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Development of the Fuzzy-Based System for Stress Intensity Factor Analysis

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Abstract

This paper describes a fuzzy-based system for analyzing the stress intensity factors (SIFs) of three-dimensional (3D) cracks. A geometry model, i.e. a solid containing one or several 3D cracks is defined. Several distributions of local node density are chosen, and then automatically superposed on one another over the geometry model by using the fuzzy knowledge processing. Nodes are generated by the bucketing method, and ten-noded quadratic tetrahedral solid elements are generated by the Delaunay triangulation techniques. The singular elements such that the mid-point nodes near crack front are shifted at the quarter-points, and these are automatically placed along the 3D crack front. The complete finite element(FE) model is generated, and a stress analysis is performed. The SIFs are calculated using the displacement extrapolation method. To demonstrate practical performances of the present system, semi-elliptical surface cracks in a inhomogeneous plate subjected to uniform tension are solved.

Key Words : Fuzzy Knowledge Processing, Finite Element Analysis, Stress Intensity Factor, Surface Crack, Automatic Mesh Generation, Fuzzy Theory, Bucketing Method, Delaunay Triangulation, Singular Element

Nomenclature

a	depth of a semi-elliptical surface crack
c	half length of a semi-elliptical surface crack
K_I	the stress intensity factor(SIF) for a Mode I
b	half width of a plate with cracks
t	plate thickness
Q	shape factor for a semi-elliptical crack
ϕ	parametric angle of the ellipse
σ	applied uniform tensile stress
E	Young's modulus
ν	Poisson's ratio
x, y, z	Cartesian coordinates

1. Introduction

Actual cracks found in practical structures are mostly 3D, i.e. surface or embedded cracks. Analyses of the 3D cracks are desirable in structural integrity studies of practical structures. Various techniques have been developed for this purpose. For example, the stress intensity factors for an elliptical or a semi-elliptical crack have been obtained by the finite element method[1-5], the alternating method[6-8], the body force method[9-10] and so on. The stress intensity factors for coplanar surface cracks have been obtained by the body force method[11] and the line spring method[12].

Among them, the finite element method (FEM) seems the most promising method to deal with 3D crack problems because of its flexibility and extensibility. However, there are still some problems to be solved. The main concern for the FEM is a relatively higher computation cost, especially when dealing with 3D crack problems. To overcome this, several techniques such as the direct method[2,13], the virtual crack extension method[14], the superposition method[15] and the special singular element method [16-17] have been proposed in conjunction with the FEM. It should be also noted here that the data preparation for 3D crack analyses require special element arrangement near the crack front, and that much efforts are necessary to generate such special meshes. Dramatic progress in computer technology now shortens computation time. However in reality, labour intensive tasks to prepare a FE model of a structural component with 3D cracks are still a bottle neck. The author has proposed an automatic FE mesh generation method for 3D structures based on the fuzzy knowledge processing and computational geometry[18-19].

In the present study, by integrating this mesh generator, one of commercial FE analysis codes and some additional techniques to calculate the SIF, a new fuzzy-based system for analyzing the SIFs of 3D cracks was developed. In order to examine accuracy and efficiency of the present system, the SIF for a

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semi-elliptical surface crack in a inhomogeneous plate subjected to uniform tension is calculated, and compared with Raju-Newman's solution[1,20].

2. Fundamental Principle and Algorithms

The biggest advantage of the present analysis system is a very simple operation to analyze complex structures such as a plate with several semi-elliptical surface cracks.

A whole analysis domain is defined using one of commercial geometry modelers, Designbase[21] which has abundant libraries which enable us to easily operate, modify and refer a solid model.

Material properties and boundary conditions are directly attached onto the geometry model by clicking the loops or edges that are parts of the geometric model using a mouse, and then by inputting values. The present system deals with displacement as well as force boundary conditions.

2.1 Control of locally-optimum mesh patterns by fuzzy knowledge

In this section, the connecting process of locally-optimum mesh images is dealt with using the fuzzy knowledge processing technique. Performances of automatic mesh generation methods based on node generation algorithms depend on how to control node spacing functions or node density distributions and how to generate nodes. The basic concept of the present mesh generation algorithm is originated from the imitation of mesh generation processes by human experts of finite element analyses. One of the aims of this algorithm is to transfer such experts' techniques to beginners.

In the present system, nodes are first generated, and then a finite element mesh is built. In general, it is not so easy to well control element size for a complex geometry. A node density distribution over a whole geometry model is constructed as follows. The present system stores several local node patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local nodal patterns, depending on their analysis purposes, and designates where to locate them.

To begin with, let us consider a mesh generation process performed by the experts on finite element method stress analyses, taking an example of an upper half portion of a cracked plate with a circular hole as shown in Fig. 1(a). Figs. 1(b) and (c) show the locally optimum mesh images around a hole and a crack-tip, respectively where stress concentration are expected. It is anticipated that the experts have attained such mesh images through theoretical studies or numerical analyses. If both mesh images are connected smoothly, we could obtain a quasi global-optimum mesh for a whole analysis domain. However, no definition of dominant area for each mesh image are too ambiguous to perform. In the present study, such a connection process of mesh images is performed by using fuzzy knowledge processing. Here, node patterns are introduced instead of mesh images as shown in Fig. 2, because they are much easier to handle in the connection process. The local node patterns as shown in Figs. 2(a) and (b) may be regarded locally-optimum around the crack tip or the hole, respectively.

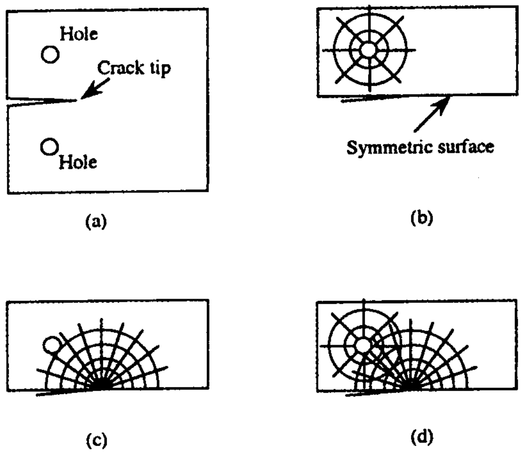


Fig. 1. Mesh images of experts on FEM stress analysis

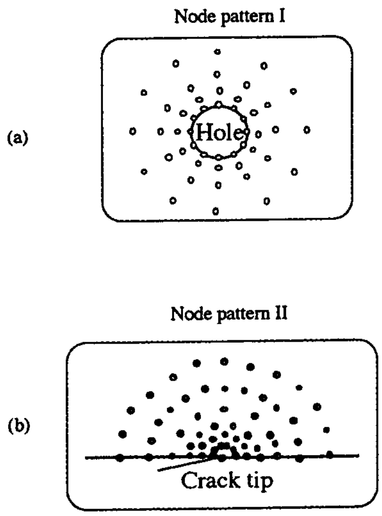


Fig. 2. Examples of locally-optimum node patterns for a crack and a hole

When these stress concentration fields exist closely to each other in the same analysis domain, a simple superposition of both local node patterns.

In the present method, the field A close to the hole and the field B close to the crack-tip are defined in terms of the membership functions used in the fuzzy set theory. For the purpose of simplicity, each membership function is given a function in the figure. In practice the membership function can be expressed as $\mu(x, y)$ in this particular example, and in 3D cases it is a function of 3D coordinates, i.e. $\mu(x, y, z)$. This procedure of node generation, i.e. the connection procedure of both node patterns, is summarized as follows :

- If $\mu A(x_p, y_p) \geq \mu B(x_p, y_p)$ for a node $p(x_p, y_p)$ belonging to the pattern A, then the node p is generated, and otherwise p is not generated.

- If $\mu A(x_q, y_q) \geq \mu B(x_q, y_q)$ for a node $q(x_q, y_q)$ belonging to the pattern B, then the node q is generated, and otherwise q is not generated.

It is apparent that the above algorithm can be easily extended to 3D problems and any number of node patterns. In addition, since finer node patterns are generally required to place near stress concentration sources, it is convenient to let the membership function correspond to node density as well.

2.2 Fuzzy control of node position

The fuzzy rules employed here can be generalized as :

$RULE^i$: IF p is A^i , THEN q is B^i

where $RULE^i$ is the i-th fuzzy rule, A^i and B^i the fuzzy variables, p the value of node, and Δp the difference of the current and the next values of p, i.e. $| p(n+1)-p(n) |$ (n : the iteration number of node), respectively. The labels of the fuzzy variables are defined as follows.

As for A^i,
LARGE → p is much larger than 1.0.
MEDIUM→ p is larger than 1.0.
SMALL → p is little larger than 1.0.

As for B^i,
LARGE → q is positive and large.
MEDIUM→ q is positive and medium.
SMALL → q is positive and small.

As shown in Fig. 3, trapezoid type membership functions are utililized as those of labes of A^i and B^i from the viewpoint of simplicity.

2.3 Mesh generation

As for 3D solid geometries, nodes are generated in the following order : vertices, edges, surfaces and domain. As an example, Fig. 4 shows a geometry model of surface crack in plate and the appearance of node generation by bucket method.

The Delaunay triangulation method[25-26] is used to generate tetrahedral elements from numerous nodes produced within a geometry. When the Delaunay triangulation method is utilized to generate elements in a geometry with cracks, mis-match elements across surface crack front tend to occur as shown in Fig. 5(a). To avoid the mis-match elements, node densities on the crack front are automatically controlled to be slightly higher than those near the crack as shown in Fig. 5(b).

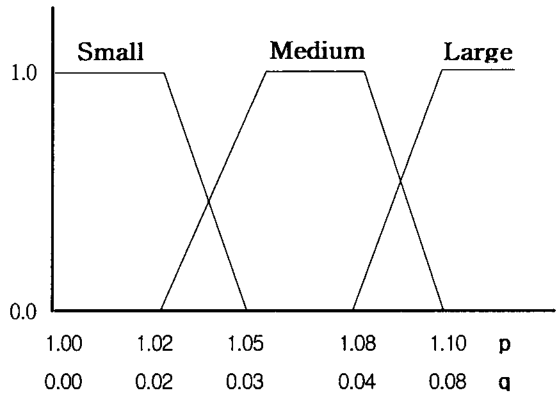


Fig. 3. Membership functions of labels of $A^i(p)$ and $B^i(q)$

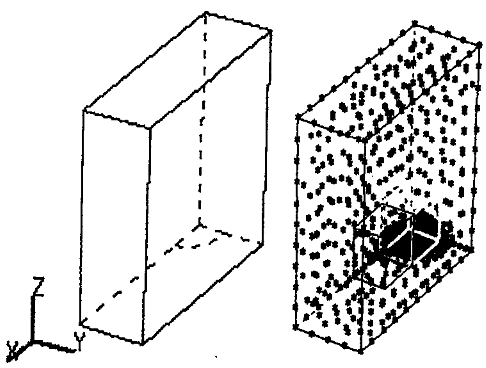


Fig. 4. Example of geometry model and node generation

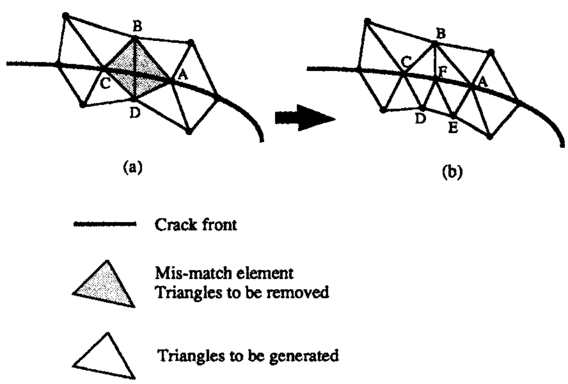


Fig. 5. Technique of avoiding mis-match elements

2.4 Calculation of Stress Intensity Factor

The FE model generated is automatically analyzed using MARC, and then displacements, strains and stresses are calculated. To obtain the stress intensity factors accurately as well as automatically, some techniques are employed.

2.4.1 Singular element

When ordinary quadratic tetrahedral elements are employed to calculated the stress intensity factor, a very fine mesh is required near crack front to capture $\sqrt{r}$ variation in displacements $1/\sqrt{r}$ and variation in stresses where r denotes the distance from crack front. To relax this situation, singular elements as shown in Fig. 6 are adopted[16,17]. In the singular element, the mid-point nodes near a crack front are shifted to the quarter-points. This conversion of ordinary tetrahedral elements along a front of 3D crack to the singular elements is automatically performed in the last stage of the creation of a FE model.

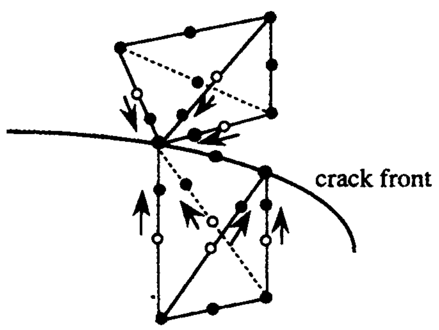


Fig. 6. Conversion of ordinary quadratic tetrahedral elements along crack front into singular elements

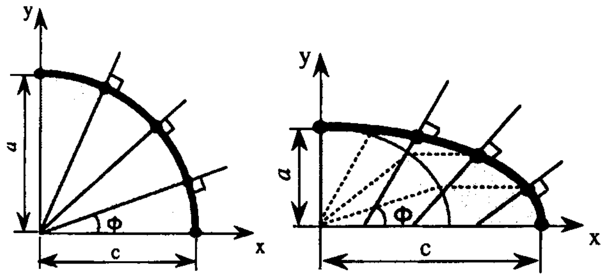
2.4.2 Calculation of stress intensity factors

Stress intensity factors are computed using the displacement extrapolation method[2,13]. Nodal displacements calculated along the crack face are substituted in the following crack tip displacement equation :

$$K = \frac{E'}{4}\sqrt{2\pi}\lim_{r\to 0}\frac{w}{\sqrt{r}} \quad E' = \begin{cases} \frac{E}{1-\nu^2} & (\text{for } \textit{plane strain}) \\ E & (\text{for } \textit{plane stress}) \end{cases} \qquad (1)$$

where w is a nodal displacement, and E' is equal to E in the plane stress condition or E/(1- ν 2) in the plane strain condition. Only positions of free surface intersection are regarded as in the plane stress condition, while other positions are in the plane strain condition. Radial lines for the semi-elliptical surface crack front and those of the semi-circular one are defined as in Fig. 7.

For each radial line with an elliptic angle ϕ, the least square method is applied to evaluate the SIF. In this least square operation, the K value evaluated at the shifted quarter point is neglected. This displacement extrapolation method is popularly used to calculate the stress intensity factor. In the present study, this process is fully automated. When a crack is designated by a user in the definition process of a geometry model, radial lines for the crack front are automatically determined. After the stress analysis using MARC, displacement distributions are interpol- ated along the radial lines, on each of which the stress intensity factor is calculated by the least square method.



(a) Half semi-circular(a=c) (b) Half semi-elliptical

Fig. 7. Definition of radial lines to calculate the stress intensity factors

3. Results and Discussion

In order to examine efficiency and accuracy of the present system, a semi-elliptical surface crack in a inhomogeneous plate of width 2b, thickness t and height 2h subjected to uniform tension as shown in Fig. 8 was analyzed.

Fig. 9 shows a typical finite element mesh of a quarter portion of a plate with a semi-elliptical surface crack generated by the present system. The mesh consists of 6,482 quadratic tetrahedral elements and 10,294 nodes. Nodes and elements are generated in about 90 seconds and in about 45 seconds, respectively. To complete this mesh, the following three node patterns are utilized : (a) the base node pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special node pattern for the semi-elliptical surface crack, and (c) a special node pattern in which node density is getting coarser departing from the bottom face including the surface crack and the ligament section.

The analysis was performed for aspect ratio of a/c=0.6 and crack depth of a/t=0.2. Young's modulus E and Poisson's ratio ν were assumed to be 210 GPa and 0.3, respectively. Fig. 10 shows the comparison between the present solutions and Newman-Raju's solution[1,20]. The SIF K for this crack configuration can be often expressed as :

$$K = \sigma\sqrt{\pi a/Q}\ F(a/c,\ \phi,\ a/t) \quad (2)$$

where Q is the squared complete elliptical integral of the second kind, and its approximate form is given as :

$$Q = 1+1.464\left(\frac{a}{c}\right)^{1.65} \quad (3)$$

It can be seen from the figure that the present results using the singular elements agree well with Raju-Newman's solutions within 2 to 3% difference.

Among a whole process, the interactive operations to be done by a user, i.e. the definition of a geometry model, the designation of local node patterns and the assignment of material properties and boundary conditions are performed in about 2 minutes. The other processes are fully automatically performed.

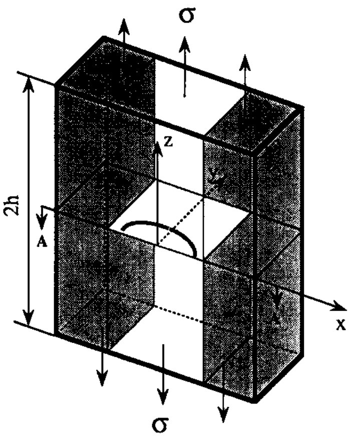


Fig. 8. Inhomogeneous plate with a semi-elliptical surface crack subjected to uniform tension

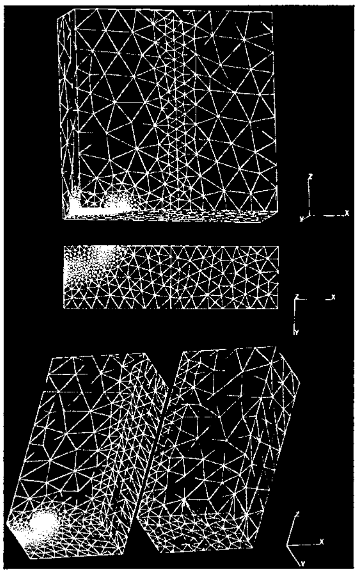


Fig. 9. A typical mesh of a quarter portion

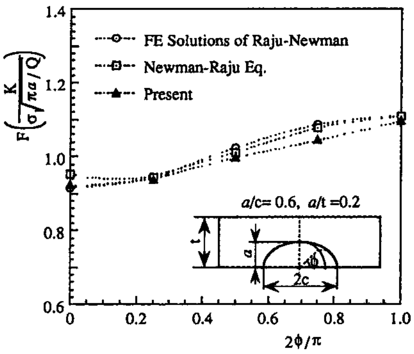


Fig. 10. Comparison of present stress intensity factor with Newman-Raju solutions

4. Conclusions

A new automated fuzzy-based system for analyzing the stress intensity factors of 3D cracks was developed in the present study. The automatic finite element mesh generation technique based on the fuzzy knowldege processing and the computational geometry techniques were integrated in the system, together with one of commercial finite element programs. Here interactive operations to be done by a user can be performed in about few minutes even for complicated problems of surface crack in a inhomogeneous plate. The other processes are fully automated being able to be performed in several minutes in a popular engineering workstation environment. To demonstrate practical performances of the present system, the system was used to the analyses of surface crack in inhomogeneous plate subjected to uniform tension.

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