韓 國 舶 用 機 關 學 會 誌
Vol 26. No3. pp.313~319. 2002

A Study on Optimal Electric Load Distribution and Generator Operating Mode Using Dynamic Programming

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동적계획법을 이용한 발전기의 운전모드 및 최적부하 배분에 관한 연구

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Key words : Dynamic Programming, Optimal Load Distribution, Parallel Operation, Automatic Load Sharing Device, Incremental Operating Cost, Lagrange multiplier

Abstract

Since the oil crisis in 1970, a great deal of effort has been made to develop automatic electric load sharing systems as a part of the efforts to save energy. A large scale electric generating system composes more than two generators whose characteristics may be different. When such a system is operated individually or in parallel, the lagrange multiplier's method has difficulty in achieving optimal load distribution because generators usually have the limitations of the operating range with inequality constraints. Therefore, a suitable operating mode of generators has to be decided according to the selection of the generators to meet electric power requirements at the minimum cost. In this study, a method which solves the optimal electric load distribution problem using the dynamic programming technique is proposed.

This study also shows that the dynamic programming method has an advantage in dealing with the optimal load distribution problem under the limitations of the operating range with inequality constraints including generator operation mode. In this study, generator operating cost curve of second order equation by shop trial test results of diesel generators are used. The results indicate that the proposed method can be applied to the ship's electric generating system.

1. Introduction

Since the oil crisis, a great deal of effort has been made to develop automatic electric load sharing systems[1),2),3)] as a part of the efforts to save energy. And in the relatively large scale electric power system, usually more than two generators are operated in parallel to supply the

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electric power requirements. Each of these generators may have different characteristics, such as different fuel consumption rates, maintenance costs, makers and operating hours. Electric power generating cost includes not only fuel cost but also maintenance cost. In this reason, some generator's operating cost becomes higher than others. In this study, It is assumed that generator's operation cost includes only the fuel cost and operating cost curves for generators follow the second-order equations[9]. The lagrange multiplier method[4),9)] is examined to solve the economic electric load distribution problem. However, it is found that the lagrange multiplier's method has difficulty in achieving optimal load distribution in case of limitation of operating range with inequality constraints and in case that the operating mode of generators has to be decided depending on the selection of generators to be operated in parallel for the electric power demands. In this study, a method which solves the optimal electric load distribution problem using the dynamic programming technique[5),6),7),8)] is proposed. It is shown that the proposed method can be applied to ship's electric generating systems.

2. Theory and Application

2.1 Fuel Cost Characteristics for Generator Operation

[Btu] unit is used as the unit of the measure for fuel energy consumed, and [Btu/h]unit is used as the unit of the measure of fuel input energy rate. Heat-energy conversion rate means the input energy to the unit electric power energy output which is expressed as unit of [Btu/watt-h]. Generally, heat-energy conversion rate(H) is represented by

$$H = \frac{a}{x} + b + cx \ \text{[Btu/w-h]} \tag{1}$$

Here, a, b and c are constants which are determined by the curve to express the relation between heat-energy conversion rate (H) and electric power energy output (x). Therefore, fuel consumption rate (C) is obtained as

$$C = H \cdot x = a + bx + cx^2 \tag{2}$$

Here, the unit of x is [watt].

Fuel cost per Btu (k_f) is used to calculate the cost per hour([cost/h]) to generate some electric power. Thus, generator's operating cost (f) is expressed by

$$f = k_f \cdot C = a\, k_f + b \cdot k_f \cdot x + c \cdot k_f \cdot x^2 \tag{3}$$

By substituting constants such as α, β and gamma for $a \cdot k_f$, $b \cdot k_f$ and $c \cdot k_f$ respectively, equation(3) is rewritten as

$$f = \alpha + \beta x + \gamma x^2 \tag{4}$$

2.2 Optimal Load Distribution

Supposing a electric power system consists of m generator sets, and the relation equations for each generator operating cost to the generator output are expressed by the equation (4), total operating cost(F) is leaded to the equation (5) in case of the operation of m generator sets to meet the required electric power(P).

$$F = f_1(x_1) + f_2(x_2) + \cdots\cdots + f_m(x_m) = \sum_{i=1}^{m} f_i(x_i) \tag{5}$$

Optimal load distribution problem is to minimize the operating cost under the condition to meet the equation (6) in which total required electric power(P) is supplied by m generator sets.

$$\sum_{i=1}^{m} x_i - P = 0 \qquad (6)$$

Lagrange multiplier method is usually applied to solve the minimum operating cost problem under the condition of constraint like this. This method adopts lagrange multiplier (λ), and makes a new cost function (F') of the equation (7), then differentiate F' by x_i.

So, equation (8) and (9) are obtained.

$$F' = F - \lambda\left(\sum_{i=1}^{m} x_i - P\right) \qquad (7)$$

Where, x_i (i=1,2,..., m) is each generators output.

$$\frac{\partial F'}{\partial x_1} = \frac{\partial}{\partial x_1}[f_1(x_1)+f_2(x_2)+\cdot\cdot\cdot+f_m(x_m)]$$
$$-\partial\lambda(x_1+x_2+x_3+\cdot\cdot\cdot+x_m-P)/\partial x_1$$
$$=\frac{\partial f_1}{\partial x_1}-\lambda=0$$
$$\frac{\partial F'}{\partial x_2}=\frac{\partial f_2}{\partial x_2}-\lambda=0$$
$$\vdots$$
$$\frac{\partial F'}{\partial x_m}=\frac{\partial f_m}{\partial x_m}-\lambda=0 \qquad (8)$$

$$\frac{\partial F'}{\partial \lambda}=\sum_{i=1}^{m}x_i-P=0 \qquad (9)$$

From the above the equation (8), it indicates that all generators must operate to have the same incremental fuel cost (λ) as equation (10) shows.

$$\lambda=\frac{\partial f_1}{\partial x_1}=\frac{\partial f_2}{\partial x_2}=\frac{\partial f_3}{\partial x_3}=\cdot\cdot\cdot=\frac{\partial f_m}{\partial x_m} \qquad (10)$$

Where $\frac{\partial f_i}{\partial x_i}$ is the incremental fuel cost of i th generator. The unit of incremental fuel cost is [cost/W-h].

2.3 Application of Lagrange Multiplier Method

Three equations in equation (11) were formed by using the data of shop trial test results for 250kw, 480kw and 550kw of under the condition of full load operation of diesel engine. Optimal load distribution problem comes into the problem to find out x_1, x_2, x_3 to minimize F(equation(13))under the condition to meet the required electric power(P)of equation (12) in case of the operation of three generators which have different capacities and characteristics.

$$f_1 = 0.095x_1^2+115x_1+6600$$
$$f_2 = 0.033x_2^2+110x_2+10000 \qquad (11)$$
$$f_3 = 0.025x_3^2+\ 95x_3+15000$$

$$x_1+x_2+x_3 = P \qquad (12)$$

$$F = f_1+f_2+f_3 \qquad (13)$$

Equation (14) is derived from the above mentioned problem with the lagrange multiplier(λ).

$$F'(x_1,x_2,x_3,\lambda)=f_1+f_2+f_3-\lambda(x_1+x_2+x_3-P) \qquad (14)$$

Optimal condition equations for F' of equation(14) can be rewritten as

$$x_1=\frac{(\lambda-115)}{0.19},\ x_2=\frac{(\lambda-110)}{0.066},\ x_3=\frac{(\lambda-95)}{0.05}, \qquad (15)$$

$$x_1+x_2+x_3 = P = 1000 \qquad (16)$$

Hence, if required electric power(P) is determined, Fmin which means optimal operating cost to minimize F can be obtained by equations (15) and (16) in case of three generators operation. But, each generator has the operating limitations of upper and low. In case of considering the operating limitations of each generator,

F' is given by

$$F'(x_1, x_2, x_3, \lambda, \mu, \gamma) = \sum_{i=1}^{3} f_i(x_i) + \lambda(\sum_{i=1}^{3} x_i - P) + \sum \mu_i(x_i - U_i) - \sum_{i=1}^{3} \gamma_i (x_i - L_i) \quad (17)$$

Where λ, μ, γ are undetermined constants, and U_i and L_i are upper limit, low limit respectively. In case of optimization with the inequality constraints like equation (17), optimization by lagrange multiplier method becomes difficult. And also actual generator's operations for the required electric power in the field have to select the operation mode which includes single operation, two generators parallel operation and three

generators parallel operation. So, approach by dynamic programming is needed to deal with this kind of optimal load distribution problem including not only the inequality constraints but also optimal combinational problem.

3. Modeling by Dynamic Programming

3.1 Formation of equations

Suppose now that number of generators are n sets ,x_k is output for kth generator and $f_k(x_k)$ is the operating cost of kth generator in the case of operation at load x_k, and $F_n(P)$is the minimum operating cost when the required electric power is P and n generator sets are operated. Then, equation (18) and (19) are made to use a dynamic programming.

$$x_1 + x_2 + x_3 + \cdot\cdot\cdot\cdot\cdot + x_n = P \quad (18)$$

$$F_n(P) = \underset{x_1, x_2, \cdot\cdot, x_n}{Min.} [f_1(x_1) + f_2(x_2) + f_3(x_3) + \cdot\cdot + f_n(x_n)] \quad (19)$$

When the limitations of generator operating range are $L_i \le x_i \le U_i$, if generator is not in operation ,$f_k(0)=0$; if generator does not operate within the operating range except not in operation, $f_i(x_i<L_i$ or $x_i>U_i)=\infty$; if generator operates within the operating range, $f_i(x_i)$ follows the equation (11). This leads to the equation (20) for equation (19).

$$F_k(P) = \min_{x_k} [F_{k-1}(P - x_k) + f_k(x_k)] \quad (20)$$

where, k is 1, 2, · · · , n, and $F_0(P)$ is zero.

3.2 Application by Dynamic Programming

Suppose that three generators which have the same capacities and characteristics already mentioned in the section 2.3 supply the electric power (refer to Fig.1); and upper and low limits for each generator are like equation (21). The optimal load distribution problem in case of consideration of generator operation mode can be written by equation (22).

$$10 \le x_1 \le 250,\ 40 \le x_2 \le 480,\ 50 \le x_3 \le 550 \quad (21)$$

$$F_3(P) = \min_{x_3} [F_2(P - x_3) + f_1(x_3)] \quad (22)$$

In case of the electric load distribution by three generators, first of all, consider dividing No.3 generator and No.1 & No.2 generator's group

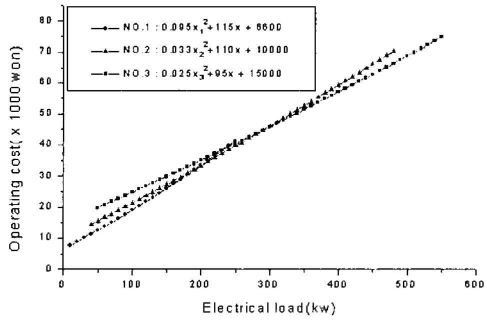

Fig.1 Operating cost of each generator

Table 1. Operating cost (No.1 and No.2 generators)

x_1 ↓	x_2 → / f_2 ↘ / f_1 ↓	0.00	10.00	20.00	30.00	40.00	50.00	60.00	→ →	480.00	490.00	→ →	1280.00
		0.00	*	*	*	14.45	15.58	6.72	→ →	70.40	*	→ →	*
0.000	0.00	0.00	*	*	*	14.45	15.58	16.72		70.40	*		*
10.00	7.76	7.76	*	*	*	22.21	23.34	24.48		78.16	*		*
20.00	8.94	8.94	*	*	*	23.39	24.52	25.66		79.34	*		*
30.00	10.13	10.14	*	*	*	24.59	25.72	26.85		80.54	*		*
40.00	11.35	11.35	*	*	*	25.80	26.93	28.07		81.76	*		*
50.00	12.59	12.59	*	*	*	27.04	28.17	29.31	→ →	82.99	*	→ →	*
60.00	13.84	13.84	*	*	*	28.29	29.42	30.56		84.25	*		*
70.00	15.12	15.12	*	*	*	29.57	30.70	31.83		85.52	*		*
80.00	16.41	16.41	*	*	*	30.86	31.99	33.13		86.81	*		*
90.00	17.72	17.72	*	*	*	32.17	33.30	34.44		88.12	*		*
100.00	19.05	19.05	*	*	*	33.50	34.63	35.77		89.45	*		*
↓													
↓													
250.00	41.29	41.29	*	*	*	55.74	56.87	58.01		116.69	*	→ →	*
260.00	*	*	*	*	*	*	*	*		*	*		*
↓													
↓													
1280.00	*	*	*	*	*	*	*	*		*	*		*

(Here, * means infinite(∞), and f_1 and f_2 are operating costs for No.1 and No.2 generators. The unit is [x 1000 won].

respectively.

Then, electric powers that have to supply by No.1 and No.2 must be optimal in order to distribute optimally through 3 generators operation. $F_2(P)$ is the minimum operating cost when considering 2 generator's operation. To develop the process of solving the optimal load distribution by a dynamic programming, first, Table1 should be made to get optimal loads with minimum operation costs when the required electric power is above 10 kw, which increases up to 1280kw. Second, $F_2(P)$ is obtained if the minimum value is founded on the cross line as shown in Table1. Therefore, optimal load x_1 and x_2 for No.1 and No.2 generators, are obtained as Table 2. As the third step, $F_3(P)$ is obtained, when a new table for $F_2(P)$ and x_3 is made by the same method in Table1. $x1$, $x2$ and x_3 became the optimal loads for No.1, No.2 and No.3 generators

Table 2. Optimal electrical load distribution in case of No.1 & No.2 generator operation

P (kw)	x_1 (kw)	x_2 (kw)	$F_2(P)$ (×1000won)
10.00	10.00	0.00	7.76
20.00	20.00	0.00	8.94
30.00	30.00	0.00	10.14
40.00	40.00	0.00	11.35
50.00	50.00	0.00	12.59
60.00	60.00	0.00	13.84
70.00	70.00	0.00	15.12
80.00	80.00	0.00	16.41
90.00	90.00	0.00	17.72
100.00	100.00	0.00	19.05
150.00	150.00	0.00	25.99
200.00	0.00	200.00	33.32
250.00	0.00	250.00	39.56
300.00	0.00	300.00	45.97
350.00	0.00	350.00	52.54
400.00	0.00	400.00	59.28
450.00	0.00	450.00	66.18
500.00	110.00	390.00	78.32
550.00	120.00	430.00	85.17
600.00	140.00	460.00	92.14
650.00	170.00	480.00	99.30
700.00	220.00	480.00	106.90
710.00	230.00	480.00	108.48
720.00	240.00	480.00	110.08
730.00	250.00	480.00	111.69

Table 3. Optimal electrical load distribution in case of three generators operation

P (kw)	x_1 (kw)	x_2 (kw)	x_3 (kw)	$F_3(P)$ (×1000won)
10.00	10.00	0.00	0.00	7.76
20.00	20.00	0.00	0.00	8.94
30.00	30.00	0.00	0.00	10.14
40.00	40.00	0.00	0.00	11.35
50.00	50.00	0.00	0.00	12.59
60.00	60.00	0.00	0.00	13.84
70.00	70.00	0.00	0.00	15.12
80.00	80.00	0.00	0.00	16.41
90.00	90.00	0.00	0.00	17.72
100.00	100.00	0.00	0.00	19.05
150.00	150.00	0.00	0.00	25.99
200.00	0.00	200.00	0.00	33.32
250.00	0.00	250.00	0.00	39.56
300.00	0.00	0.00	300.00	45.75
350.00	0.00	0.00	350.00	51.31
400.00	0.00	0.00	400.00	57.00
450.00	0.00	0.00	450.00	62.81
500.00	0.00	0.00	500.00	68.75
550.00	0.00	0.00	550.00	74.81
600.00	50.00	0.00	550.00	87.40
650.00	100.00	0.00	550.00	93.86
700.00	150.00	0.00	550.00	100.80
750.00	0.00	200.00	550.00	108.13
800.00	0.00	250.00	550.00	114.38
850.00	0.00	300.00	550.00	120.78
900.00	0.00	350.00	550.00	127.35
950.00	0.00	400.00	550.00	134.09
1000.00	0.00	450.00	550.00	140.99
1050.00	110.00	390.00	550.00	153.13
1100.00	120.00	430.00	550.00	159.98
1150.00	140.00	460.00	550.00	166.96
1200.00	170.00	480.00	550.00	174.11
1210.00	180.00	480.00	550.00	175.59
1220.00	190.00	480.00	550.00	177.10
1230.00	200.00	480.00	550.00	178.62
1240.00	210.00	480.00	550.00	180.16
1250.00	220.00	480.00	550.00	181.71
1260.00	230.00	480.00	550.00	183.29
1270.00	240.00	480.00	550.00	184.89
1280.00	250.00	480.00	550.00	186.50

respectively. Table 3 shows the results.

4. Efficiency of Generator and Operating Mode

Generator's operation mode have to be determined to operate the priority generator which has higher efficiency than the others in case of increasing the required electric power. Suppose now that generator's efficiency is operating cost(unit : won) to generator's output(unit :kw), μ_1, μ_2 and μ_3 are the efficiency for three generators.

Thus, the efficiencies for each generator can be written by equations (23), (24) and (25).

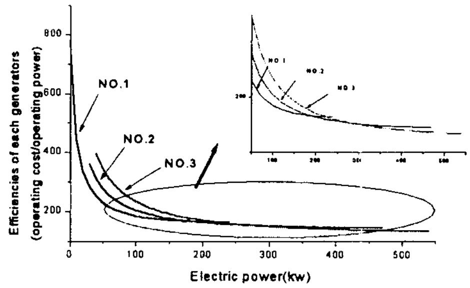

Fig.2 Efficiencies of three generators

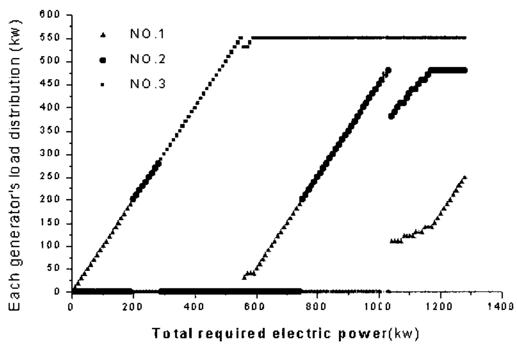

Fig.3 Three generator's load distribution

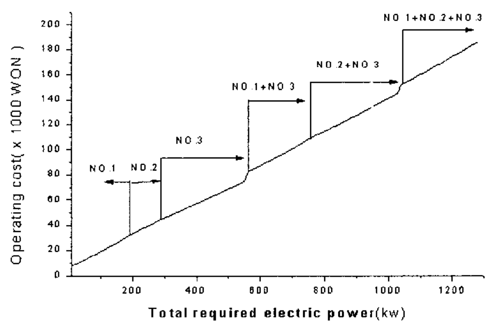

Fig.4 Generator's operating mode

No.1 generator :

$$\mu_1 = f_1 / x_1 = 0.095\, x_1 + 115 + 6600/x_1 \quad (23)$$

No.2 generator :

$$\mu_2 = f_2 / x_2 = 0.033\, x_2 + 110 + 10000/x_2 \quad (24)$$

No.3 generator :

$$\mu_3 = f_3 / x_3 = 0.025\, x_3 + 95 + 15000 / x_3 \quad (25)$$

Fig.2 shows the efficiencies of three generators. Fig.3 indicates the optimal distribution value for three generators in the basis of results to get using a dynamic programming. Fig.4 indicates the minimum operating cost and optimal operating mode according to the required electric power. From Fig.3 and Fig.4, it is revealed that the

generator with higher efficiency is operated in preference to any generator within the operation limit of that generator as the required electric power increases.

5. Conclusion

In this study, it was revealed that the lagrange multiplier method is useful in the case of all generator's operation to supply required electric power, but is not proper in the case of having the limitation of operating range with the inequality constraints. This study also found that the dynamic programming method has an advantage in dealing with the optimal load distribution problem under the above mentioned condition including generator operation mode. In this study, generator operating cost curve of second order equation by shop trial test results of diesel generators were used, and in examples, diesel generators were examined. But it is expected that the proposed method can be applied to the ship's optimal load distribution problem in which many generators of different kind and various characteristics are adopted if their detailed data are given. So, It is expected that the results of this study could be useful as the basic data for the development of automatic electric load sharing device, and as the basic materials for the decision of generator operation mode.

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