Prediction of Dimensional Instability Resulting from Layer Removal of an Internally Stressed Orthotropic Composite Cylinder

Shang Hyon Shin*
Department of Marine Engineering, Mokpo Maritime University, Chonnam 530-729, Korea

When a layer of cylindrical composite component containing an axisymmetric residual stress state is removed from the inner or outer surface, the dimension of the remaining material changes to balance internal forces. Therefore, in order to machine cylindrical composite components within tolerances, it is important to know dimensional changes caused by residual stress redistribution in the body. In this study, analytical solutions for dimensional changes and the redistribution of residual stresses due to the layer removal from a residually stressed cylindrically orthotropic cylinder were developed. The cylinder was assumed to have axisymmetric radial, tangential and axial residual stresses. The result of this study is useful in cases where the initial residual stress distribution in the component has been measured by a non-destructive technique such as neutron diffraction with no information on the effect of layer removal operation on the dimensional changes.

Key Words : Dimensional Instability, Residual Stress, Layer Removal, Orthotropic Composite Cylinder

Nomenclature

r	: Radial coordinate
a_o, b_o	: Original inner and outer radii
L_o	: Original length
L	: New length after layer removal
a_1, b_1	: The radial locations where layer removal is to be reached
a, b	: New inner and outer radii after layer removal
u_r, u_z	: Radial and axial displacements
σ_{ro}, $\sigma_{\theta o}$, σ_{zo}	: Original radial, tangential and axial residual stresses
σ_r', σ_θ', σ_z'	: Radial, tangential and axial stresses resulting from applied stresses
σ_r, σ_θ, σ_z	: Redistributed radial, tangential and axial residual stresses
ε_r, ε_θ, ε_z	: Radial, tangential and axial strains resulting from applied stresses
E_r, E_θ, E_z	: Moduli of elasticity in radial, tangential and axial directions
v_{ij}	: Poisson's ratio for transverse strain in the j-direction when stressed in the i-direction

* **E-mail** : shshin@mail.mmu.ac.kr
TEL : +82-61-240-7086; **FAX** : +82-61-240-7282
Department of Marine Engineering, Mokpo Maritime University, Chonnam 530-729, Korea. (Manuscript **Received** March 20, 2001; **Revised** March 14, 2002)

1. Introduction

Residual stresses are self-equilibrated internal stresses in a component after fabrication. The non-homogeneous plastic deformation developed during manufacturing cylindrical parts is the main cause of residual stresses. Residual stresses are most dramatically observed as any shape change when a part of residually stressed material is removed by machining.

Methods for measuring residual stresses fall under two major categories : destructive and non-destructive. Destructive are the layer removal method, the sectioning method and the hole-drilling method. The hole-drilling method (Joo and Pank, 1998) is often described as "semi-destructive" because the damage by the method is very localized and in many cases does not significantly affect the capability of the specimen. Diffraction, ultrasonic, and magnetic methods are non-destructive. Non-destructive determination for residual stresses is limited to near surface measurement techniques such as X-ray diffraction and ultrasonic testing. X-ray diffraction technique (Boo et al., 1998) is most widely used for measuring surface residual stresses and can be used in conjunction with the removal of a layer of material in order to detect subsurface residual stresses. Neutron diffraction method is recently developed to the level of precision where it can be used for engineering applications, and it is the only measuring technique by which residual stress distributions within the interior of materials can be determined nondestructively (Barrera and Tello, 1993).

Axisymmetric metal-forming operations used to produce cylindrical bodies typically induce axisymmetric residual stresses. Several researchers (Voyiadjis et al., 1985 ; Voyiadjis and Hartley 1987) investigated methods for determining residual stresses in a cylindrically orthotropic material. However, a method for determining the dimensional changes due to axisymmetric layer removal from an orthotropic cylinder has not been presented.

Honda et al. (1988) performed a numerical and experimental study of the interaction of residual stresses with the material removal by investigating the distortions from three holes that are drilled consecutively around a center-welded part of a circular plate. Fujiwara et al. (1988) showed that a circular disk deformed into elliptical shape when it was cut from the center of a rectangular plate containing biaxial residual stresses, and also investigated the effect of machining on the residual stress redistribution and dimensional changes of a circular plate containing an axisymmetric residual stress pattern.

The purpose of this study was to formulate a procedure for the determination of the dimensional changes resulting from the removal of a cylindrical layer from the inner or outer surface of a cylindrically orthotropic cylinder with known radial, tangential and longitudinal residual stress states. A cylindrically orthotropic material is one with three mutually orthogonal planes of elastic symmetry perpendicular to the radial, tangential and axial directions. Zircaloy-2 is an orthotropic material frequently used in cylindrical nuclear-reactor cores (Olson and Bert, 1966).

In this study it was assumed that the stresses relaxation is a linearly elastic process regardless of the deformation process by which the residual stresses were induced. When the redistribution process of residual stress involves the plastic flow, the developed solutions are invalid. It was also assumed that the material was removed without creating appreciable residual stresses at the exposed material surface utilizing such as Electro-Chemical Machining or Electric Discharge Machining.

2. Analysis of Layer Removal from Inner Surface

As a part of material is removed from the inner surface of a cylinder, the remaining body changes its dimension to attain the equilibrium of forces. Similar to the original inner surface, the radial residual stress at the new inner surface is also equal to zero. Therefore, the layer removal process can be simulated by superposition of an equal but opposite radial stress to the radial residual stress at the radial location to which the removal process is reached, and an average axial stress of the removed material at the end section of the remaining body.

Consider a cylinder of uniform geometry as shown in Fig. 1, which is for cylindrical orthotropy. Assume that the inner surface of the cylinder of length L_o is to be machined from a radius of $r=a_o$ to $r=a_1$, where a_o is the original inner radius.

The layer removal process can be thought of first as the superposition of the radial stress, $-\sigma_{ro}|_{r=a_1}$, to the original radial residual stress at $r=a_1$, $\sigma_{ro}|_{r=a_1}$, such that the radial residual stress at the new inner surface is equal to zero,

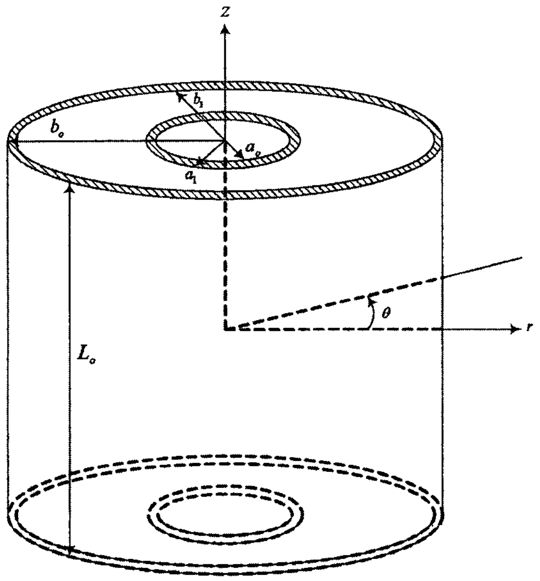

Fig. 1 Schematic diagram for illustrating the material removal processes of the orthotropic cylinder

and then as applying the removed axial force, $\pi(a_1^2-a_o^2)\bar{\sigma}_{zo}$, at the end section of the remaining material, where $\bar{\sigma}_{zo}$ is the averaged axial residual stress through the thickness of the removed layer. Thus, the radial and axial displacements of the remaining body would be equal to those experienced by a cylinder with inner radius of a_1 and outer radius of b_o subjected to an internal pressure $p=\sigma_{ro}|_{r=a_1}$ and axial force of $\pi(a_1^2-a_o^2)\bar{\sigma}_{zo}$.

The radial and tangential stresses caused by the application of the internal pressure p to the cylinder of a cylindrically orthotropic material are given by (Lekhnitskii et al., 1968);

$$\sigma_r'=\frac{pc^{k+1}}{1-c^{2k}}\left[\left(\frac{r}{b_o}\right)^{k-1}-\left(\frac{b_o}{r}\right)^{k+1}\right] \quad (1)$$

$$\sigma_\theta'=\frac{pkc^{k+1}}{1-c^{2k}}\left[\left(\frac{r}{b_o}\right)^{k-1}+\left(\frac{b_o}{r}\right)^{k+1}\right] \quad (2)$$

where $k=\sqrt{\frac{E_\theta}{E_r}}$ and $c=\frac{a_1}{b_o}$

And the axial stress due to the axial force can be given as;

$$\sigma_z'=\frac{(a_1^2-a_o^2)\bar{\sigma}_{zo}}{b_o^2-a_1^2} \quad (3)$$

Substituting Eqs. (1), (2), and (3) into Hooke's law for a material exhibiting cylindrical orthotropy,

$$\begin{aligned}\varepsilon_r&=(\sigma_r'-\nu_{r\theta}\sigma_\theta'-\nu_{rz}\sigma_z')/E_r\\ \varepsilon_\theta&=(\sigma_\theta'-\nu_{\theta r}\sigma_r'-\nu_{\theta z}\sigma_z')/E_\theta\\ \varepsilon_z&=(\sigma_z'-\nu_{z\theta}\sigma_\theta'-\nu_{zr}\sigma_r')/E_z\end{aligned} \quad (4)$$

and utilizing the axisymmetric strain-displacement relations, the radial and axial displacements of the remaining body can be expressed as;

$$u_r=\frac{b_opc^{k+1}}{E_\theta(1-c^{2k})}\left[(k-\nu_{\theta r})\left(\frac{r}{b_o}\right)^k+(k+\nu_{\theta r})\left(\frac{b_o}{r}\right)^k\right]-\frac{\nu_{\theta z}(a_1^2-a_o^2)\bar{\sigma}_{zo}}{E_\theta(b_o^2-a_1^2)} \quad (5)$$

$$u_z=\frac{L_o}{E_z}\left[\frac{(a_1^2-a_o^2)\bar{\sigma}_{zo}}{b_o^2-a_1^2}-\frac{pc^{k+1}}{1-c^{2k}}\left\{\left(\frac{r}{b_o}\right)^{k-1}(k\nu_{z\theta}+\nu_{zr})+\left(\frac{b_o}{r}\right)^{k+1}(k\nu_{z\theta}-\nu_{zr})\right\}\right] \quad (6)$$

Therefore, the new dimensions of the cylinder due to layer removal up to $r=a_1$ can be written as;

$$a=a_1+u_r|_{r=a_1} \quad (7)$$

$$b=b_o+u_r|_{r=b_o} \quad (8)$$

$$L=L_o+u_z \quad (9)$$

where a, b and L are the new inner, outer radius and length of the cylinder after the layer removal, respectively.

The new state of residual stresses in the remaining cylinder can be obtained by the superposition of the stresses resulting from the application of internal pressure $p=\sigma_{ro}|_{r=a_1}$ and the axial stress σ_z' to the original residual stress distribution. Thus, the redistributed residual stresses in the remaining cylinder can be expressed as;

$$\sigma_r=\sigma_{ro}+\sigma_r' \quad (10)$$

$$\sigma_\theta=\sigma_{\theta o}+\sigma_\theta' \quad (11)$$

$$\sigma_z=\sigma_{zo}+\sigma_z' \quad (12)$$

3. Analysis of Layer Removal from the Outer Surface

Assuming that the cylinder is to be machined from the outer surface to $r=b_1$, similar to the analysis of the case of material removal from the inner surface, the radial and tangential stress due to the application of the external pressure

$q=\sigma_{ro}|_{r=b_1}$ can be given by ;

$$\sigma_r'=\frac{q}{1-d^{2k}}\left[d^{2k}\left(\frac{b_1}{r}\right)^{k+1}-\left(\frac{r}{b_1}\right)^{k-1}\right] \quad (13)$$

$$\sigma_\theta'=\frac{-qk}{1-d^{2k}}\left[d^{2k}\left(\frac{b_1}{r}\right)^{k+1}+\left(\frac{r}{b_1}\right)^{k-1}\right] \quad (14)$$

where $d=\frac{a_o}{b_1}$.

The axial stress caused by the application of the removed axial force at the end of the cylinder can be expressed as ;

$$\sigma_z'=\frac{b_o^2-b_1^2}{b_1^2-a_o^2}\bar{\sigma}_{zo} \quad (15)$$

The radial and axial displacements can then be given by ;

$$u_r=\frac{-bq}{E_\theta(1-d^{2k})}\left[(k-\nu_{\theta r})\left(\frac{r}{b_1}\right)^k+(k+\nu_{\theta r})d^{2k}\left(\frac{b_1}{r}\right)^k\right] -\frac{r\nu_{\theta z}\bar{\sigma}_{zo}(b_o^2-b_1^2)}{E_\theta(b_1^2-a_o^2)} \quad (16)$$

$$u_z=\frac{L_o}{E_z}\left[\frac{(b_o^2-b_1^2)\bar{\sigma}_{zo}}{b_1^2-a_o^2}+\frac{q(k\nu_{z\theta}+\nu_{zr})}{1-d^{2k}}\left(\frac{r}{b_1}\right)^{k-1} +\frac{qd^{2k}(k\nu_{z\theta}-\nu_{zr})}{1-d^{2k}}\left(\frac{b_1}{r}\right)^{k+1}\right] \quad (17)$$

Hence, the new inner, outer radius and length of the cylinder after the layer removal from the outer surface up to $r=b_1$ can be written as ;

$$a=a_o+u_r|_{r=a_o} \quad (18)$$

$$b=b_1+u_r|_{r=b_1} \quad (19)$$

$$L=L_o+u_z \quad (20)$$

The redistributed residual stresses in the remaining material due to the layer removal from the outer surface can also be obtained by utilizing the Eqs. (10), (11), and (12).

4. Applications

The principle of the analysis is that the residual stress distribution in resultant forces and moments of the material removed possesses exactly equals in magnitude and opposite in sign to those which cause the dimensional changes of the rest of material. And p and q in the Eqs. (1), (2), (13), and (14) can be either positive or negative. Therefore, the residual stresses in the analysis, which are in equilibrium, can be tensile or compressive.

In practice, the developed equations, in conjunction with measurements of radial residual stress at the location of the machine-planning radius and the averaged axial residual stress in the material removed, can be used to guide the fabrication process of precision parts. Although the material was assumed to be elastically orthotropic, the developed equations can also be applied for isotropic materials with some modifications in material properties.

In internal combustion engines, cylinder liner surface is an important wear-prone area. Designing the cylinder liner is one of areas where the developed closed-form solutions can be utilized. A beneficial residual stress state can be intentionally introduced in the cylinder liner in order to improve the cylinder liner performance. Compressive hoop residual stress in the inner surface area of the cylinder liner would be of a role as a crack resistor, and, as liner wear progresses the new cylinder liner surface would be displaced either inward or outward depending mainly on the radial residual stress pattern. The inward displacement of the inner surface of the cylinder liner brings about the effect of diminishing cylinder liner wear. A priori knowledge of the redistributed residual stress pattern in the cylinder liner due to the liner wear can play an important role in pre-estimating the performance of the cylinder liner.

Further applications may be found in the area of Zircaloy tubes of an orthotropic material frequently used as nuclear reactor cores. The tubes are formed by the Pilger rolling process which leaves an axisymmetric pattern of residual stresses in the tube. Composite materials fabricated by impregnating fibers wound around a mandrel with polymeric filler may be a potential area of the application of the results.

5. Conclusions

The object of this study is to develop a methodology of prediction of dimensional changes and redistribution of residual stresses due to the layer removal of a cylindrical body containing axisymmetric radial, tangential and axial residual stresses. The material was assumed to be cylindrically orthotropic in elasticity.

It is interesting to note that in predicting the dimensional changes due to axisymmetric material removal the initial radial residual stress at the radial point to which the material removal process is to be extended and the averaged axial residual stress across the thickness to be removed are required except material properties. The results of this study are more useful in cases where the initial residual stress distribution in the component has been obtained by a nondestructive technique such as the neutron diffraction with no information on the effect of material removal operation on the dimensional changes and the redistribution of residual stresses in the material In this study it was also assumed that the relaxation of the residual stresses is a linearly elastic process. If changes of the residual stresses give rise to plastic flow, the solutions obtained in the study are not valid. Hence, future study should be directed towards analysis of cases in which the redistribution of residual stress involves plastic deformation during the removal of material.

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