theories.

This paper is organized as follows. The second section briefly reviews related works. The new formulation and corresponding solution algorithm are proposed in the third section. Numerical scenarios



(a) No Directional Movements (b) Considering Directional Movements (c) Considering Directional and U-Turn Movements

⟨Figure 1⟩ Network Representations

are discussed in the forth section. This paper is completed with some conclusions.

II. User-Optimal Traffic Equilibrium Assignment with Asymmetric Cost Functions

In this paper, directional delays are assumed to be interactively affected by directional flows on the other road segments. This section briefly reviews literatures on the UO traffic assignment problems related to asymmetric cost functions.

Denote **f** and **c** as the link flow and cost vectors in the transportation network and f_a and, c_a, respectively the link flow and cost function of link a.

$$\mathbf{f} = (f_1, \cdots, f_a, \cdots, f_L) \qquad (1)$$

$$\mathbf{c} = (c_1(\mathbf{f}), \cdots, c_a(\mathbf{f}), \cdots, c_L(\mathbf{f})) \qquad (2)$$

In the UO traffic assignment model, demands between OD pairs are loaded in the network so that no driver can reduce his or her travel cost by unilaterally changing his or her route(Wardrop, 1952). The link cost functions are assumed to be separable if the Jacobian matrix of equation (2) is symmetric. It follows that

$$\frac{\partial c_i(\mathbf{f})}{\partial f_j} = \frac{\partial c_j(\mathbf{f})}{\partial f_i} \qquad i \neq j \qquad (3)$$

Then, the convergence and uniqueness of the UO traffic assignment problem is solvable as an equivalent mathematical programming problem. If, however, the link cost functions is asymmetric, no equivalent mathematical program can be developed (Nagurney, 1993). The asymmetric nature of link cost function can be represented as follows:

$$\frac{\partial c_i(\mathbf{f})}{\partial f_j} \neq \frac{\partial c_j(\mathbf{f})}{\partial f_i} \qquad i \neq j \qquad (4)$$

The variational inequalities(VI) were proposed by Smith(1979a) and Dafermos(1980) to represent asymmetric UO traffic assignment problem. The VI is to find a user-optimal link flow vector $\mathbf{f}^*$ of the following inequality:

$$\mathbf{c}(\mathbf{f}^*) \cdot (\mathbf{f} - \mathbf{f}^*) \geq 0 \qquad \forall \mathbf{f} \in F \qquad (5)$$

where F is the feasible link flow sets.

Smith(1979a) proved that a VI solution exists based on the assumptions that **F** is a closed and convex set and **c** is continuous and that the solution is unique if **c** is strictly monotone on **F**. Dafermos (1980) proved uniqueness of the equilibrium when strong monotonicity condition holds.

When considering directional delay functions of intersections, the Jacobian matrix has a block-diagonal structure if directional movements of any intersection are numbered consecutively,

$$\boldsymbol{J} = \begin{vmatrix} J_1 & 0 & 0 \\ 0 & J_m & 0 \\ 0 & 0 & J_M \end{vmatrix} \qquad (6)$$

where $\boldsymbol{J}_m$ represents the Jacobian matrix of the travel costs for the $\boldsymbol{m}$th intersections, and $\boldsymbol{M}$ is the total number of intersections in the network.

Smith(1982) showed that the uniqueness of solution can be guaranteed when each $\boldsymbol{J}_k$ is positive definite for all $\mathbf{f} \in \boldsymbol{F}$ by proving that **c** is strictly monotone if and only if each $\mathbf{c}_m$, vector of link travel costs at intersection m, is strictly monotone.

Heydecker(1983) addressed that Smith's strict assumptions can be weakened while preserving the "good behavior" of route choice models in many real traffic conditions. He proved that a necessary condition for the existence of a stable unique solution is that the determinants of all principal sub-matrices of the Jacobian have nonnegative value at every $\mathbf{f} \in \boldsymbol{F}$.

Megeguzzer(1995) incorporated intersection delay models, signalized, all-way stop-controlled, and priority controlled intersection delay models into UO traffic assignment model and demonstrated model convergences through the practical point of views.

III. A VI Formulation and Diagonalization Algorithm

This section proposes the new approach to preclude employment of expanded network structures for a UO traffic assignment model in terms of VI formulation and diagonalization algorithm.

1. VI Formulation With Network Expansion

The VI formulation in equation (5), which requires network expansion to consider directional movements, is represented using link cost function and a set of constraints. It follows that

$$\sum_{ij} \mathbf{c}_{ij}(\mathbf{f}^*)\cdot(\mathbf{f}-\mathbf{f}^*)\geq 0 \qquad \forall \mathbf{f}\in F$$
$$s.t.$$
$$\sum_{ij\in A(j)} f_{ij} - \sum_{jk\in B(j)} f_{jk} = \sum_r T_j^r \tag{7}$$
$$f_{ij}, f_{jk}\geq 0, \qquad A(j)\subset L, B(j)\subset L$$

where T_j^r is the demand between origin node r and node j(destination).

$A(j)$ is the set of links whose tail node is j.
$B(j)$ is the set of links whose head node is j.
L is the whole link set.

In equation (7), since there is no cost term associated with directional delays, to deal with directional cost, the considered network needs to be expanded to treat all directional movements as links. Thus the network expansion is unavoidable in equation (7). First constraint means node conservation condition. Centering to a destination node j, sum of inflows departing from origin r and arriving at node j is equal to outflows leaving from node j plus flows whose destination is node j.

2. Link-Based Route Choice Condition Considering Directional Delays

The variational inequality model is derivable from a new link-based route choice condition in which directional delays can be considered. In this section, we introduce a new set of user optimal (UO) route choice conditions based on link and node variables associated with directional delays.

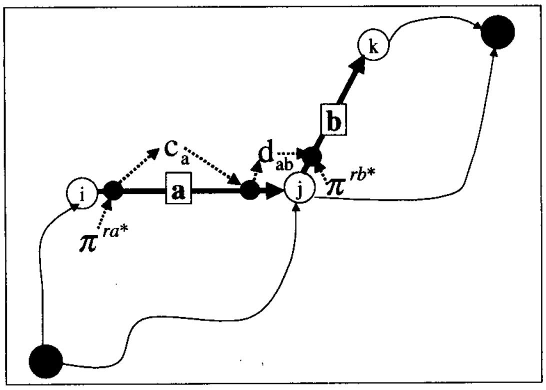
The definition of the UO route choice(Wardrop, 1952) is as follows.

"For each O-D pair, at user equilibrium, the travel time on all used paths is equal, and(also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path."

We write the equivalent mathematical inequalities for the above definition using variables associated directional movements. In this case, any route from origin r to destination s, two adjacent links a and b are defined as being used if directional volumes between links a and b, v_{ab}^{rs}, has positive values(⟨Figure 2⟩). Define π^{ra*} as the minimal travel time experienced by vehicles departing origin r to link a, the asterisk denoting that the travel time is computed in the UO condition. For two links(a,b) the minimal time π^{rb*} from origin r to link a should b equal to or less than the minimal travel time, π^{ra*}, from origin r to node I plus the travel time of link a, c_a, plus the directional delays between links a and b, d_{ab}. It follows that

$$\pi^{ra*} + c_a + d_{ab} \geq \pi^{rb*}, \qquad \forall a=(i,j), b=(j,k), r$$

If, for each O-D pair rs, when directional flows from origin r through links a and b, v_{ab}^{rs} has positive value, then the UO route choice conditions require that links a and b is on the minimal travel time



〈Figure 2〉 Directional Movements Between Two Adjacent Links

route. In other words, the minimal travel time, π^{rb*}, from origin r to link b should equal the minimal travel time, π^{ra*}, from origin r to link b plus the travel time of link a, c_a, plus the directional delays between links a and b, d_{ab}. It follows that

$$\pi^{ra*} + c_a + d_{ab} = \pi^{rb*}, \quad if \;\; v_{ab}^{rs} > 0 \qquad \forall a = (i,j), b = (j,k), r, s$$

The above equation is equivalent to the following:

$$\pi^{ra*} + c_a + d_{ab} \geq \pi^{rb*}, \quad if \;\; v_{ab}^{rs} = 0 \qquad \forall a = (i,j), b = (j,k), r, s$$

Thus, the link-based UO route choice conditions can be summarized as follows:

$$\left(\pi^{ra*} + c_a + d_{ab} - \pi^{rb*}\right) \geq 0, \qquad \forall a = (i,j), b = (j,k), r \tag{8}$$

$$v_{ab}^{rs*} \cdot \left(\pi^{ra*} + c_a + d_{ab} - \pi^{rb*}\right) = 0, \qquad \forall a = (i,j), b = (j,k), r, s \tag{9}$$

$$v_{ab}^{rs} \geq 0, \qquad \forall a = (i,j), b = (j,k), r, s \tag{10}$$

3. VI Formulation Without Network Expansion

Denote Ω_{ab}^{rb*} as the difference of the minimal travel time from r to link b and the travel time from r to link b via minimal travel time route from r to link a and link a for vehicles departing from origin. It follows that

$$\Omega_{ab}^{rb*} = \pi^{ra*} + c_a + d_{ab} - \pi^{rb*} \qquad \forall a = (i,j), b = (j,k), r \tag{11}$$

We than rewrite the link-based UO route choice conditions as:

$$\Omega_{ab}^{rb*} \geq 0, \qquad \forall a = (i,j), b = (j,k), r \tag{12}$$

$$v_{ab}^{rs*} \cdot \Omega_{ab}^{rb*} = 0, \qquad \forall a = (i,j), b = (j,k), r, s \tag{13}$$

$$v_{ab}^{rs} \geq 0, \qquad \forall a = (i,j), b = (j,k), r, s \tag{14}$$

Then, the equivalent variational inequality formulation of link-based UO route choice conditions (12)~(14) may be stated as follows.

[Theorem 1]

The traffic flow pattern is in a link-based UO route choice state if and only if it satisfies the variational inequalities problem:

$$\sum_{rs}\sum_{a}\sum_{b}\left(c_a + d_{ab} + \pi^{ra*} - \pi^{rb*}\right)(\mathbf{v} - \mathbf{v}^*) \geq 0 \qquad \forall \mathbf{v} \in \boldsymbol{F}$$

$$\begin{aligned} s.t.\quad & \sum_a f_a^{rs} - \sum_a\sum_b v_{ab}^{rs} = \sum_r T_j^{rs} \\ & f_a^{rs} = \sum_a\sum_b v_{ab}^{rs} \\ & f_a^{rs},\; v_{ab}^{rs} \geq 0 \\ & \forall a, b, r, s; a = (i,j), b = (j,k) \end{aligned} \tag{15}$$

The new VI formulation is proposed in equation (15). In the formulation, directional delay functions are directly embedded into the VI formulation and directional flows are incorporated in the node constraints. Thus, directional delays can be implicitly considered in both the VI formulation and constraints.

〔Proof of necessity〕

We need to prove that link-based UO route choice condition (12)~(14) imply variational equality (15). For any adjacent links a and b, a feasible directional flow is

$$v_{ab}^{rs} \geq 0 \quad (16)$$

Multiplying equation (16), and equation (12) we have

$$v_{ab}^{rs} \cdot \ \Omega_{ab}^{rb*} \geq 0 \qquad \forall a,b,r; a=(i,j), b=(j,k) \quad (17)$$

We subtract the second UO route choice condition (13) from equation (17)

$$\left[v_{ab}^{rs} - v_{ab}^{rs*}\right] \cdot \Omega_{ab}^{rb*} \geq 0 \qquad \forall a,b,r,s; a=(i,j), b=(j,k) \quad (18)$$

Summing above equation for all directional movement a and b and all OD pairs rs, we obtain variational equality (15)

$$\sum_{rs}\sum_{a}\sum_{b}\left[v_{ab}^{rs} - v_{ab}^{rs*}\right] \cdot \Omega_{ab}^{rb*} \geq 0 \quad (19)$$

〔Proof of sufficiency〕

We need to prove that any solution v_{ab}^{rs*} to variational equality (18) satisfies link-based UO route choice condition (12)~(14). We know that the first and third DUO route choice conditions (12) and (14) hold by definition. Thus, we need to prove that the second UO route choice condition (13) also holds.

Assume that the second DUO route choice condition (13) does not hold only for a directional movement between link e=(m,l) and f=(l,n) for an origin g to destination h, i.e.,

$$v_{ef}^{gh} > 0 \text{ and } \Omega_{ef}^{gd*} > 0$$

Thus, we have

$$v_{ef}^{gh} \cdot \ \Omega_{ef}^{gf*} > 0$$

where

$$\Omega_{ef}^{gf*} = \pi^{ge*} + c_e + d_{ef} - \pi^{gf*} > 0$$

$$where \ e=(m,l) \ \ and \ \ f=(l,n)$$

Note that the second UO route choice condition (13) holds for all directional movements other than $ef=(m,l,n)$ for origin g and destination h. Equation (13) also holds for directional movements between links e and f for origin g and destination h. It follows that

$$\sum_{rs}\sum_{a}\sum_{b}\Omega_{ab}^{rb*} \cdot \ v_{ab}^{rs} = \Omega_{ef}^{gf*} \cdot \ v_{ef}^{gh} > 0 \quad (20)$$

We note that all other terms in the above equation vanish because of UO route choice condition (13).

For each O-D pair rs, we can always find one minimal travel time route p for vehicles departing origin r to destination s, where route p was evaluated under optimal directional flow pattern $\{v_{ab}^{rs*}\}$.

This will generate a set of feasible directional flow patterns $\{v_{ab}^{rs*}\}$ which always satisfies equations (12)~(14) because flows are not assigned to routes with non-minimal travel times which were evaluated under the optimal flow pattern $\{v_{ab}^{rs*}\}$. It follows that

$$v_{ab}^{rs} \cdot \ \Omega_{ab}^{rb*} = 0 \qquad \forall a,b,r;s \ ; a=(i,j), b=(j,k)$$

Summing above equations for all directional movements between links a and b and all origins r and destinations s, we have

$$\sum_{rs}\sum_{a}\sum_{b} v_{ab}^{rs} \cdot \ \Omega_{ab}^{rb*} = 0$$
$$\text{where } a = (i,j),\ b = (j,k) \qquad (21)$$

We subtract equation (20) from equation (22) and obtain

$$\sum_{rs}\sum_{a}\sum_{b} \Omega_{ab}^{rb*} \ \left[v_{ab}^{rs} - v_{ab}^{rs*}\right] < 0 \qquad (22)$$

The above equation contradicts variational inequality (19). Therefore, any optimal solution $\{v_{ab}^{rs*}\}$ to variational equality (19) satisfies the second DUO route choice condition (13). Since we proved the necessity and sufficiency of the equivalence of variational inequality (19) to link-based UO route choice condition (12)~(14), the proof is complete.

4. Diagonalization Algorithm

Several algorithms have been proposed for solving the asymmetric UO traffic assignment model (Dafermos, 1982; Fisk and Nguyen, 1982; Smith, 1983b; Lawphongpanich and Hearn, 1984). The diagonalization algorithm is most widely used among these because the standard UO traffic assignment problem is applicable. The diagonalization algorithm solves a UO traffic assignment problem by using two iteration steps: inner and outer iterations. The inner iteration solves the standard separable the UO traffic assignment problem and in the outer iteration, the impact of flows of other links, which is implanted in the cost function of this link, is fixed as constant value to make the UO traffic assignment problem separable in the inner iteration.

In the proposed formulation (15), directional delay and flow terms are in both the VI and the constraints. To solve this problem, the diagonalization algorithm needs to calculate directional delays and update flows at every inner iteration. If there is no directional delay terms in the formulation, conventional shortest path algorithms(Moore, 1957; Dijkstra, 1959; Dial, 1979) can play this role. However these algorithms cannot be used solve our problem. To track directional delays and flows in the original networks, the feature of vine-based shortest path algorithms can be exploited because these algorithms take two links or three nodes simultaneously into considerations(Kirby and Potts, 1969, and Ziliaskopoulos and Mahmassani, 1996), directional delays are implicitly counted in the shortest path searching process. In equation (24) proposed by Ziliaskopoulos and Mahmassani(1996), directional penalty $\xi(i,j,m_k)$ is counted when the next node searching procedure is processed from link (i,j) toward link (j,k) with directional movement m_k.

$$\lambda_{j,m_k} = \min_{\forall i \in \Gamma^{-1}(j)} \{\xi(i,j,m_k) + \tau(i,j) + \lambda_{i,m_j}\} \ \forall k \in \Gamma(j) \qquad (23)$$

where $\Gamma(j)$ and $\Gamma^{-1}(j)$ are the set of successor and predecessor nodes of node j, respectively; λ_{j,m_k} is the best path to the preceding node j for movement m_k; $\xi(i,j,m_k)$ is the directional delay of link (i,j) for movement m_k; $\tau(i,j)$ is the link cost of link (i,j).

In the paper, to calculate $\xi(i,j,m_k)$ as a directional delays d_{ab} between two links $a=(i,j)$ and $b=(j,k)$ in equation (15), the link-based vine shortest algorithm proposed by NamGoong(1996) is applied. As noted by(Choi, 1995; NamGoong, 1996), the link-based vine algorithm guarantees an optimal solution while the node-based vine algorithm cannot.

The directional flows can be updated in all-or-nothing assignment. In the all-or-nothing assignment of the separable UO traffic assignment problem, directional flows can be calculated by loading all demands on the shortest path tree established by vine-based shortest path algorithms considering directional delays for movements embedded at junction nodes.

The modified diagonalization algorithm is summarized as follows.

Step 0. Initialization;

Find a feasible initial solution $\mathbf{f}^1$ and $\mathbf{v}^1$. Set the outer iteration counter m=1.

Step 1. Relaxation.

Fix the impact of flows from other links in link cost functions and solve the standard UO traffic assignment problem.

[1.1] : Update link costs and directional delays.

[1.2] : At iteration n, based on the current link flow pattern $\mathbf{f}^n$ and $\mathbf{v}^n$, generate $\mathbf{f}^{n+1}$ and and $\mathbf{v}^{n+1}$ by solving a standard UO traffic assignment problem as follows:

$$\min_{f_{\tilde{a}}, v_{\tilde{a}\tilde{b}}} \sum_{\tilde{a}} \left\{ \int_0^{f_{\tilde{a}}} c_{\tilde{a}}(\omega) d\omega + \sum_{\tilde{b}} \int_0^{v_{\tilde{a}\tilde{b}}} d_{\tilde{a}\tilde{b}}(v^n_{\tilde{a}\tilde{1}}, v^n_{\tilde{a}\tilde{2}}, \ldots, v^n_{\tilde{a}\tilde{b}-\tilde{1}}, \omega, v^n_{\tilde{a}\tilde{b}+\tilde{1}}, \ldots, v^n_{\tilde{a}\tilde{D}}) d\omega \right\}$$

$$\begin{aligned} s.t. \quad & \sum_{ij \in \tilde{a}} f_{ij} - \sum_{\tilde{a}} \sum_{jk \in \tilde{b}} v_{ijk} = \sum_r T_j^r \\ & f_{\tilde{a}} = \sum_{\tilde{a}} \sum_{\tilde{b}} v_{\tilde{a}\tilde{b}}, \quad v_{\tilde{a}\tilde{b}} \geq 0 \\ & \forall \tilde{a} = (i,j) \in \tilde{A}(j), \tilde{b} = (j,k) \in \tilde{B}(j) \end{aligned} \tag{24}$$

where

$\tilde{a}$ is an used link by flow, i.e., $f_{\tilde{a}} > 0$;

$\tilde{a}\tilde{b}$ is an used directional movement from link $\tilde{a}$ to $\tilde{b}$;

i.e., $v_{\tilde{a}\tilde{b}} > 0$ for $\tilde{a} \in \tilde{A}(j), \tilde{b} \in \tilde{B}(j)$;

$\tilde{A}(j)$ is a set of used links which tail node is j.

$\tilde{B}(j)$ is a set of used links which head node is j.

* The proof of equation (24) based on Equation (15) is in Appendix A.

[1.3] : Inner iteration convergence test : If it converges, go to step 2; otherwise, set n=n+1 and go to step [1.1].

Step 2. Outer iteration convergence test;

If it converges, stop; otherwise, set m=m+1 and go to step [1.1].

Ⅳ. Numerical Examples

1. Data Input

The revised Sioux Fall network is employed for numerical tests. The network has 78 links and 26 nodes. It is assumed that all links are 2-lane roads and every node is a junction point and there exists a directional delay for each turning movement. ⟨Figure 3⟩ depicts the network with distance (miles) on the link.

The path travel time is assumed to consist of two cost functions; link travel time and directional delay functions. Equation (25) represents the BPR function employed in this paper to estimate link travel time.

$$c_a(\mathbf{f}) = FT_a \cdot \left(1 + 0.15 \cdot \left(\frac{f_a}{\mathrm{Cap}_a} \right)^4 \right) \tag{25}$$

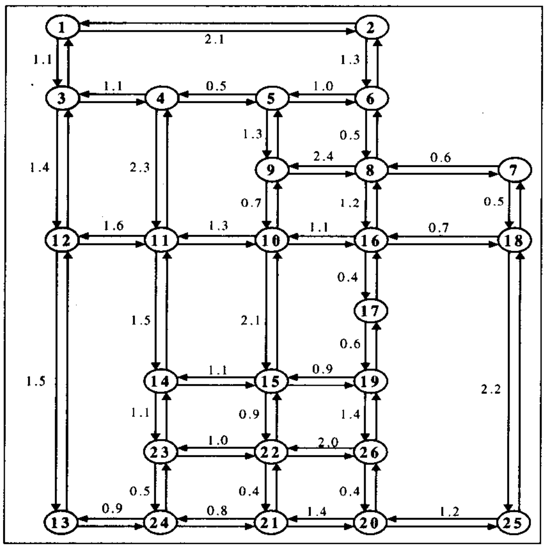
where

c_a : cost function of link a

$\mathbf{f}$: link flow vector

FT_a : free flow travel time of link a(minutes)

Cap_a : capacity of link a(vehicles/hour)



⟨Figure 3⟩ Revised Sioux Fall Network

Equation (26) represents four directional delay functions employed based on the following directional movements: U-Turn, Left-Turn, Through Movement and Right-Turn.

$$d_{ab}^{m}(\mathbf{v}_m) = FT_{ab}^{m} \cdot \left(1 + \boldsymbol{\beta} \cdot \left(\frac{\mathbf{v}_m}{\mathrm{DCap}_{ab}^{m}}\right)^4\right) \quad (26)$$

where

d_{ab}^{m} : directional delay function from link a to b at intersection m

FT_{ab}^{m} : free flow passing time from link a to link b at intersection m

DCap_{ab}^{m} : capacity of directional movement from link a to link b at intersection m

$\mathbf{v}_m$: directional flow vector at intersection m

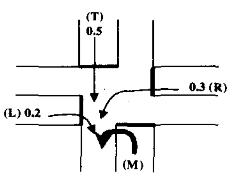
$\boldsymbol{\beta}$: parameter.

⟨Table 1⟩ describes parameters or default values utilized in equations (25) and (26) in the paper. Different capacity values and parameters are utilized to reasonably reflect realistic traffic conditions. In each directional delay function, impact of flows from other directional movements and the major movement is considered in the directional delay functions and these are represented in ⟨Figure 4⟩.

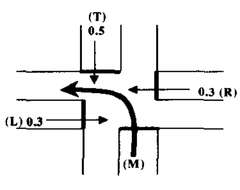
As denoted in equation (6), in case of a transportation network with intersection control, the Jacobian matrix usually has a block-diagonal structure. The reason that the Jacobian has this structure is that the delay at an intersection usually depends only on the flows approaching the same intersection. In order to guarantee an optimal solution, a sufficient condition for the solution to be unique is that the directional delay function d_{ab}^{m} is strictly monotone. The strict monotonicity is satisfied if and only if the Jacobian matrix of the directional delay function d_{ab}^{m} is positive definite(Stewart, 1973). As Smith(1982) proved that the Jacobian matrix is positive definite if and only if each of the blocks on the diagonal is positive definite. This condition is guaranteed when the impact of other directional flows is not dominating, i.e., the movement delay function depends mainly on that directional flow. As Meneguzzer

⟨Table 1⟩ Parameters and Default Values

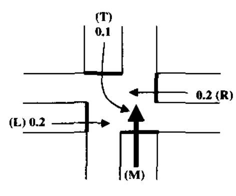
Functions	FT_a (minutes/mile)	FT_{ab}^{m} (seconds)	Cap_a (vehicles/hour)	DCap_{ab}^{m} (vehicles/hour)	$\boldsymbol{\beta}$	$\mathbf{v}_m$
Link	2	-	2200	-	-	-
U-Turn	-	5	-	300	0.60	Figure 4-a
Left Turn	-	5	-	1400	0.30	Figure 4-b
Thru. Turn	-	3	-	2200	0.15	Figure 4-c
Right Turn	-	3	-	1000	0.10	Figure 4-d
Turn Prohi.	-	$+\infty$	-	-	-	-



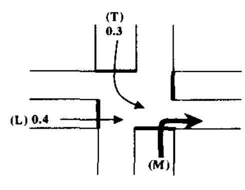
$\mathbf{v}_m = M + 0.2L + 0.5T + 0.3R$
(a) U-Turn



$\mathbf{v}_m = M + 0.3L + 0.5T + 0.3R$
(b) Left-Turn



$\mathbf{v}_m = M + 0.2L + 0.1T + 0.2R$
(c) Through Movement



$\mathbf{v}_m = M + 0.4L + 0.3T$
(d) Right Turn

⟨Figure 4⟩ Impact of Directional Flows

(1995) pointed out, the question arises whether the sufficient conditions for the uniqueness of the solution are too strict. In 〈Figure 4〉, in case of left-turn movement, this movement can be dominated by the impact of other movement such as left, through, and right turns, and thus multiple solutions can be existed. However, from the computational results from Meneguzzer(1995), it was reported that a unique solution does existed with weaker sufficient conditions. The examples of the turn movement in the paper focus more on the weaker sufficient conditions.

Three scenarios are employed to validate the proposed formulation. Scenario-I assumes that U-Turn movements are permitted in all links and 8 directional movements infinite delay values, which means Turn-Prohibitions. 〈Figure 5〉 represents Scenario-I. In Scenario-II, it is assumed that U-Turn movements for all links have infinite delays and 10 directional movements have infinite delay values. 〈Figure 7〉 represents Scenario-II. The proposition of Scenario-III is that U-Turn is prohibited except one directional movement and 14 directional movements have infinite delay values. In all junction nodes, except infinite delay cases, delay is calculated based on directional delay function described in equation (11). 〈Figure 8〉 represents Scenario-III. Only one origin and destination pair is considered for each scenario.

2. Analysis of Results

1) Result of Scenario-I

〈Table 3〉 summarizes flows, costs, and node sequences for used paths in the solution algorithm. Between origin 3 and destination 4, 5 used paths are generated and user optimal is achieved based on the fact that cost on used paths are almost identical and directional delays in junction nodes are embedded in the calculated path costs. U-Turn movements are detected at junctions 13 and 14 to minimize route costs by avoiding directional movement 3->12->11 and 4->11->10.

〈Figure 6〉 illustrates convergence pattern of the solution algorithm. After 5 outer iterations, an optimal solution is achieved by meeting the stopping criteria. Objective values of the diagonalized sub-problem increase at the second iteration, and then monotonically decrease. As described by Meneguzzer (1995), because asymmetric network equilibrium

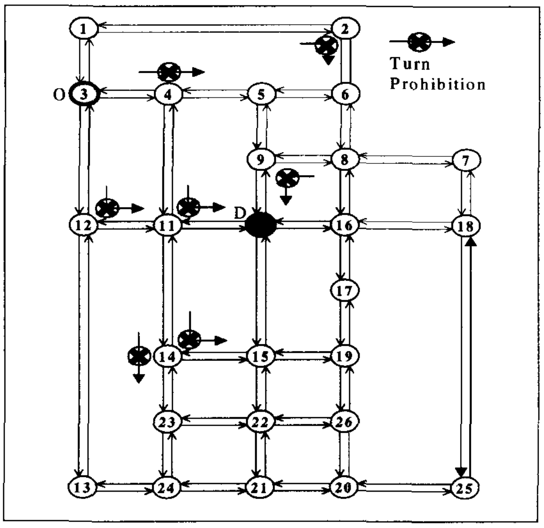
〈Table 2〉 Three Study Scenarios

Scen.	Scenarios Descriptions	Trip Demand
I	- Permitted U-Turn for all links - Turn prohibition of 8 directions	5000 vehicles from node 3 to 10
II	- U-Turn prohibitions for all links - Turn prohibitions of 10 directions	5000 vehicles form node 3 to 9
III	- U-Turn permitted for only one direction - Turn prohibitions of 14 directions	5000 vehicles form node 3 to 9

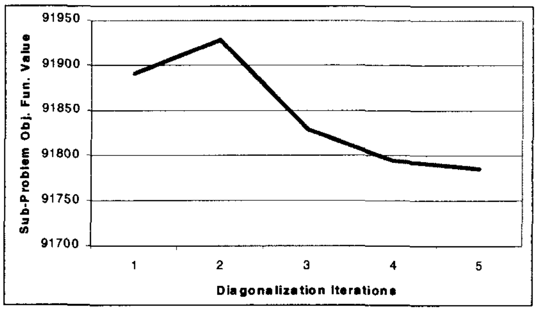
〈Table 3〉 Result of Scenario-I

Used Path	Path Flow (vehcle)	Path Cost (minute)	Node Sequence for Each Used Path for Scenario-I
0	801.4	18.0	3->12->*(0.1)->13->*(2.6)->12->*(0.1)->11->*(0.1)->10 (*)
1	593.5	18.5	3-> 4->*(0.1)->11->*(0.1)->14->*(2.9)->11->*(0.1)->10 (**)
2	2351.9	18.0	3->12->*(0.1)->13->*(0.7)->24->*(0.1)->21->*(0.3)->22->*(0.1)->15->*(0.1)->10
3	764.8	18.1	3->12->*(0.1)->13->*(0.7)->24->*(0.1)->23->*(0.1)->14->*(0.1)->11->*(0.1)->10
4	488.3	18.4	3-> 4->*(0.1)->11->*(1.8)-> 4->*(0.1)-> 5->*(0.1)-> 9->*(0.1)->10 (***)

a->*()->b: Turn delay (minutes) between node a and b.
(*) : U-Turn at node 13; (**) : U-Turn at node 14; (***) : U-Turn at node 11



〈Figure 5〉 Scenario-I

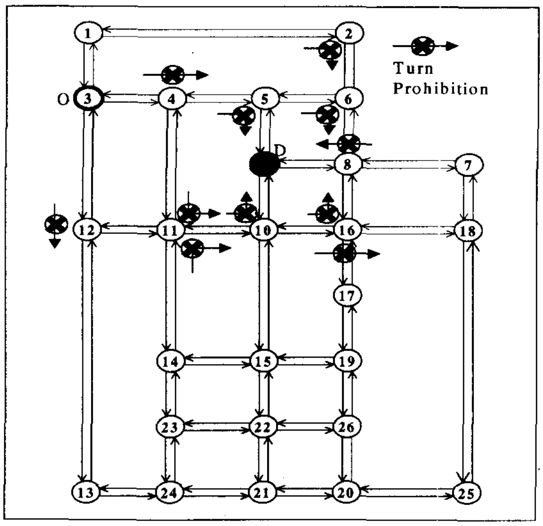


〈Figure 6〉 Convergence of Sub-Problem Objective Function

problem does not have a global objective function formulated as an equivalent minimization problem, the diagonalization algorithm does not necessarily perform as a descent type of algorithm.

2) Result of Scenario-II

〈Table 4〉 summarizes flows, costs, and node



〈Figure 7〉 Scenario-II

sequences for used paths for Scenario-II in the solution algorithm. Between origin 3 and destination 9, 6 used paths are generated and user optimal is achieved which is based on the fact that costs on used paths are almost identical. In path 4 and 5, long loop movements, which are generated when a visited node is revisited, are observed at nodes 10 and 15 to minimize route costs because U-Turn movements are prohibited for all links and thus cannot contribute to reduce route cost in Scenario-II. In path 5, P-Turn movement, which is frequently observed in real traffic situation, is included.

3) Result of Scenario-III

The difference between Scenario-III and Scenario-II is in that except one directional movement, 11->14->11, all U-Turn movements are prohibited.

〈Table 4〉 Result of Scenario-II

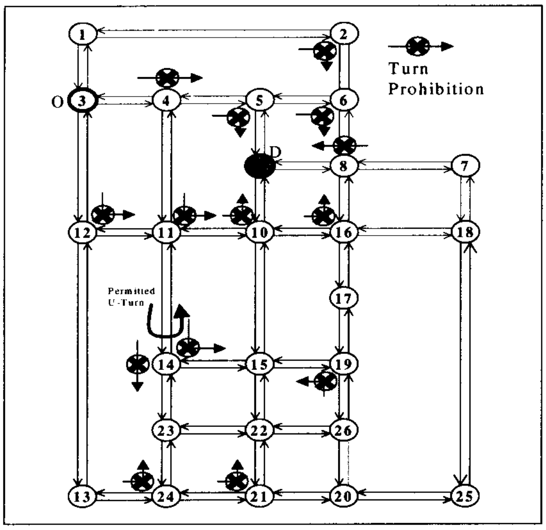
Used Path	Path Flow(vehcle)	Path Cost(minute)	Node Sequence for Each Used Path
0	2226.4	22.4	3->12->11->14->15->10-> 9
1	247.8	22.7	3-> 4->11->14->23->22->15->10-> 9
2	154.2	22.4	3-> 4->11->14->15->19->17->16->10-> 9
3	1547.3	22.4	3-> 4->11->14->15->10-> 9
4	573.6	22.4	3->12->11->10->15->19->17->16->10-> 9 (*)
5	250.7	22.4	3->12->11->10->16->17->19->15->10-> 9 (**)

(*) : loop at node 10; (**) : P-Turn at node 10

〈Table 5〉 Result of Scenario-III

Used Path	Path Flow(vehcle)	Path Cost(minute)	Node Sequence for Each Used Path
0	3569.5	30.4	3-〉12-〉13-〉24-〉21-〉20-〉26-〉19-〉17-〉16-〉10-〉 9
1	770.6	30.2	3-〉 4-〉11-〉14-〉11-〉10-〉16-〉17-〉19-〉15-〉10-〉 9 (*)
2	659.9	30.4	3-〉12-〉13-〉24-〉21-〉20-〉25-〉18-〉16-〉10-〉 9
3	0.0	30.8	3-〉12-〉13-〉24-〉21-〉20-〉26-〉19-〉17-〉16-〉 8-〉 9

(*) : U-Turn at node 14 and P-Turn at node 10



〈Figure 8〉 Scenario-III

〈Table 5〉 summarizes flows, costs and node sequences for used paths for Scenario-III in the solution algorithm. Between origin 3 and destination 9, 4 used paths are generated and user optimal is achieved which is based on the fact that used paths cost are almost identical. In path 1, U-Turn and P-Turn movements are observed at nodes 14 and 10, respectively.

V. Conclusion

This paper proposed the VI formulation for a UO traffic assignment model to explicitly consider directional delays embedded in junction points without expanding network. The directional delay functions and flow terms are directly implanted into the VI formulation and the flow conservation constraints. A vine-based shortest path algorithm is efficiently applied to calculate and update link flows and directional delays and flows in the sub-problem of the diagonalization algorithm. The results demonstrated that the solution converges and the user-optimal condition is achievable. Various loop-related movements such as U-Turn and P-Turn are observable in the process of searching minimal routes while avoiding Turn-Prohibition. With the proposed schemes, network-building complexities are reduced, and computational performances in terms of memory and computational efficiency have been improved.

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♧ 주 작 성 자 : 신성일
♧ 논문투고일 : 2001. 11. 6
논문심사일 : 2001. 12. 31 (1차)
2002. 2. 7 (2차)
심사판정일 : 2002. 2. 7

Appendix A

When rewrite Equation (15), it follows:

$$\sum_{rs}\sum_{a}\sum_{b}\left(c_a + d_{ab} + \pi^{ra*} - \pi^{rb*}\right)(\mathbf{v} - \mathbf{v}^*) \geq 0 \tag{A1}$$

From the VI formulation in Equation (A1), in each outer iteration step, UO traffic assignment is formulated as following NLP problem where M_1 and M_2 is other impact but its main flows. Then it follows:

$$\min_{v_{ab}^{rs}} \sum_{rs}\sum_{a}\sum_{b}\int_0^{v_{ab}^{rs}}\left[c_{\tilde{a}}(M_1+\omega) + d_{ab}(M_2+\omega) + \pi^{ra*} - \pi^{rb*}\right]d\omega \tag{A2}$$

Considering a used link ã and its associated directional movement $\tilde{a}\tilde{b}$ in terms of origin r and destination s, the objective value of equation (A2) become zero. We have:

$$\min_{v_{\tilde{a}\tilde{b}}^{rs}} \sum_{rs}\sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[c_{\tilde{a}}(M_1+\omega) + d_{\tilde{a}\tilde{b}}(M_2+\omega) + \pi^{r\tilde{a}*} - \pi^{r\tilde{b}*}\right]\ d\omega = 0 \tag{A3}$$

Furthermore, $\pi^{rb*} - \pi^{ra*}$ is always equal to $c_{\tilde{a}} + d_{\tilde{a}\tilde{b}}$. Thus, it is as follows:

$$\min_{v_{\tilde{a}\tilde{b}}^{rs}} \sum_{rs}\sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[c_{\tilde{a}}(M_1+\omega) + d_{\tilde{a}\tilde{b}}(M_2+\omega)\right]d\omega > 0 \tag{A4}$$

$$\min_{v_{\tilde{a}\tilde{b}}^{rs}} \sum_{rs}\sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[c_{\tilde{a}}(M_1+\omega)\right]d\omega + \sum_{rs}\sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[d_{\tilde{a}\tilde{b}}(M_2+\omega)\right]d\omega \tag{A5}$$

If assume that $c_{\tilde{a}}$ depends on mainly its link flow and there is no interaction phenomenon between its directional flows on link $\tilde{a}$, then $M_1 = 0$. Then we have:

$$\min_{f_{\tilde{a}}, v_{\tilde{a}\tilde{b}}} \sum_{\tilde{a}}\int_0^{f_{\tilde{a}}}\left[c_{\tilde{a}}(\omega)\right]d\omega + \sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[d_{\tilde{a}\tilde{b}}(M_2+\omega)\right]d\omega \tag{A6}$$

In Equation (A6), when M_2 is expressed as fixed impact of other directional flows at every outer iterations. It follows that:

$$\min_{f_{\tilde{a}}, v_{\tilde{a}\tilde{b}}} \sum_{\tilde{a}}\int_0^{f_{\tilde{a}}}\left[c_{\tilde{a}}(\omega)\right]d\omega + \sum_{\tilde{a}}\sum_{\tilde{b}}\int_0^{v_{\tilde{a}\tilde{b}}^{rs}}\left[d_{\tilde{a}\tilde{b}}(v_{\tilde{a}\tilde{1}}^n, v_{\tilde{a}\tilde{2}}^n, \ldots, v_{\tilde{a}\tilde{b}-\tilde{1}}^n, v_{\tilde{a}\tilde{b}+\tilde{1}}^n, \ldots, v_{\tilde{a}\tilde{B}}^n + \omega)\right]d\omega \tag{A7}$$

Since (A7) is the same equation as Equation (24), the proof is complete.