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The degrees of fuzzy net-convergences on complete MV-algebras

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Abstract

In this paper, we introduce the degrees of fuzzy net-convergences in L-fuzzy topologies using complete MV-algebras. We investigate the relationships among the degrees of fuzzy convergent, fuzzy cluster and fuzzy adherent points. We study the properties of net-convergences.

Key Words : Complete MV-algebra, Neighborhood systems, The degrees of fuzzy adherent points (fuzzy cluster points,fuzzy convergent points)

1. Introduction

Ward and Dilworth [10] introduced residuated lattices as the foundation of the algebraic structures of fuzzy logics. Hajeck [2] introduced a BL-algebra which is a general tool of a fuzzy logic. Recently, Hohle [3,4] extended the fuzzy set $f:X\to L$ where L is a complete MV-algebra in stead of an unit interval I or a lattice L. The complete MV-algebra is an important tool which generalizes a completely distributive lattice, a Boolean Agebra, a continuous t-norm and t-conorm. It is a remarkable work to apply fuzzy topologies to fuzzy logics. On the other hand, Pu Pao-Ming and Ying-Ming Liu [8] introduced the convergence theory in fuzzy topologies with quasi coincident neighborhood systems. Ying [11] introduced the neighborhood systems as a new method. Kim and Ko [5] introduced neighborhood systems in L-fuzzy topologies in a view of [11] using complete MV-algebras.

In this paper, we introduce the degrees of fuzzy net-convergences in L-fuzzy topologies using complete MV-algebras. We investigate the relationships among the degrees of fuzzy convergent, fuzzy cluster and fuzzy adherent points. We study the properties of net-convergences.

2. Preliminaries

Definition 2.1 [4.9] A lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$ is called a residuated lattice if it satisfies the following conditions: for each $x,y,z\in L$,

(R1) $(L,\odot,1)$ is a commutative monoid,

(R2) if $x\le y$, then $x\odot z\le y\odot z$ ($\odot$ is an isotone operation),

(R3) (Galois correspondence): $(x\odot y)\le z$ iff $x\le y\to z$.

In a residuated lattice L, $x^*=(x\to 0)$ is called complement of $x\in L$.

Definition 2.2 [4,9] A residuated lattice $(L,\le,\wedge,\vee,\odot,\to,0,1)$is called a BL-algebra if it satisfies the following conditions:for each $x,y\in L$,

(B1) $x\wedge y=x\odot(x\to y)$,

(B2) $x\vee y=[(x\to y)\to y]\wedge[(y\to x)\to x]$,

(B3) $(x\to y)\vee(y\to x)=1$.

Definition 2.3 [4,9] A BL-algebra L is called an MV-algebra if $x=x^{**}$ for each $x\in L$.

Definition 2.4 [4,9] An MV-algebra L is called complete if $\bigwedge_{i\in\Gamma}x_i\in L$ and $\bigvee_{i\in\Gamma}x_i\in L$ for any $x_i\in L$.

Theorem 2.1 [4,9] Let L be a complete MV-algebra.

For each $x\in L$, $\{y_i\mid i\in\Gamma\}\subset L$, we have the following properties.

(1) $\bigwedge_{i\in\Gamma}y_i^*=(\bigvee_{i\in\Gamma}y_i)^*$.

(2) $\bigvee_{i\in\Gamma}y_i^*=(\bigwedge_{i\in\Gamma}y_i)^*$.

(3) $\bigwedge_{i\in\Gamma}(x\odot y_i)=x\odot(\bigwedge_{i\in\Gamma}y_i)$.

(4) $\bigvee_{i\in\Gamma}(x\odot y_i)=x\odot(\bigvee_{i\in\Gamma}y_i)$.

(5) $x\odot y=(x\to y^*)^*$.

(6) $x\le y$ iff $x^*\ge y^*$.

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Throughout this paper, let L be a complete MV-algebra. The class of all fuzzy sets on a set X will be denoted by L^X and the fuzzy sets by the Greek symbols λ, μ, ν, etc.

Definition 2.5 [4] All algebraic operations on L can be extended pointwise to the set L^X as follows:

$$\mu \rightarrow \rho \text{ iff } \mu(x) \rightarrow \rho(x), \text{ for all } x \in X,$$

$$(\mu \odot \rho)(x) = \mu(x) \odot \rho(x), \text{ for all } x \in X.$$

The set of all fuzzy points in X is denoted by $Pt(X)$. For $x_t \in Pt(X)$, $x_t \in \lambda$ iff $t \leq \lambda(x)$. All the other notations and the other definitions are standard in fuzzy set theory.

Definition 2.6 [1] A subset T of L^X is called an L-fuzzy topology on X if it satisfies the following conditions:

(O1) $\overline{0}, \overline{1} \in T$, where $\overline{0}(x) = 0$ and $\overline{1}(x) = 1$ for all $x \in X$.

(O2) If $\mu_1, \mu_2 \in T$, $\mu_1 \wedge \mu_2 \in T$.

(O3) If $\mu_i \in T$ for each $i \in \Gamma$, $\bigvee_{i \in \Gamma} \mu_i \in T$.

The pair (X, T) is called an L-fuzzy topological space.

Definition 2.7 [5] Let $\lambda \in L^X$ and $x_p \in Pt(X)$. Then the degree to which x_p belongs to λ is

$$[x_p \rightarrow \lambda] = p \rightarrow \lambda(x).$$

Definition 2.8 [5] Let (X, T) be an L-fuzzy topological space, $\mu \in L^X$ and $e \in Pt(X)$. Then the degree to which λ is a neighborhood of e is defined by

$$N_e(\lambda) = \bigvee \{ [e \rightarrow \mu] \mid \mu \leq \lambda, \ \mu \in T \}.$$

A mapping $N_e : L^X \rightarrow L$ is called the fuzzy neighborhood system of e.

Theorem 2.2 [5] Let (X, T) be an L-fuzzy topological space and N_e the fuzzy neighborhood system of e. For $\lambda, \mu \in L^X$, it satisfies the following properties:

(1) $N_e(\overline{0}) = [e \rightarrow \overline{0}]$ and $N_e(\overline{1}) = 1$.

(2) $N_e(\lambda) \leq [e \rightarrow \lambda]$.

(3) $N_e(\lambda) \leq N_e(\mu)$, if $\lambda \leq \mu$.

(4) $N_e(\lambda) \wedge N_e(\mu) \leq N_e(\lambda \wedge \mu)$.

(5) $N_e(\lambda)$
$\leq \bigvee \{ N_e(\mu) \mid \mu \leq \lambda, [d \rightarrow \mu] \leq N_d(\mu, r) \, \forall d \in Pt(X) \}$.

(6) $N_{x_p}(\lambda) = p \rightarrow N_{x_1}(\lambda)$, for each $x_p \in Pt(X)$.

Definition 2.9 [5] Let (X, T) be an L-fuzzy topological space , $\lambda \in L^X$ and $e \in Pt(X)$. Then the degree to which e is an adherent point of λ is defined by

$$Ad_e(\lambda) = N_e(\lambda^*)^*.$$

Theorem 2.3 [5] Let (X, T) be an L-fuzzy topological space .For each $\lambda \in L^X$, we define operators $C_T, I_T : L^X \rightarrow L^X$ as follows:

$$C_T(\lambda) = \bigwedge \{ \rho \in L^X \mid \lambda \leq \rho, \rho^* \in T \},$$

$$I_T(\lambda) = \bigvee \{ \nu \in L^X \mid \nu \leq \lambda, \nu \in T \}.$$

For each $\lambda \in L^X, e, x_t \in Pt(X)$, we have the following properties.

(1) $I_T(\lambda^*) = C_T(\lambda)^*$.

(2) $[e \rightarrow I_T(\lambda)] = N_e(\lambda)$.

(3) $[e \rightarrow C_T(\lambda)^*] = Ad_e(\lambda)^*$.

(4) $Ad_{x_t}(\lambda) = [x_t \odot Ad_{x_1}(\lambda)]$.

3. The degees of fuzzy net convergences

Definition 3.1[7,8] Let D be a directed set. A function $S: D \rightarrow Pt(X)$ is called a fuzzy net. Let $\lambda \in I^X$.

(1) S is a fuzzy net in λ if $S(n) \in \lambda$ for every $n \in D$.

(2) S is often in λ if for each $n \in D$, there exists $n_0 \in D$ such that $n_0 \geq n$ and $S(n_0) \in \lambda$.

(3) S is finally in λ if there exists $n_0 \in D$ such that for each $n \in D$ with $n \geq n_0$, we have $S(n) \in \lambda$.

Definition 3.2 [7,8] Let $S: D \rightarrow Pt(X)$ and $T: E \rightarrow Pt(X)$ be two fuzzy nets. A fuzzy net T is called a subnet of S if there exists a function $N: E \rightarrow D$, called by a cofinal selection on S, such that:

(1) $T = S \circ N$.

(2) For every $n_0 \in D$, there exists $m_0 \in E$ such that $N(m) \geq n_0$, for $m \geq m_0$.

Definition 3.3 Let (X, T) be an L-fuzzy topological space, $\lambda \in I^X$ and $e \in Pt(X)$.

(1) The fuzzy convergent degree of S to e is defined by

$$Con_e(S) = \bigwedge \{ N_e(\lambda)^* \mid S \text{ is often in } \lambda^* \},$$

(2) The fuzzy cluster degree of S to e is defined by

$$Cl_e(S) = \bigwedge \{ N_e(\lambda)^* \mid S \text{ is finally in } \lambda^* \}.$$

Theorem 3.1 Let (X, T) be an L-fuzzy topological space.Let $S: D \rightarrow Pt(X)$ be a fuzzy net and $W: E \rightarrow Pt(X)$ a subnet of S. Then we have the following properties.

(1) $Con_e(S) \le Cl_e(S)$.

(2) $Cl_e(W) \le Cl_e(S)$.

(3) $Con_e(S) \le Con_e(W)$.

(4) $Con_{x_t}(S) = [x_t \odot Con_{x_1}(S)]$.

(5) $Cl_{x_t}(S) = [x_t \odot Cl_{x_1}(S)]$.

Proof. (1) If S is finally in λ^*, S is often in λ^*. Hence

$$Con_e(S) = \bigwedge\{N_e(\lambda)^* \mid S \text{ is often in } \lambda^*\}$$
$$\le \bigwedge\{N_e(\lambda)^* \mid S \text{ is finally in } \lambda^*\}$$
$$= Cl_e(S).$$

(2) If S is finally in λ^*, W is finally in λ^*. Hence

$$Cl_e(W) = \bigwedge\{N_e(\lambda)^* \mid W \text{ is finally in } \lambda^*\}$$
$$\le \bigwedge\{N_e(\lambda)^* \mid S \text{ is finally in } \lambda^*\}$$
$$= Cl_e(S).$$

(3) Let W be often in λ^*. We will show that S is often in λ^*. Let $n \in D$. Since $W: E \to Pt(X)$ is a subnet of S, there exists a cofinal selection $N: E \to D$. For each $n \in D$, there exists $m \in E$ such that $N(k) \ge n$ for $k \ge m$. Since W is often in λ^*, for $m \in E$, there exists $m_0 \in E$ such that $m_0 \ge m$ for $W(m_0) \in \lambda^*$.

Put $n_0 = N(m_0)$. Then $n_0 \ge n$ and

$$S(n_0) = S(N(m_0)) = W(m_0) \in \lambda^*.$$

Thus, S is often in λ^*. Hence

$$Con_e(S) = \bigwedge\{N_e(\lambda)^* \mid S \text{ is often in } \lambda^*\}$$
$$\le \bigwedge\{N_e(\lambda)^* \mid W \text{ is often in } \lambda^*\}$$
$$= Cl_e(W).$$

(4)

$$Con_{x_t}(S) = \bigwedge\{N_{x_t}(\lambda)^* \mid S \text{ is often in } \lambda^*\}$$
$$= \bigwedge\{(t \to N_{x_1}(\lambda))^* \mid S \text{ is often in } \lambda^*\}$$
(by Theorem 2.2(6))
$$= \bigwedge\{[x_t \odot N_{x_1}(\lambda)^*] \mid S \text{ is often in } \lambda^*\}$$
(by Theorem 2.1(5))
$$= [x_t \odot \{\bigwedge N_{x_1}(\lambda)^* \mid S \text{ is often in } \lambda^*\}]$$
(by Theorem 2.1(3))
$$= [x_t \odot Con_{x_1}(S)]$$

(5) It is similarly proved as (4).

Example 3.1 Let $L = ([0,1], \le, \wedge, \vee, \odot, \to, 0, 1, ^*)$ be a complete MV-algebra defined by (called a Lukasiewicz logic, ref.[6,9])

$$a \to b = \min\{1, 1 - a + b\}$$
$$a \odot b = \max\{0, a + b - 1\}.$$

Then $a^* = a \to 0 = 1 - a$.

Let $X = \{x, y\}$ be a set and $\mu \in L^X$ as follows:

$$\mu(x) = 0.3, \quad \mu(y) = 0.4.$$

We define an L-fuzzy topology as follows:

$$T = \{\overline{0}, \overline{1}, \mu\}.$$

(1) In general, $Con_e(S) \ne Cl_e(S)$.

Let N be a natural number set. Define a fuzzy net $S: N \to Pt(X)$ by

$$S(n) = x_{a_n}, \quad a_n = 0.6 + (-1)^n 0.2.$$

Let $e = x_{0.5}$. Since S is often in μ^* or $\overline{1}$, for $\mu, \overline{0} \in T$, we have

$$Con_e(S) = \bigwedge\{N_e(\lambda)^* \mid S \text{ is often in } \lambda^*\}$$
$$= (\bigvee\{N_e(\lambda) \mid S \text{ often in } \lambda^*\})^*$$
(by Theorem 2.1(1))
$$= N_e(\mu)^*$$
$$= 1 - N_e(\mu)$$
$$= 1 - [x_{0.5} \to \mu] = 0.2.$$

Similarly, since S is finally in $\overline{1}$,

$$Cl_e(S) = N_e(\overline{0})^*$$
$$= 1 - N_e(\overline{0})$$
$$= 1 - [x_{0.5} \to \overline{0}] = 0.5.$$

Hence $Con_e(S) \le Cl_e(S)$ but $Con_e(S) \ne Cl_e(S)$.

(2) In general, $Cl_e(W) \ne Cl_e(S)$, for a fuzzy subnet W of S. By (1), we define a subnet $W: N \to Pt(X)$ of S by

$$W(n) = S(2n+1) = 0.4.$$

Since W is finally in μ^* or $\overline{1}$,

$$Cl_e(W) = N_e(\mu)^*$$
$$= 1 - N_e(\mu)$$
$$= 1 - [x_{0.5} \to \mu] = 0.2.$$

Hence $Cl_e(W) \le Cl_e(S)$ but $Cl_e(W) \ne Cl_e(S)$.

(3) In general, $Con_e(U) \ne Con_e(S)$, for a fuzzy subnet U of S. By (1), we define a subnet $U: N \to Pt(X)$ of S by

$$U(n) = S(2n) = 0.8.$$

Since W is often in $\overline{1}$.

$$\begin{aligned} Con_e(U) &= N_e(\overline{0})^* \\ &= 1 - N_e(\overline{0}) \\ &= 1 - [x_{0.5} \to \overline{0}] = 0.5. \end{aligned}$$

Hence $Con_e(S) \le Con_e(U)$ but $Con_e(S) \ne Con_e(U)$.

(4) $Con_{x_{0.5}}(S) = [x_{0.5} \odot Con_{x_1}(S)]$.

By (1), $Con_{x_{0.5}}(S) = 0.2$ and $Con_{x_1}(S) = 0.7$

Thus,

$$\begin{aligned} Con_{x_{0.5}}(S) &= [x_{0.5} \odot Con_{x_1}(S)] \\ &= \max\{0, 0.5 + 0.7 - 1\} = 0.2. \end{aligned}$$

(5) $Cl_{x_{0.5}}(S) = [x_{0.5} \odot Cl_{x_1}(S)]$.

By (1), $Cl_{x_{0.5}}(S) = 0.5$ and $Cl_{x_1}(S) = 1$

Thus,

$$\begin{aligned} Cl_{x_{0.5}}(S) &= [x_{0.5} \odot Cl_{x_1}(S)] \\ &= \max\{0, 0.5 + 1 - 1\} = 0.5. \end{aligned}$$

Definition 3.4 [7] A lattice L is called order dense if for each $x, y \in L$ such that $x < y$, there exists $z \in L$ such that $x < z < y$.

Theorem 3.2 Let (X, T) be an L-fuzzy topological space and $S, U: D \to Pt(X)$ fuzzy nets such that for each $n \in D$. $S(n) \vee U(n), S(n) \wedge U(n) \in Pt(X)$.

Define fuzzy nets $S \vee U, S \wedge U: D \to Pt(X)$ by, for each $n \in D$,

$$(S \vee U)(n) = S(n) \vee U(n),$$

$$(S \wedge U)(n) = S(n) \wedge U(n).$$

Then the following properties hold:

(1) If $S(n) \le U(n)$ for all $n \in D$, then $Cl_e(S) \le Cl_e(U)$, $Con_e(S) \le Con_e(U)$.

(2) $Cl_e(S \wedge U) \le Cl_e(S) \wedge Cl_e(U)$.

(3) $Con_e(S \vee U) \ge Con_e(S) \vee Con_e(U)$.

(4) $Con_e(S \wedge U) \le Con_e(S) \wedge Con_e(U)$.

(5) If L is an order dense totally ordered lattice, we have

$$Cl_e(S \vee U) = Cl_e(S) \vee Cl_e(U).$$

Proof. (1) Let U be finally (often) in λ.
Then S be finally (often) in λ, respectively.
Thus it is trivial.
(2),(3) and (4) are easily proved.
(5) Since $S \le S \vee U$ and $U \le S \vee U$, by (1),we have

$$Cl_e(S \vee U) \ge Cl_e(S) \vee Cl_e(U).$$

Suppose

$$Cl_e(S \vee U) \not\le Cl_e(S) \vee Cl_e(U).$$

Since L is an order dense totally ordered lattice, there exist $t \in L$ such that

$$Cl_e(S \vee U) > t > Cl_e(S) \vee Cl_e(U). \qquad \textbf{(A)}$$

Since $Cl_e(S) < t$ and $Cl_e(U) < t$, by the definition Cl_e, there exist $\lambda, \mu \in L^X$ such that S and U are finally in λ^* and μ^*, respectively, with

$$Cl_e(S) \vee Cl_e(U) \le N_e(\lambda)^* \vee N_e(\mu)^* < t.$$

Since S is finally in λ^*, there exists $n_1 \in D$ such that

$$S(n) \in \lambda^* \text{ for every } n \in D \text{ with } n \ge n_1.$$

Since U is finally in μ^*, there exists $n_2 \in D$ such that

$$S(n) \in \mu^* \text{ for every } n \in D \text{ with } n \ge n_2.$$

Let $n_3 \in D$ such that $n_3 \ge n_1$ and $n_3 \ge n_2$.
For $n \ge n_3$, we have

$$(S \vee U)(n) \in \lambda^* \vee \mu^* = (\lambda \wedge \mu)^*.$$

Thus, $S \vee U$ is finally in $(\lambda \wedge \mu)^*$.
It implies

$$\begin{aligned} Cl_e(S \vee U) &\le N_e(\lambda \wedge \mu)^* \\ &\le (N_e(\lambda) \wedge N_e(\mu))^* \\ &\quad \text{(by Theorem2.2(4) and Theorem2.1(6))} \\ &= N_e(\lambda)^* \vee N_e(\mu)^* < t. \\ &\quad \text{(by Theorem2.1(2))} \end{aligned}$$

It is a contradiction for the equation **(A)**. Hence we have

$$Cl_e(S \vee U) \le Cl_e(S) \vee Cl_e(U).$$

Theorem 3.3 Let (X, T) be an L-fuzzy topological space and L be a totally ordered lattice.For $\lambda \in I^X$ and $e \in Pt(X)$, we have:

$$\begin{aligned} Ad_e(\lambda) &= \bigvee\{Cl_e(S) \mid S \text{ is a fuzzy net in } \lambda\} \\ &= \bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\}. \end{aligned}$$

Proof. Since S is a fuzzy net in λ, S is finally in λ.
Thus, $Cl_e(S) \le N_e(\lambda^*)^*$.
We easily show

$$\begin{aligned} Ad_e(\lambda) &= N_e(\lambda^*)^*. \\ &\ge \bigvee\{Cl_e(S) \mid S \text{ is a fuzzy net in } \lambda\} \\ &\ge \bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\} \\ &\quad \text{(by Theorem 3.2(1))} \end{aligned}$$

We only show that

$$Ad_e(\lambda) \leq \bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\}.$$

If $Ad_e(\lambda)=0$. It is trivial.

Let $Ad_e(\lambda)=t>0$. Then $N_e(\lambda^*)^*=t$.

Put $D=\{\mu\in I^X \mid N_e(\mu)>t^*\}$. Define a relation on D by

$$\mu_1<\mu_2 \Leftrightarrow \mu_1\geq\mu_2,\ \forall\mu_1,\mu_2\in D.$$

For each $\mu_1,\mu_2\in D$, since, by Theorem 2.2(4),

$$N_e(\mu_1\wedge\mu_2)\geq N_e(\mu_1)\wedge N_e(\mu_2)>t^*,$$

Thus $\mu_1\wedge\mu_2\in D$ and $\mu_1,\mu_2<\mu_1\wedge\mu_2$.

Hence $(D,<)$ is a directed set.

For each $\mu\in D$, that is, $N_e(\mu)>t^*$, we have $\mu\not\leq\lambda^*$. (If not, $N_e(\mu)\leq N_e(\lambda^*)=t^*$.) Since L is a totally ordered lattice, there exists $x\in X$ such that $\lambda(x)>\mu^*(x)$. Thus, we can define a fuzzy net $S:D\rightarrow Pt(X)$ by

$$S(\mu)=x_{\lambda(x)}$$

where $S(\mu)\in\lambda$ and $\lambda(x)=S(\mu)(x)>\mu^*(x)$. **(B)**

We will show that if $\mu\in D$, then S is not often in μ^*. Suppose S is often in μ^*.For $\mu\in D$, there exists $\rho\in D$ with $\mu<\rho$ such that , by the definition of S and **(B)**,

$$S(\rho)=y_{\lambda(y)}\in\mu^* \text{ and } \lambda(y)=S(\rho)(y)>\rho^*(y). \quad \textbf{(C)}$$

Since $\mu<\rho$ implies $\mu\geq\rho$, by Theorem 2.1(6),

$$\lambda(y)\leq\mu^*(y)\leq\rho^*(y).$$

It is contradiction for the equation **(C)**.

Thus, if S is often in μ^*, then $\mu\notin D$, that is, $N_e(\mu)\leq t^*$. It implies $N_e(\mu)^*\geq t$ from Theorem2.1(6). Therefore,

$$\begin{aligned} t &= Ad_e(\lambda)\\ &\leq \bigwedge\{N_e(\mu)^* \mid S \text{ is often in } \mu^*\}\\ &= Con_e(S)\\ &\leq \bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\}.\end{aligned}$$

Thus,

$$Ad_e(\lambda)\leq\bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\}.$$

Hence

$$\begin{aligned} Ad_e(\lambda) &= N_e(\lambda^*)^*.\\ &\geq \bigvee\{Cl_e(S) \mid S \text{ is a fuzzy net in } \lambda\}\\ &\geq \bigvee\{Con_e(S) \mid S \text{ is a fuzzy net in } \lambda\}\\ &\geq Ad_e(\lambda).\end{aligned}$$

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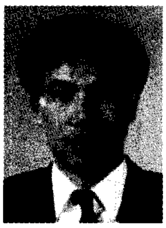
저 자 소 개



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