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Design of a Fuzzy Logic Controller for a Rotary-type Inverted Pendulum System

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Abstract

Various inverted pendulum systems have been frequently used as a model for the performance test of the proposed control system. We first identify a rotary-type inverted pendulum system by the Euler-Lagrange method and then design a FLC (Fuzzy Logic Controller) for the plant. FLC's are one of useful control schemes for plants having difficulties in deriving mathematical models or having performance limitations with conventional linear control schemes. Many FLC's imitate the concept of conventional PD (Proportional-Derivative) or PI (Proportional-Integral) controller. That is, the error e and the change-of-error $\dot{e}$ are used as antecedent variables and the control input u or the change of control input Δu is used as its consequent variable for FLC's. In this paper we design a simple-structured FLC for the rotary inverted pendulum system. We also perform some computer simulations to examine the tracking performance of the closed-loop system.

Key Words : Fuzzy Logic Control, simple-structured FLC, switching line, signed distance, rotary inverted pendulum system

I. Introduction

Most physical systems are nonlinear and such property makes difficult to identify the system mathematically. So, a nonlinear system has been frequently used to verify the performance of the designed system. Various inverted pendulum systems have been widely used for a test model. One of those is the rail-cart type which consists of a cart running on a rail and a pendulum attached to the cart. In this case it has the moving limitation of its cart as a restriction of the control system. Recently, there has been introduced many kinds of inverted pendulum systems. We consider a rotary-type inverted pendulum system.

Fuzzy Logic Controllers are one of useful control schemes for plants having difficulties in deriving mathematical models or having performance limitations with conventional linear control schemes. Most works in fuzzy control fields use the error e and the change-of-error $\dot{e}$ as antecedent variables of the fuzzy control rule regardless of complexity of controlled plants. Either control input u (PD-type) or incremental control input Δu (PI-type) is typically used as its consequent variable [1-2]. This scheme naturally comes from the concept of the conventional PD or PI control algorithm. Although such FLC's are proper to simple second order plants, all process states are typically required as fuzzy input variables for complex higher order plants. That is, all state variables must be used to represent contents of the rule antecedent ("if" part of a rule). However it requires a huge number of control rules, which makes difficult to design a FLC with good performance.

Many researches has been introduced to improve the performance of FLC's. Tang and Mulholland [3] developed a relation between the scaling factors of fuzzy controller and the control gains of equivalent linear PI controller. They also showed that the FLC can be used as a multiband control. Li and Gatland [4-5] proposed a more systematic design method for the PD and PI-type FLC's. They also presented a simplified rule generation method using two two-dimensional spaces instead of a three-dimensional space for a PID-type FLC. Palm [6] proposed a sliding mode fuzzy controller which generates the absolute value of a switching magnitude in the sliding mode control law using the error and the change-of-error. As mentioned above most of researches use two or more as antecedent variables for a control rule.

Strugen and Loscutoff [7] presented a design of a dynamic stabilizer for a double inverted pendulum system. They linearized around target and then designed a linear controller. Furuta and Yamakita [8] proposed a swing up controller for a rotary-type inverted pendulum system.

In this paper, we suggest a simple-structured fuzzy logic control for a rotary-type inverted pendulum system. We first identify it mathematically. And then we explain the design of a simple-structured FLC in Section III. The conventional 2-input FLC has a control rule table which is established on a two-dimensional space of the phase plane (e, $\dot{e}$). Then the table shows the skew-symmetry for controlled plants with the minimum phase property. This property makes possible to design a simple-structured FLC for a rotary-type inverted pendulum system. In Section IV, we compare the control

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performance between the conventional 2-input FLC and a simple-structured FLC for a rotary-type inverted pendulum system. Finally, we present a concluding remarks in Section V.

II. Rotary-type Inverted Pendulum System

The inverted pendulum system has been widely used as a test model to show the performance of the proposed control system. We here consider a rotary-type inverted pendulum system that deals successfully with some moving limitation of a rail-cart type. Figure 1 illustrates the structure of a rotary-type inverted pendulum system. A servo motor is used as an actuator and the pendulum is attached to the rotating shaft of the motor.

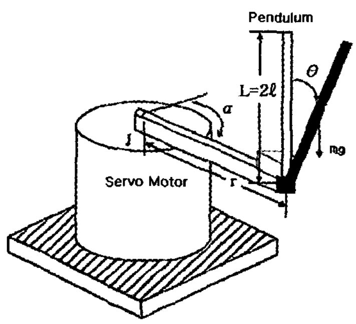


Fig. 1. Structure of a rotary-type inverted pendulum system.

Consider the identification of a rotary-type inverted pendulum system. In Fig. 1, α and θ represent deflections of rotating arm and pendulum, respectively. r and J represent a total length and inertia of the arm, respectively. l denotes a distance to the gravity center of the pendulum. A mathematical model can be obtained by the Euler-Lagrange method [9]. The kinetic energy K_i and the potential energy P_i are obtained as follows. Here subscripts 0 and 1 stand for variables for the arm and pendulum, respectively.

The kinetic and potential energy in the pendulum:

$$\begin{aligned} K_1 &= \frac{1}{2} m[(\dot{\alpha} r + \dot{\theta} l \cos\theta)^2 + (\dot{\theta} l \sin\theta)^2] \\ &= \frac{1}{2} m[(\dot{\alpha}^2 r^2 + \dot{\theta}^2 l^2 \cos^2\theta + 2\dot{\alpha}\dot{\theta} rl\cos\theta) + \dot{\theta}^2 l^2 \sin^2\theta] \\ &= \frac{1}{2} m[\dot{\alpha}^2 r^2 + \dot{\theta}^2 l^2 + 2\dot{\alpha}\dot{\theta} rl\cos\theta] \end{aligned} \tag{1}$$

$$P_1 = mgl\cos\theta. \tag{2}$$

The kinetic and potential energy in the arm:

$$K_2 = \frac{1}{2} J\dot{\alpha}^2 \tag{3}$$

$$P_2 = 0. \tag{4}$$

Then the Lagrangian is as follows:

$$\begin{aligned} L &= \text{kinetic energy}(K_i) - \text{potential energy}(P_i) \\ &= (K_1 + K_2) - (P_1 + P_2) \\ &= \frac{1}{2}(J + mr^2)\dot{\alpha}^2 + m\dot{\alpha}\dot{\theta} rl\cos\theta + \frac{1}{2} m\dot{\theta}^2 l^2 - mgl\cos\theta \end{aligned} \tag{5}$$

Thus, the Lagrangian equation is determined as followed.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0 \tag{6}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\alpha}}\right) - \frac{\partial L}{\partial\alpha} = \tau \tag{7}$$

Substituting Eq.(5) for Eq.(6)

$$\frac{\partial L}{\partial\dot{\theta}} = ml^2\dot{\theta} + m\dot{\alpha} rl\cos\theta$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = ml^2\ddot{\theta} + m\ddot{\alpha} rl\cos\theta - m\dot{\alpha}\dot{\theta} rl\sin\theta \\ \frac{\partial L}{\partial\theta} = -m\dot{\alpha}\dot{\theta} rl\sin\theta + mgl\sin\theta \end{cases},$$

we get the following equation:

$$ml^2\ddot{\theta} + m\ddot{\alpha} rl\cos\theta - mgl\sin\theta = 0 \tag{8}$$

Substituting Eq.(5) for Eq.(7)

$$\frac{\partial L}{\partial\dot{\alpha}} = (J + mr^2)\dot{\alpha} + m\dot{\theta} rl\cos\theta$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\alpha}}\right) = (J + mr^2)\ddot{\alpha} + m\ddot{\theta} rl\cos\theta - m\dot{\theta}^2 rl\sin\theta \\ \frac{\partial L}{\partial\theta} = 0 \end{cases},$$

we get the following equation:

$$(J + mr^2)\ddot{\alpha} + m\ddot{\theta} rl\cos\theta - m\dot{\theta}^2 rl\sin\theta = \tau \tag{9}$$

Consequently, the rotary-type inverted pendulum system is modeled by Eqs. (8) and (9).

III. Simple-structured Fuzzy Logic Controller

A simple-structured FLC is designed for FLC's with the skew-symmetric property in the control rule table [10]. We first consider a 2-nd order plant and then extend to n-th order case. Let the controlled plant be a system with n-th order (linear or nonlinear) state equation:

$$\begin{aligned} x^{(n)} &= f(\mathrm{x}, t) + b(\mathrm{x}, t)u(t) + d(t), \\ y &= x, \end{aligned} \tag{10}$$

with

$$\begin{aligned} \mathrm{x} &= [x_1, x_2, \cdots, x_n]^T \\ &= [x, \dot{x}, \cdots, x^{(n-1)}]^T, \end{aligned} \tag{11}$$

where $f(\mathrm{x}, t)$ and $b(\mathrm{x}, t)$ are partially known continuous functions, $d(t)$ is the unknown external disturbance, and $u(t) \in R$ and $y(t) \in R$ are the input and output of the system, respectively. $x(t) \in R^n$ is the process state vector.

The control problem is to force $y(t)$ to follow a given bounded reference input signal $x_d(t)$. Let $e(t)$ be the tracking error vector as follows

$$\begin{aligned} e(t) &= x(t) - x_d(t) \\ &= [e, \dot{e}, \cdots, e^{(n-1)}]^T. \end{aligned} \tag{12}$$

The rule form for the conventional 2-input FLC using two fuzzy input variables of the error and the change-of-error is as follows:

R^{ij}_{old} : If e is LE_i and $\dot{e}$ is LDE_j, then u is LU_{ij}

where $i = 1, 2, \cdots, M$, $j = 1, 2, \cdots, N$, and LE, LDE, and LU are the linguistic values taken by the process state variables e, $\dot{e}$, and u, respectively. Here the number of control rules is $M \times N$.

We now consider a rule table of the conventional 2-input FLC with the control rule form R^{ij}_{old}. As each number of linguistic values for error, change-of-error and control input is five, a typical rule table is established on the space of the error and the change-of error like Table 1 with 25 rules.

Table 1. Rule table for the conventional 2-input FLC.

$\dot{e}$ \ e	LE_{-2}	LE_{-1}	LE_0	LE_1	LE_2
LDE_2	LU_0	LU_{-1}	LU_{-1}	LU_{-2}	LU_{-2}
LDE_1	LU_1	LU_0	LU_{-1}	LU_{-1}	LU_{-2}
LDE_0	LU_1	LU_1	LU_0	LU_{-1}	LU_{-1}
LDE_{-1}	LU_2	LU_1	LU_1	LU_0	LU_{-1}
LDE_{-2}	LU_2	LU_2	LU_1	LU_1	LU_0

In Table 1, subscripts -2, -1, 0, 1, and 2 denote fuzzy linguistic values of Negative Big (NB), Negative Small (NS), ZeRo (ZR), Positive Small (PS), and Positive Big (PB), respectively.

Most rule tables for the minimum phase plants have a skew-symmetric property like Table 1, namely, $\tilde{u}_{ij} = -\tilde{u}_{ji}$. Note that the boundaries of $(e, \dot{e})$ for the same control input LU_k have staircase shapes and the absolute magnitude of the control input $|u|$ is approximately proportional to the distance from the main diagonal line. If the quantization levels of the independent variables become infinitesimal, the boundaries of Table 1 become straight lines as shown in Fig. 2.

Then the control law describes the multilevel relay controller with five bands. Also, note that the absolute magnitude of the control input is proportional to the distance from the following straight line called the switching line.

$$s_l: \quad \dot{e} + \lambda e = 0, \tag{13}$$

where $\lambda > 0$ is a slope of the switching line. Note that the control inputs above and below the switching line have opposite signs.

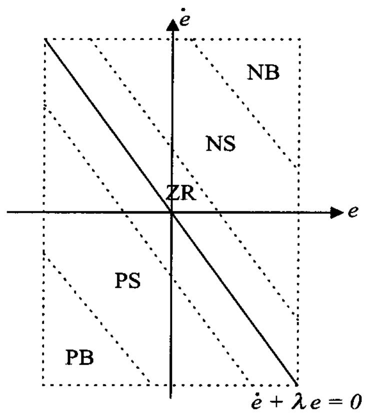


Fig. 2. Rule table with infinitesimal quantization levels.

We now introduce a new variable called the signed distance, d_s. That is, it is a distance between the switching line and an operating point and has a sign as follows:

$$d_s = \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}}, \tag{14}$$

Since the sign of the control input is negative for $s_l > 0$ and positive for $s_l < 0$ and its absolute magnitude is proportional to the distance from the line $s_l = 0$, we can reduce the relation between the control input and the distance as follows:

$$u \propto -d_s. \tag{15}$$

Now the control rule table can be established on an one-dimensional space of d_s instead of a two-dimensional space of e and $\dot{e}$. That is, the control action is now determined by d_s only. The rule form for a simple-structured FLC is given as follows:

R^k_{new}: If d_s is LDL_k then u is LU_k,

where LDL_k is the linguistic value of the signed distance in the k-th rule. In the simple-structured FLC, the number of rules is greatly reduced compared to the case of the conventional 2-input FLC. Hence, we can easily add or modify rules for fine control.

Table 2. Rule table for the simple-structured FLC.

d_s	LDL_{-2}	LDL_{-1}	LDL_0	LDL_1	LDL_2
u	LU_2	LU_1	LU_0	LU_{-1}	LU_{-2}

Above scheme can be extended to the case of a general n-input FLC. The general n-input FLC has rules of the following form:

R^k_{GO} : If e_1 is LE^1_k, e_2 is LE^2_k, $\cdots$, and e_n is LE^n_k then u is LU_k,

$$k = 1, 2, \cdots, m^n,$$

where m is the number of fuzzy sets for each fuzzy input variable and LE_k^i ($i = 1, 2, \cdots, n$) is the linguistic value taken by the process state variable e_i $(= x^{(i-1)} - x_d^{(i-1)})$ in the k-th rule. In this case, the rule table is established on n-dimensional space of e_1, e_2, $\cdots$, and e_n.

Similar to the two-dimensional rule table of Table 1, the n-dimensional one for R_{GO}^k also satisfies the skew-symmetric property and the absolute magnitude of the control input is proportional to the distance from its main diagonal hyperplane (instead of the diagonal line in the two-dimensional table). That is, the switching line s_l is changed to the following switching hyperplane S_l.

$$S_l : e^{(n-1)} + \lambda_{n-1} e^{(n-2)} + \cdots \lambda_2 \dot{e} + \lambda_1 e = 0. \quad (16)$$

Also, d_s of Eq. (14) is changed to the generalized signed distance D_s as follows:

$$D_s = \frac{e^{(n-1)} + \lambda_{n-1} e^{(n-2)} + \cdots \lambda_2 \dot{e} + \lambda_1 e}{\sqrt{1 + \lambda_{n-1}^2 + \cdots \lambda_2^2 + \lambda_1^2}}. \quad (17)$$

That is, D_s represents the signed distance from the operating point to the switching hyperplane of Eq. (16). Then the rule table is equivalent to Table 2 except D_s instead of d_s. From Eq. (17) we can see that the generalized signed distance, D_s, contains knowledge of all process states as well as the error and the change-of-error.

IV. Simulation Example

Now we compare the control performance of two FLC's for the rotary-type inverted pendulum system via computer simulations. Parameters of Eqs. (8) and (9) are given as Table 3.

Table 3. Parameters for a rotary-type inverted pendulum system.

symbol	value [unit]
r	0.145 [m]
J	0.0044 [kg · m²]
m	0.21 [kg]
l	0.305 [m]

Fuzzy sets for simulations are shown in Fig. 3. Namely, we used all the same fuzzy sets for error, change-of-error, signed distance and control input. The scaling factor K for error, change-of-error, signed distance and control input is 0.05, 6, 6, and 1, respectively. We also use the product inference and the center-average defuzzification method.

We consider a tracking performance for the rotary-type inverted pendulum system. Here (a) and (b) are the cases of the conventional 2-input FLC and the simple-structured FLC, respectively. Here we used 0.83×10^{-2} for λ in the

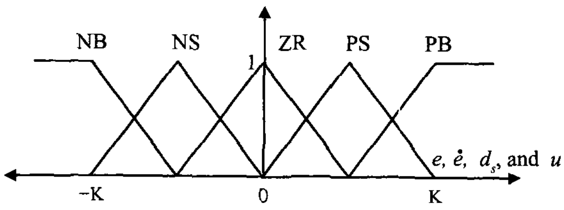
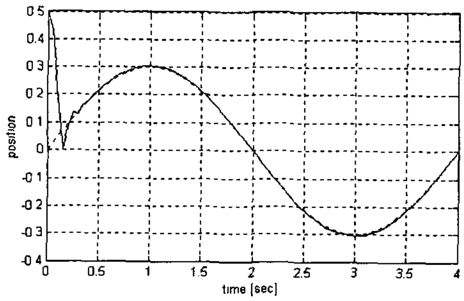
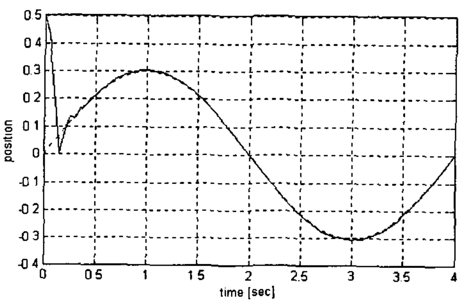


Fig. 3. The fuzzy sets for simulations.

simple-structured FLC. Figures 4, 5 and 6 show the simulation results of tracking performances, control inputs, and tracking errors, respectively. As shown in figures, the control performances are almost the same even if the simple-structured FLC has only 5 control rules.

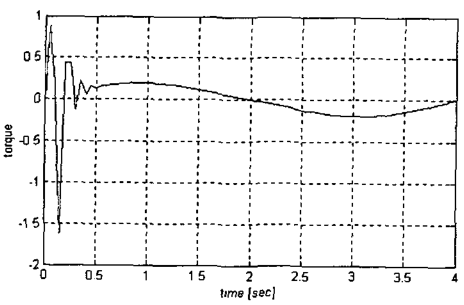


(a) 2-input FLC.

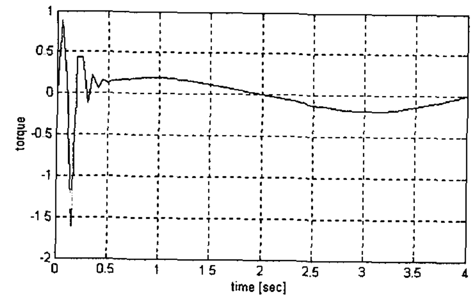


(b) Simple-structured FLC.

Fig. 4. Comparison of tracking performances.

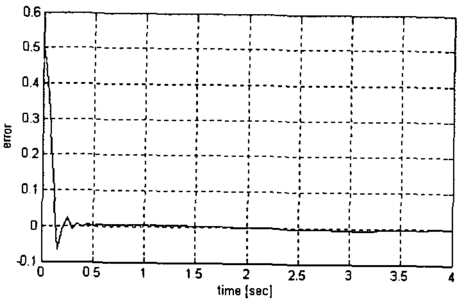


(a) 2-input FLC.

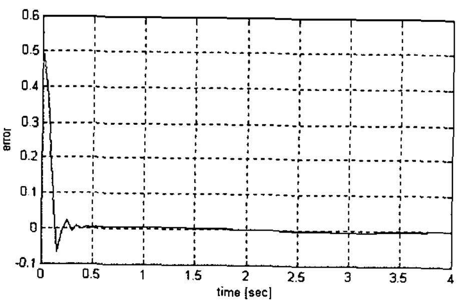


(b) Simple-structured FLC.

Fig. 5. Comparison of control inputs.



(a) 2-input FLC.



(b) Simple-structured FLC.

Fig. 6. Comparison of tracking errors.

V. Concluding Remarks

We compared the performance of the conventional 2-input FLC and the simple-structured FLC for a rotary-type inverted pendulum system. We first identified the plant using the Euler-Lagrange method. Next, we observed some properties of the rule table for the conventional PD or PI-type FLC. It represented the skew symmetry and the absolute magnitude of control input was proportional to the distance from its main diagonal line. These properties were also satisfied in the general n-input FLC. This facts allowed us to derive a simple-structured FLC which uses only a single antecedent variable. Finally, we showed that the control performance of the simple-structured FLC is nearly the same as that of the conventional 2-input FLC through computer simulations. Thus, the simple-structured FLC has some advantages: The number of fuzzy rules was greatly reduced and thus the computational complexity was mitigated. Also, generation, modification, and tuning of control rules were much easier.

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